1 (5 points)

One mole of a monoatomic perfect gas initially at temperature T_0 expands from volume V_0 to $2V_0$, (a) at constant temperature, (b) at constant pressure.
Calculate the work of expansion and the heat absorbed by the gas in each case.

2 (15 points)

A thermally insulated cylinder, closed at both ends, is fitted with a frictionless heat-conducting piston that divides the cylinder in two parts. Initially, the piston is fixed at the center, with 1 liter of air at 200K and 2 atm of pressure on one side and 1 liter of air at 300K and 1 atm on the other side. The piston is released and the system adiabatically reaches equilibrium in pressure and temperature, with the piston in a new position. (a) Compute the final pressure and temperature. (b) Compute the total increase of entropy.

3 (5 points)

A particle of mass m moves in one dimension such that it has the Lagrangian

$$L = \frac{m^2\dot{x}^4}{12} - m\dot{x}^2 V(x) - V^2(x)$$

where V is some differentiable function of x. Find the equation of motion for x(t).

4 (3 points)

A nucleus, originally at rest, decays radioactively by emitting an electron of momentum 1.73 MeV/c, and at a right angle to the direction of the electron a neutrino with momentum 1.00 MeV/c. In what direction does the nucleus recoil? What is its momentum in MeV/c? If the mass of the residual nucleus is 3.90×10^{-25} kg what is its kinetic energy, in electron volts?

5 (10 points)

(a) For a one-dimensional system with the Hamiltonian $H = \frac{p^2}{2} - \frac{1}{2q^2}$, where q is a generalized coordinate and p the conjugate momentum, show that there is a constant of the motion $D = \frac{pq}{2} - Ht$.

(b) As a generalization of part (a), for motion in a plane with the Hamiltonian $H = |\mathbf{p}|^n - ar^{-n}$, where $\mathbf{p}$ is the vector of the momenta conjugate to the Cartesian coordinates (in three dimensions), show that there is a constant of the motion $D = \frac{\mathbf{p}\cdot\mathbf{r}}{n} - Ht$.

6 (total 37 points)

Consider the quantum mechanics of a one-dimensional motion of a particle with mass m in a harmonic oscillator potential. The coordinate and momentum are denoted by x and p, respectively. Then the Hamiltonian is given by

$$H = \frac{p^2}{2m} + \frac{k}{2}x^2 \qquad (1)$$

where k is the spring constant. Here we use natural units where the Plank constant is set to unity, $\hbar = 1$.

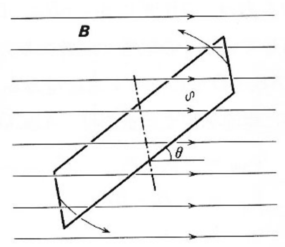
(a) (**11 points**) Show that the ground state wave function in the coordinate space, $\psi_0(x) = N \exp(-\alpha x^2)$ with N the normalization constant, satisfies the Schrodinger equation, $H\psi_0 = E\psi_0$, when the eigenenergy E and the parameter α are chosen appropriately. Determine E, α, and the normalization constant N.

(b) (**5 points**) Draw the graphs of the wave functions of the ground and first excited states as functions of x.
(c) (**5 points**) By the coordinate (scale) transformation, $x \rightarrow x \rightarrow \tilde{x} = ax$ and $p \rightarrow \tilde{p} = p/a$, show that the Hamiltonian is written as

$$h = \frac{\omega}{2}(\tilde{p}^2 + \tilde{x}^2) \qquad (2)$$

Determine a and ω, and then write the wave function using the new variable $\tilde{x}$.
(d) (**3 points**) Observing that the Hamiltonian (2) is symmetric in $\tilde{x}$ and $\tilde{p}$, find the ground state wave function in the momentum space as a function of $\tilde{p}$, $\varphi_0(\tilde{p})$.
(e) (**5 points**) Calculate the expectation values $\langle \tilde{x}^2 \rangle$ and $\langle \tilde{p}^2 \rangle$.
(f) (**3 points**) Using the results of (e), find the expectation values $\langle x^2 \rangle$ and $\langle p^2 \rangle$.
(g) (**5 points**) Show that the ground state wave function of the harmonic oscillator minimizes the Heisenberg's uncertainty relation, $\Delta x \Delta p \geq 1/2$.

7 (total 25 points)



(a) A circuit is rotated with a constant angular velocity in an external magnetic field as shown in the figure. The external magnetic field is constant in time and uniform in space and the strength of the magnetic field is B. The surface area of the circuit is S, the rotation frequency is ω, and the angle made by the circuit and the magnetic field is θ.
(I) **(5 points)** Derive the voltage induced in the circuit.
(II) **(5 points)** Based on the above result, evaluate the voltage defined as follows. We have a flat circular coil of diameter 10 cm and made of 2000 turns and rotate it in the Earth magnetic field at 15 rotations per second. The magnetic field of the Earth is assumed to be 5×10^{-5} T. Evaluate the maximum voltage generated in the coil.
(b) Let us assume that neutral atoms or molecules can be modeled as harmonic oscillators in some cases. Then, the equation of the displacement between nucleus and electron cloud can be written as

$$\mu\left(\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^{\,2}x\right) = qE \, ,$$

where x is the displacement in the direction of the external electric field E, μ is the effective mass, γ is the damping constant, ω_0 is the oscillation frequency of the harmonic potential, and q is the effective charge of the neutral atom or molecule. Assuming that E is a single frequency electric field with angular frequency ω, answer the following questions.
(I) **(2.5 points)** Obtain the amplitude of the stationary solution for the equation of the displacement.
(II) **(2.5 points)** Assuming that the material contains such atoms or molecules with density n_0, obtain the current density.
(III) **(2.5 points)** Obtain the dielectric constant $\varepsilon(\omega)$, which will be in complex form.
(IV) **(2.5 points)** Obtain the real and imaginary parts of the dielectric constant. Assume that $\gamma \ll \omega$ and calculate approximate solutions.
(V) **(5 points)** Introducing the following constants, $\varepsilon_{st} = \varepsilon(\omega = 0)$ and $\varepsilon_\infty = \varepsilon(\omega = \infty)$, draw the curves for the real and imaginary parts of the dielectric constant as functions of the frequency ω.