Field theory and Topological Phases

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High-energy theory group

Quantum field theory is a very useful tool

Quantum field theory (QFT)

- describes a system with infinite d.o.f by small number of d.o.f

- have particle picture:
    1. creation and annihilation
    2. mass eigenstates as particles

- is applied to high-energy physics, condensed matter system, Atomic, molecular, optical physics

Topological phases can be studied by QFT

A new type of phases of matter
(quantum phases)

- has distinctive ground states
  by topological invariant in momentum space

- appears in low temperature —> quasiparticle
  excitations

- has very very large number of d.o.f → QFT

The summary of my research

Boundary condition analysis

1. Boundary conditions in topological phases

2. New exotic states—edge-of-edge states

3. Equivalence between wave function formalism(TKNN) and field theoretical formalism

The benefit of boundary conditions analysis

- It’s a first principle calculation

- Helps us understand edge states in topological phases

- Helps us understand the relation between theories in different dimensions

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1 Boundary Conditions in Topological Phases

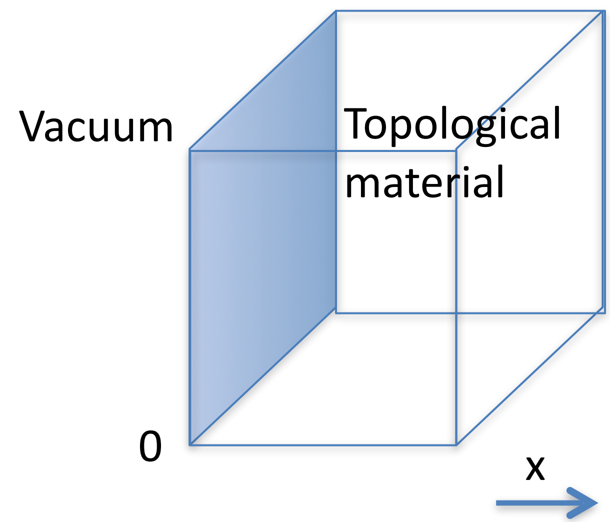
Xi Wu (Osaka Univ.)

In collaboration with

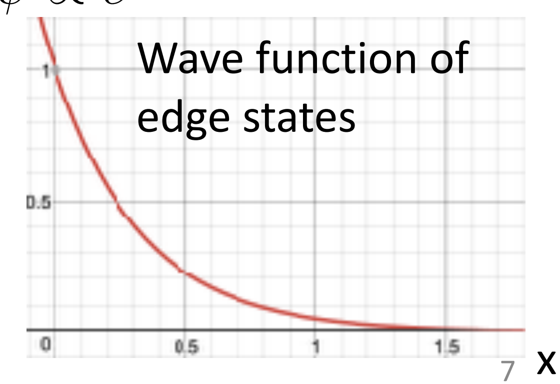
Koji Hashimoto(Osaka U.) and Taro Kimura(Keio U.)

Do boundary conditions affect edge states?

- Do boundary conditions affect topological phases in general
- Topological phases of matter are characterized by topological invariant in momentum space
- Edge states are protected by the topological invariant



$$\psi \propto e^{-\alpha x}$$



Do boundary conditions affect edge states?

Our approach: Continuum analysis + lattice simulation

(my part)

- Least action principle
- Hermiticity of Hamiltonian
- Hamiltonian eigenequation

⇒

Boundary condition

Edge states solution

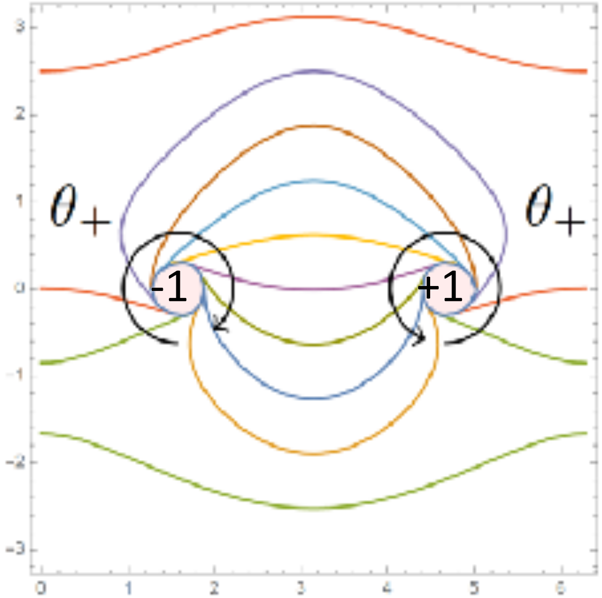
Boundary conditions affect edge states

Our findings:

1. Boundary conditions of Weyl semimetals are dictated by one single parameter
2. This parameter rotates edge states dispersion
3. The rotation direction is dictated by the topological number

Boundary conditions affect edge states

a new explanation of bulk-edge correspondence



Constant energy slice

- Curves with different colors correspond to different θ_+
- Boundary conditions don't change the existence of edge states.

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2 Edge-of-edge States

Xi Wu (Osaka Univ.)

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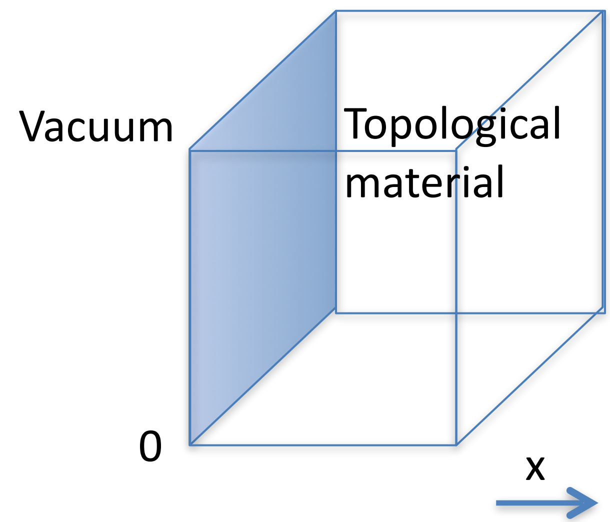
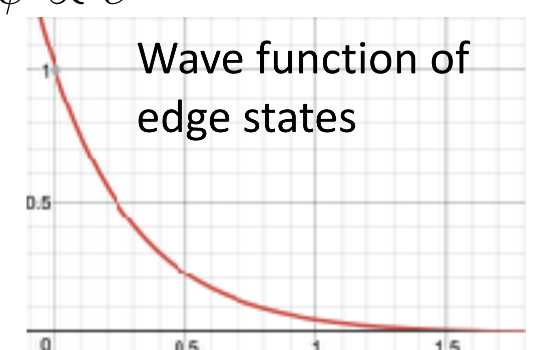


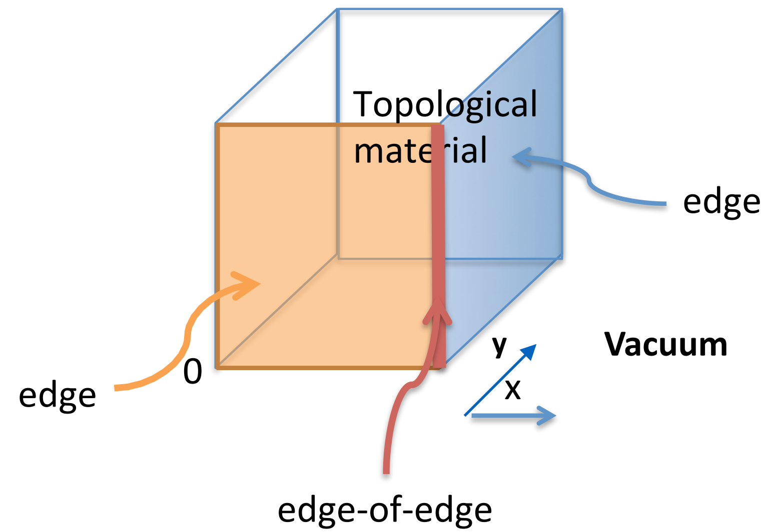
$\psi \propto e^{-\alpha x}$



What is an edge-of-edge state?

If some edge state has a nontrivial topology in momentum space

$$n \neq 0$$



⇨ $\text{Bulk} \longrightarrow \text{edge} \overset{?}{\longrightarrow} \text{e-o-e}$

What we expect for the e-o-e:

1 Energy dispersion differs from bulk and edge states

2 $\psi \propto e^{-\alpha x - \beta y}$

3 TKNN number and Ishikawa-Matsuyama(IM) Relations

An equivalence between wave function formalism and field theory formalism

Xi Wu

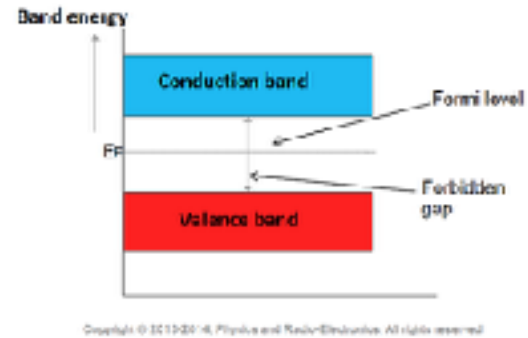
In collaboration with Prof. T Onogi(Osaka U.)

Two formalisms of Hall conductivity

- Wave function formalism: Hall conductivity is related with wave functions

$$\sigma_{xy} = \sum_{E_I < \epsilon_F} \int_{BZ} \frac{d^2 p}{(2\pi)^2} (\partial_x a^I_y - \partial_y a^I_x);$$

$$\text{where } a^I_i = -i \langle E_I | \frac{\partial}{\partial p^i} | E_I \rangle$$

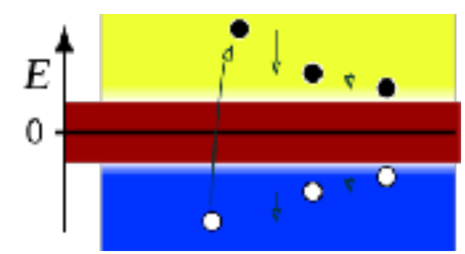


Energy bands

- Field theory formalism: Hall conductivity is related with the free fermion propagator

$$\sigma_{xy} \propto \epsilon^{\alpha\mu\nu} \int \frac{d^3 p}{(2\pi)^3} \mathrm{tr}(S \partial_\alpha S^{-1} S \partial_\mu S^{-1} S \partial_\nu S^{-1})$$

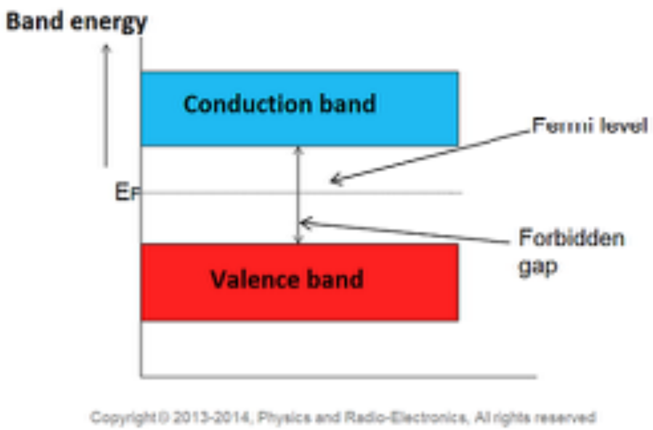
$$S(p) : \text{fermion propagator}$$



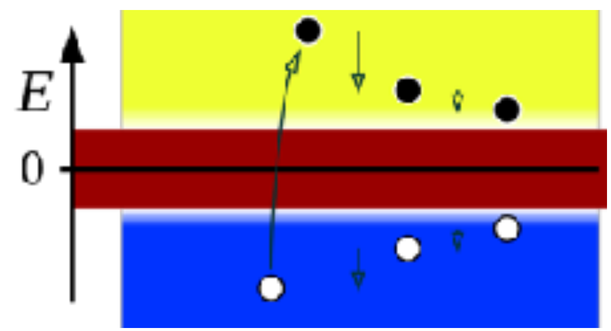
Dirac sea

Our contribution

1 Wave function formalism and field theory formalism are proved to be equivalent rigorously



Energy bands

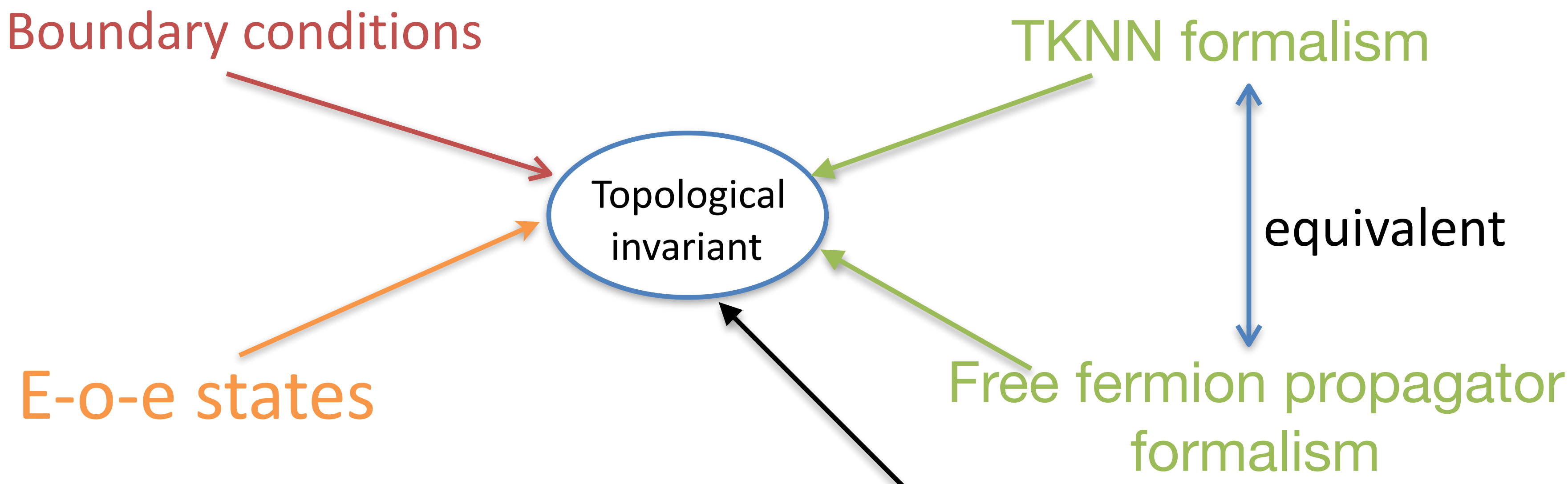


Dirac sea

2 Field theory formula is shown to be valid for arbitrary band Hamiltonian, not limited to Dirac fermions

Summary and Future plan

- Free theory—no electron-electron coupling



- Interacting theory? ⇨ Green's function

Appendix: Boundary condition rotates edge states dispersion

- Obtaining edge states by solving

$$(p_i \sigma_i - \epsilon)\psi = 0$$
$$\begin{pmatrix} 1 & e^{-2i\theta_+} \end{pmatrix} \psi \Big|_{x^3=0} = 0$$

- Edge states dispersion is a plane tangent to the bulk dispersion

edge dispersion: $\epsilon = -p_1 \cos 2\theta_+ - p_2 \sin 2\theta_+$

- B.C. parameter appears in the dispersion as a rotation angle