The $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ E2 cross section at stellar energies

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1. General presentation of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$
2. Microscopic calculations
3. R-matrix fits
4. Conclusions
1. General presentation of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

**Astrophysics**
- Very important in stellar models
- $2^{\text{nd}}$ step of He burning
- Determines the $^{12}\text{C}/^{16}\text{O}$ ratio
- Cross section needed near $E_{\text{cm}}=300$ keV (barrier $\sim 2.5$ MeV)
  \[\rightarrow \text{cannot be measured in the Gamow peak}\]

**Nuclear Physics**
- $1^-$ and $2^+$ subthreshold states
  \[\rightarrow \text{extrapolation difficult}\]
- E1 and E2 important (E1 forbidden when $T=0$)
- Interferences between $1^-_1, 1^-_2$ and between $2^+_1, 2^+_2$
- Capture to gs dominant but also cascade transitions
1. General presentation of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

Many experiments

- **Direct** $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ (angular distributions are necessary: E1 and E2)
- **Indirect**: spectroscopy of $1^-_1$ and $2^+_1$ subthreshold states
- **Constraints**
  - $\alpha+^{12}\text{C}$ phase shifts ($1^- \rightarrow E1$, $2^+ \rightarrow E2$)
  - E1: $^{16}\text{N}$ beta decay  
    probes $J=1^- \rightarrow E1$
  - E2: ???
1. General presentation of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

Current situation

![Graph showing S-factor as a function of $E_{cm}$ (MeV) with data points and fitted curves for different authors.]

S-factor

$S(E) = \sigma(E) \cdot E \cdot \exp(2\pi\eta)$
S(300 keV): current situation for E1

\[ ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \quad \text{E1} \]

- Weisser 74
- Dyer 74
- Koonin 74
- Humblet 76
- Descouvemont 87
- Redder 87
- Kremer 88
- Filippone 89
- Ouellet 92
- Ouellet 96
- Trautvetter 97
- Gialanella 01
- Fey 03

\[ ^{16}\text{N} \quad \text{data available} \]
$S(300 \text{ keV})$: current situation for E2

![Graph showing the results of various experiments on $12C(\alpha,\gamma)^{16}O$ E2 transitions over the years from 1980 to 2010. The graph plots $S_{E2}$ (keV-barn) on the y-axis and year on the x-axis. Significant data points include Kettner 82, Descouvemont 84, Redder 87, Langanke 85, Descouvemont 93, Buchmann 96, Fey 03, and Dufour 08.](image-url)
Present work: E2 S-factor


Two approaches

Microscopic calculation
- Based on an NN interaction
- All nucleons included
- Calculation of $S(E), S(300 \text{ keV})$
- Difficult for $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

R-matrix fits
- **Fit** of the $^{12}\text{C}+\alpha$ phase shift the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ cross section
- Extrapolation to 300 keV
- Relies on experimental data!
2. Microscopic calculation

Based on

- a 16-nucleon **Hamiltonian**
  \[ H = \sum_{i}^{16} T_i + \sum_{i<j=1}^{16} V_{ij} \]
  (\( T_i \)=kinetic energy, \( V_{ij} \)=NN interaction)

- a microscopic **wave function**
  \[ \Psi^{JM\pi} = A\phi_\alpha \phi_C g^{JM\pi}(\rho) \]
  (\( A \)=antisymmetrizer)
  
  with \( \phi_\alpha, \phi_C \)=shell-model wave functions (multichannel: several \( ^{12}\text{C} \) states)

- \( \Psi \) written as combination of Slater determinants
  - Bound states (\( ^{16}\text{O} \) ground state + excited states)
  - Scattering states at any c.m. energy (with the « calculable » R-matrix method)

- Calculation of the **phase shifts** (essentially \( J=2^+ \))
  \[ \sigma(E) \sim | < \Psi_{^{16}\text{O}} | E2 | \Psi_{\alpha+C}(E) > |^2 \]
Microscopic results

S(300 keV) \approx 50 \text{ keV} \cdot \text{b}
3. R-matrix fits

E2 → 2+ states (=poles) are involved
• 2+1 subthreshold state
• 2+2, 2+3 narrow resonances at 2.68 and 4.36 MeV
• background (high energy): other resonances

« Observed » (experimental) data:
- $E_{ri}$: energy of pole $i$
- $\Gamma_{\alpha i}$: $\alpha$ width (reduced $\gamma^2$)
- $\Gamma_{\gamma i}$: $\gamma$ width

« R-matrix » data: $\tilde{E}_{ri}$, $\tilde{\gamma}_i^2$, $\tilde{\Gamma}_\gamma$

$$R(E) = \sum_{i=1}^{4} \frac{\tilde{\gamma}_i^2}{\tilde{E}_{ri} - E} \quad \rightarrow \text{phase shift}$$

$$\sigma(E') \sim \left| \sum_{i=1}^{4} \frac{\tilde{\gamma}_i [\tilde{\Gamma}_\gamma]^{1/2}}{\tilde{E}_{ri} - E} \right|^2 \quad \rightarrow \text{capture cross section}$$
Strategy:

4 poles, 3 parameters for each pole ➔ 12 parameters

<table>
<thead>
<tr>
<th>pole</th>
<th>$E_i$</th>
<th>$\gamma^{2 _i}$</th>
<th>$\Gamma_{\gamma _i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i=1</td>
<td>-0.24</td>
<td>fitted</td>
<td>exp.</td>
</tr>
<tr>
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<td>2.68</td>
<td>exp.</td>
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</tr>
<tr>
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</tr>
<tr>
<td>i=4</td>
<td>variable</td>
<td>fitted</td>
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➔ 3 parameters (+background)

1. Fix $E_4$ to different values
2. Fit of $\gamma^{2 \_1}$, $\gamma^{2 \_4}$ (on the 2$^+$ phase shift)
3. Fit of $\Gamma_{\gamma \_4}$ (on the E2 S-factor)
Fit for several values of the background energy $E_{r4}$

- only an upper limit can be deduced: $S(300 \text{ keV}) < 190 \text{ keV.b}$
- can we constrain the R-matrix fit with microscopic results?
  yes, with the ANC of the $2^+$ subthreshold state
Constrain from ANC

• Stands for Asymptotic Normalization Constant

• Defined from the asymptotic properties of the radial wave function

\[ g_{\ell}(\rho) \longrightarrow C_{\ell} W_{-\eta_B,\ell+1/2}(2k_B\rho) \]

\[ W(x) = \text{Wittakher function} \]

• ANC proportional to \( \gamma^2 \) \( \Rightarrow \) for pole 1, \( \gamma^2 \) is taken from the theory

• Can be computed from microscopic models

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\[ \Rightarrow \text{only 2 parameters (background energy)} \]
- Virtually independent of the background energy
- $S(300 \text{ keV}) = 42 \pm 2 \text{ keV.b}$
ANC can be obtained from

- indirect (transfer) measurements:
  example $^{12}\text{C}(^{6}\text{Li},d)^{16}\text{O}$
  $C \sim 1.5 \times 10^5 \text{ fm}^{-1/2}$

- capture to $2^+$ state (« cascade »):
  $2^+$ weakly bound $\rightarrow$ external capture ($\sigma \sim |C|^2$)
  $C \sim 3.2 \times 10^5 \text{ fm}^{-1/2}$

- present:
  Solution of the 16-nucleon problem
  $C = 1.3 \times 10^5 \text{ fm}^{-1/2}$

- capture to the $2^+$ state
- external ($\sigma \sim |C|^2$ at low E)
- 2.5 MeV resonance (E1)
- amplitude inconsistent with indirect measurements

$\Rightarrow$ new experiment??
4. Conclusion

Two (complementary) approaches for the E2 contribution

1. Microscopic calculation: \( S(300 \text{ keV}) = 50 \text{ keV.b} \)
2. R-matrix fit: \( S(300 \text{ keV}) < 190 \text{ keV.b} \)

\( \Rightarrow \) a constraint is necessary (as for E1)

External constraint on the R-matrix fit: ANC of the 2+ subthreshold state
\( \Rightarrow S(300 \text{ keV}) = 42 \pm 2 \text{ keV.b} \)

\( \Rightarrow \) importance of the ANC to improve the E2 extrapolation

but: direct and indirect data are not consistent with each other!
\( \Rightarrow \) new measurement of cascade transitions?