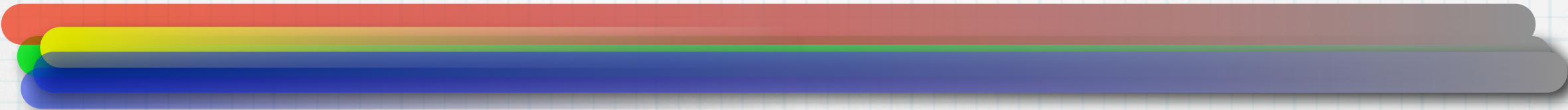


# Exotic hadrons from a quark-model's point of view



Sachiko Takeuchi  
(Japan College of Social Work)

in collaboration with

K. Shimizu (Sophia)

V.E. Lyubovitskij, Th. Gutsche, Amand Faessler  
(Institut für Theoretische Physik, Univ Tübingen)

# Exotic hadrons

- Exotic hadrons v.s. baryons & mesons

$q^3$   $\longleftrightarrow$  Baryons

$q^4\bar{q}$   $\longleftrightarrow$  Baryons with meson cloud + more...?

$q\bar{q}$   $\longleftrightarrow$  Mesons

$q^2\bar{q}^2$   $\longleftrightarrow$  Two-meson systems

↑  
Exotic hadrons

↓  
+ more...?

# Exotic hadrons

- Exotic hadrons v.s. baryons & mesons

$q^3$   $\longleftrightarrow$  Baryons

$q^4\bar{q}$   $\longleftrightarrow$  Baryons with quark cloud

$\Lambda(1405)$

+ more...?



Exotic hadrons



$q\bar{q}$   $\longleftrightarrow$  Mesons

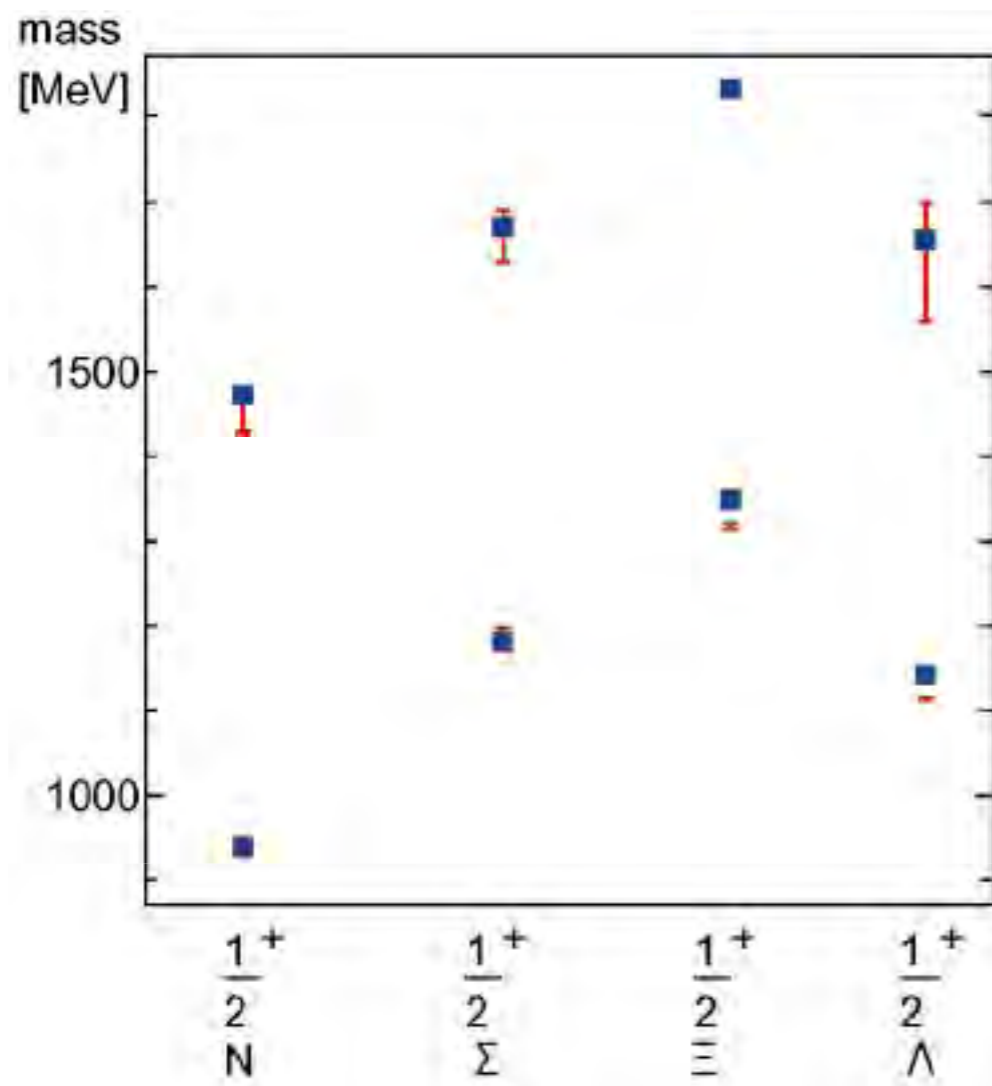
$q^2\bar{q}^2$   $\longleftrightarrow$  Two-meson systems

$X(3872)$

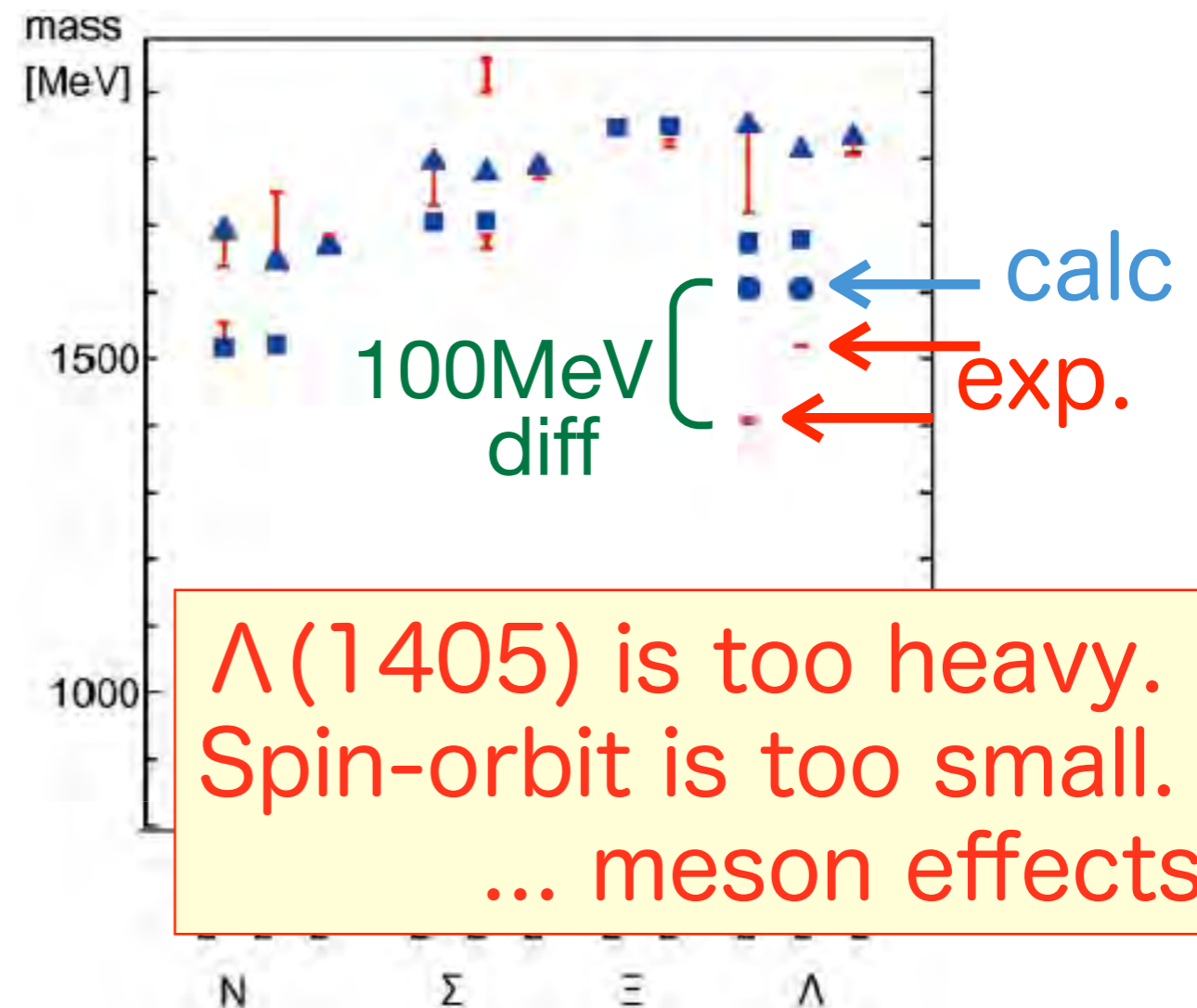
+ more...?

# $\Lambda(1405)$

- mass spectrum of  $q^3$  baryons



S-wave



$\Lambda(1405)$  is too heavy.  
Spin-orbit is too small.  
... meson effects?

P-wave

# $\Lambda(1405)$

- Flavor-singlet **P-wave**  $q^3$  state ?
  - Observed  $\Lambda_8 - \Lambda_1$  splitting
  - Observed large LS splitting
    - are difficult to reproduce...
- **S-wave**  $q^4 \bar{q}$  state ?
  - CMI  $(\lambda \cdot \lambda)(\sigma \cdot \sigma)$  can be strongly attractive in 2 states of  $T=0$   $J^P = 1/2^-$ 
    - but also in  $T=1$   $1/2^-$



# Problems in $\Lambda(1405)$

From a quark model's viewpoint

mass diff.	config.	origin	Theo.	Exp.
$M_{\Delta} - M_N$	$q^3$	$(\sigma \cdot \sigma)$	300 MeV	300 MeV
$M_{\Lambda 8} - M_{\Lambda 1}$	$q^3$	$(\sigma \cdot \sigma)$	150 MeV	200 MeV
	$(q^4 \bar{q})$	flavor sym.	Larger	
$M_{N(3/2-)} - M_{N(1/2-)}$	$q^3$	(LS)	0 MeV	0 MeV
$M_{\Lambda(1520)} - M_{\Lambda(1405)}$	$q^3$	(LS)	0 MeV	115 MeV
	$(q^4 \bar{q})$	$(\sigma \cdot \sigma)$	Larger	

# $\Lambda(1405)$

- Flavor-singlet **P-wave**  $q^3$  state ?
  - Observed  $\Lambda_8 - \Lambda_1$  splitting
  - Observed large LS splitting
    - are difficult to reproduce...
- **S-wave**  $q^4 \bar{q}$  state ?
  - CMI  $(\lambda \cdot \lambda)(\sigma \cdot \sigma)$  can be strongly attractive in 2 states of  $T=0$   $J^P = 1/2^-$ 
    - but also in  $T=1$   $1/2^-$

# Baryon-meson scattering (QCM)

- From Schrödinger eq for quarks:

$$(H_q - E)\phi = 0$$

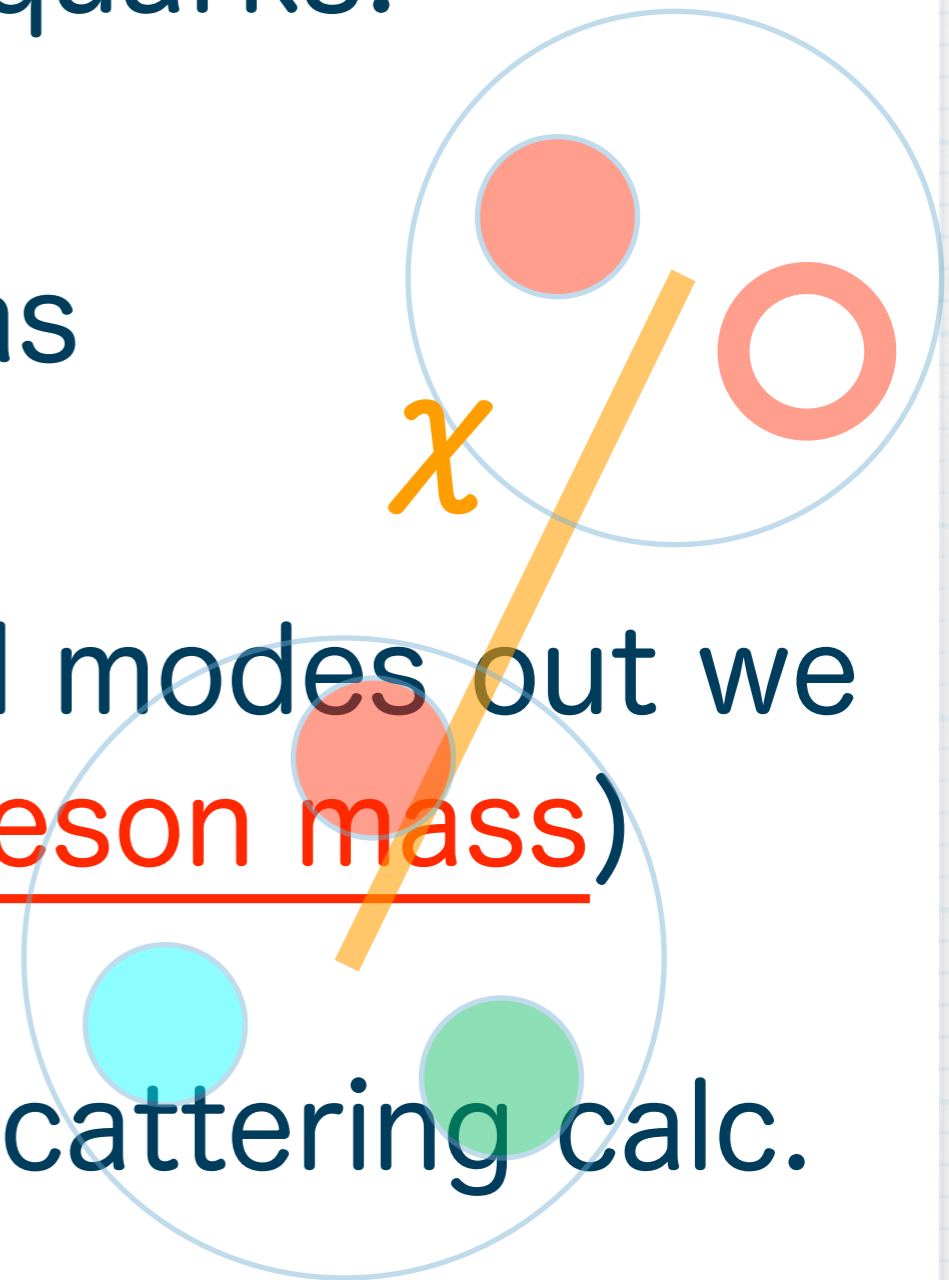
- Assuming wave function as

$$\Psi = \phi_B \phi_M \chi$$

- By integrating the internal modes out we get RGM eq (using real meson mass)

$$(H - EN)\chi = 0$$

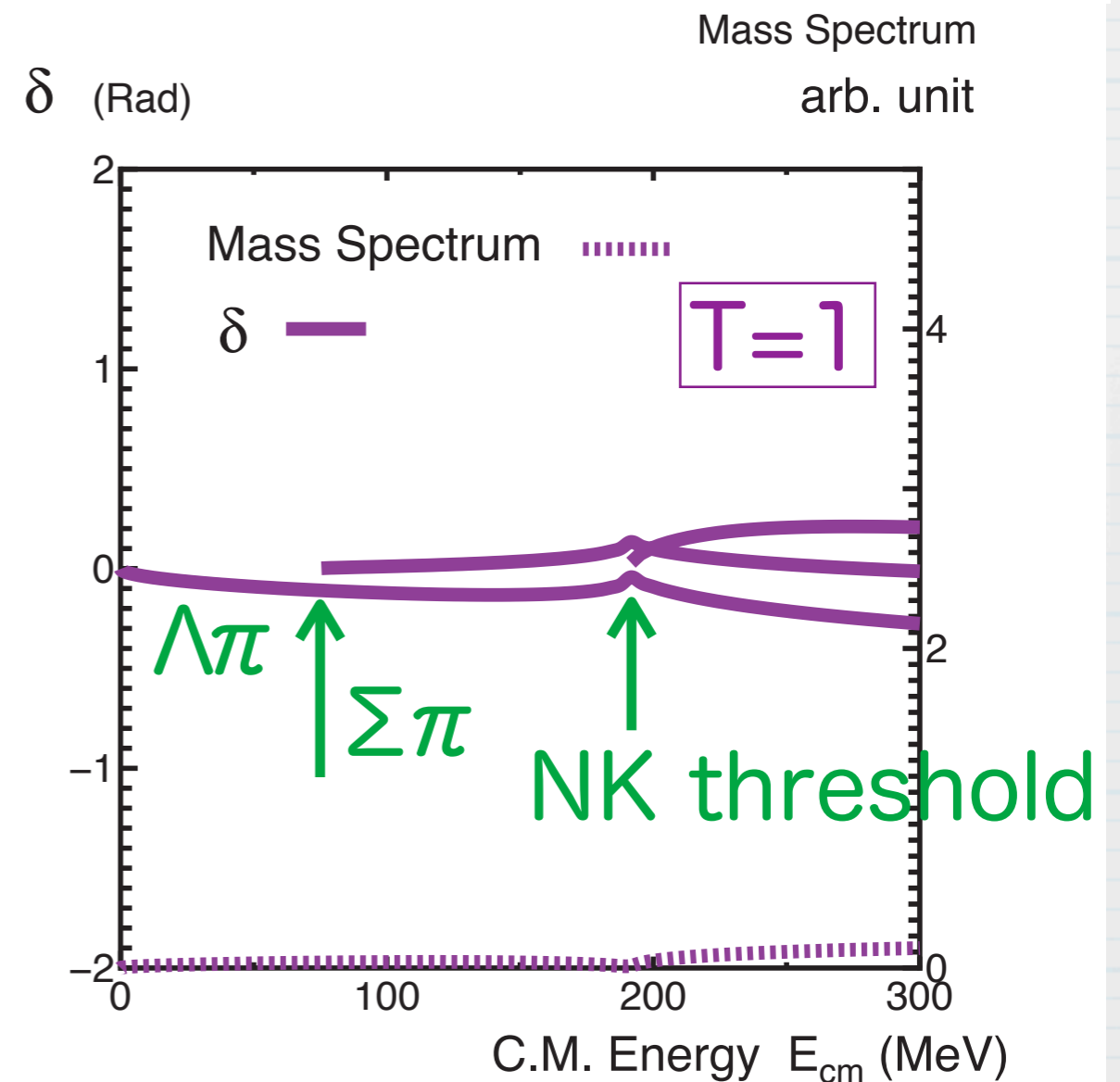
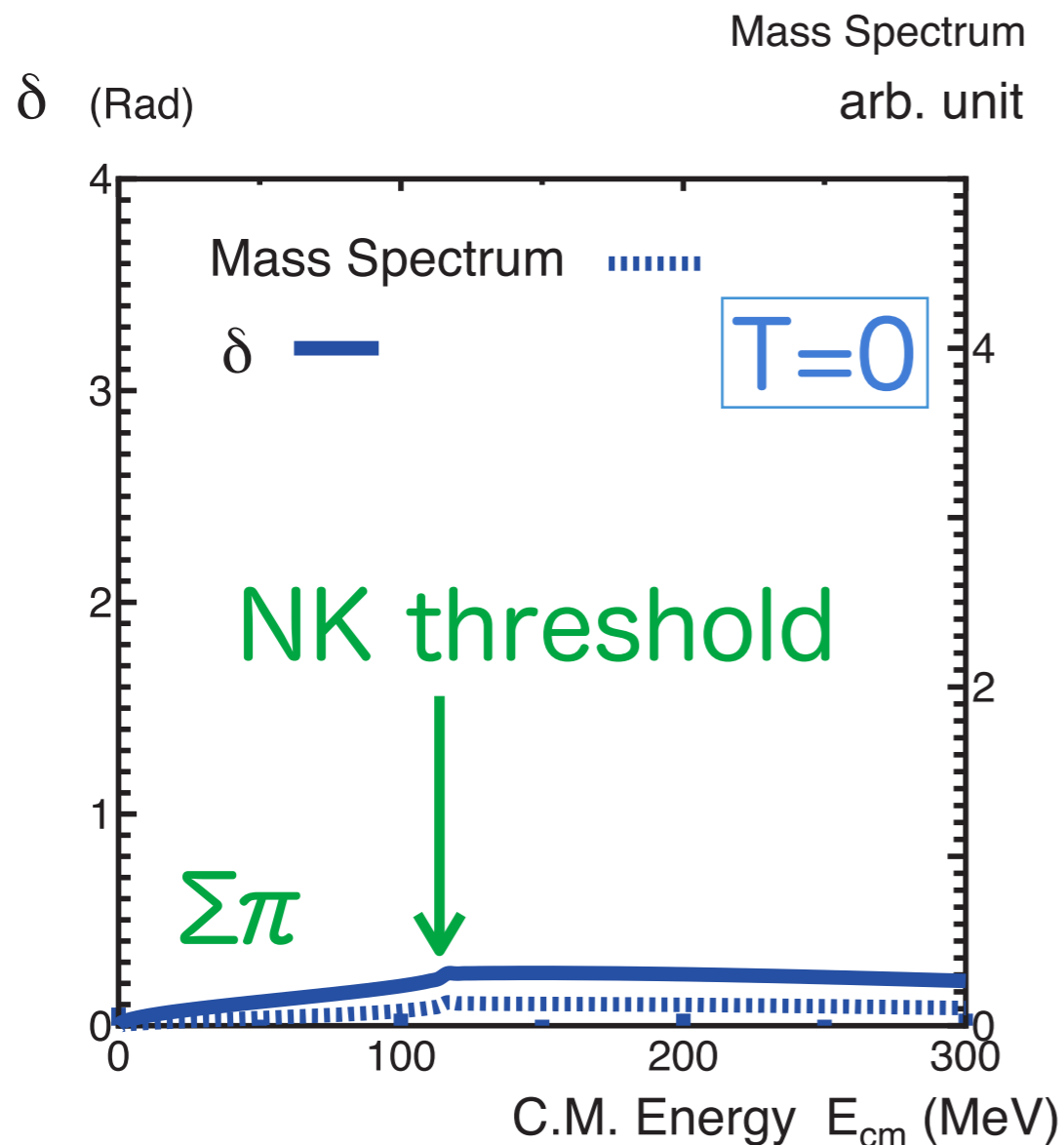
- 3-channel coupled QCM scattering calc. for  $m_u \neq m_s$





# No peak is found for $q^4\bar{q}$ !!

- Reduced mass of  $\Sigma\pi$  is small  $\rightarrow$  Kinetic term is large  $\rightarrow$  Short range attraction is suppressed.
- No attraction in the NK channel.

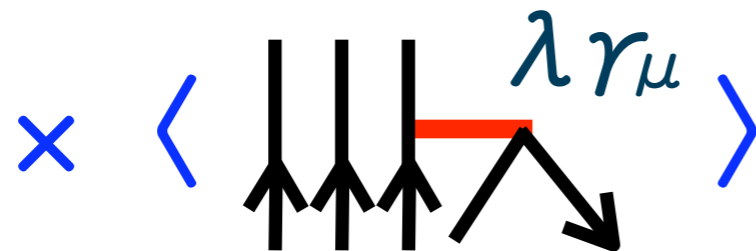


# With $q^3$ -pole ...

- $\Lambda(1405) = |q^3\rangle + |q^4\bar{q}\rangle$  (Very rough estimate)

Transition pot.  $\langle q^3 | V | q^4\bar{q} \rangle$  : estimate)

$$V = | \Lambda_1 q^3 (0s)^2 0p \rangle \langle \text{BM } q^4 \bar{q} (0s)^5 |$$



pair-annihilating  
diagram



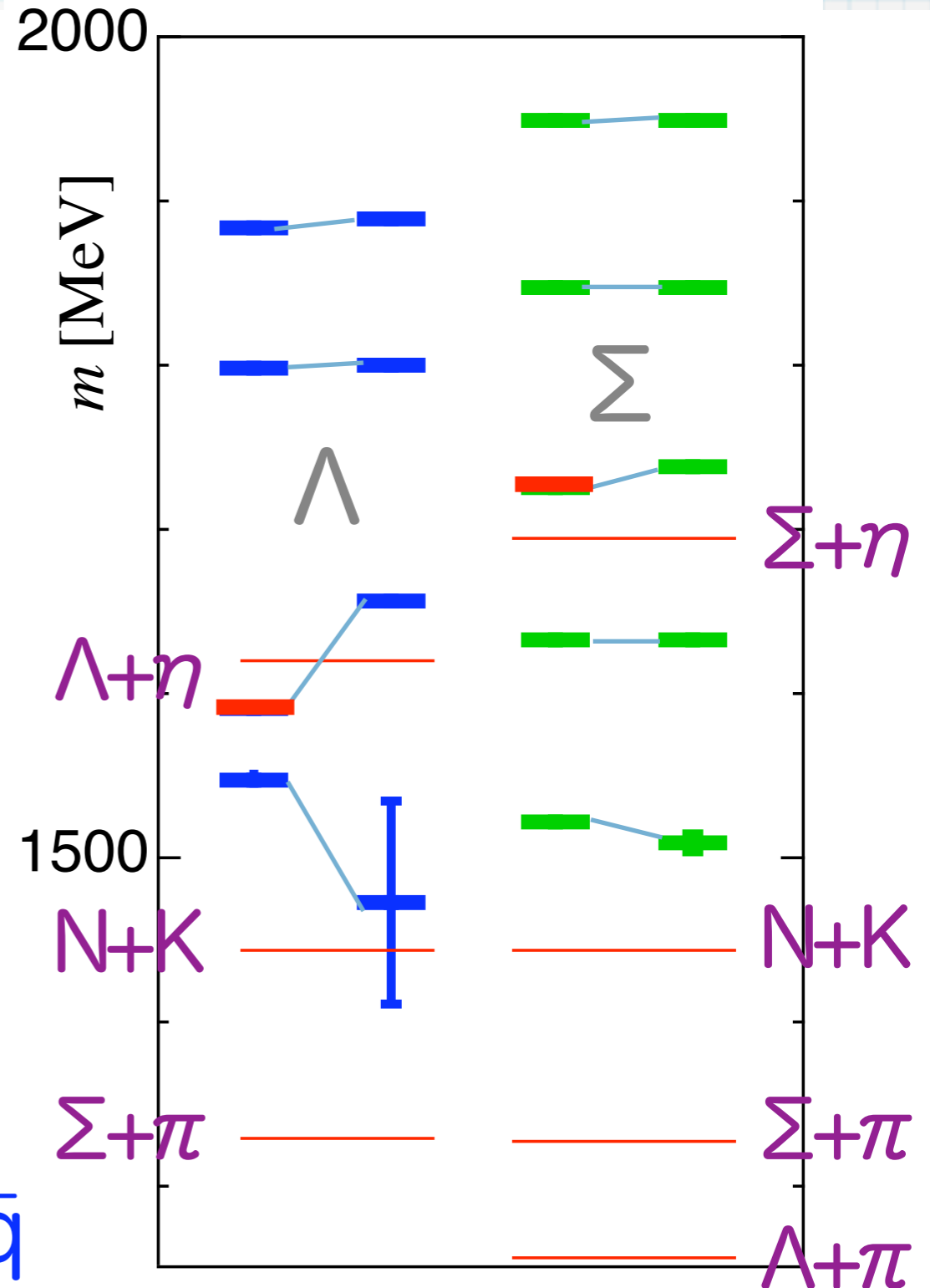
use smaller  $\alpha_s : \times 1/2$

$\Lambda_1 1/2^-$		$\Sigma_8 1/2^-$	
$\Sigma \pi$	-178.1	$\Lambda \pi$	47.6
NK	117.1	$\Sigma \pi$	61.4
$\Lambda \eta$	57.5	NK	-85.0
(in MeV)		$\Sigma \eta$	-43.4

# Coupling to $q^3$

- $(0s)^5 + (0s)^2 0p$ 
  - The mixing is larger in  $\Lambda 1/2^-$ .
  - Width  $\sim 100$  MeV.
- $\Lambda$  may be seen; while  $\Sigma$  does not give a peak ???

—  $q^3$   
 —  $q^4 \bar{q}$



# $q^3$ - $q\bar{q}$ scattering with $q^3$ -pole

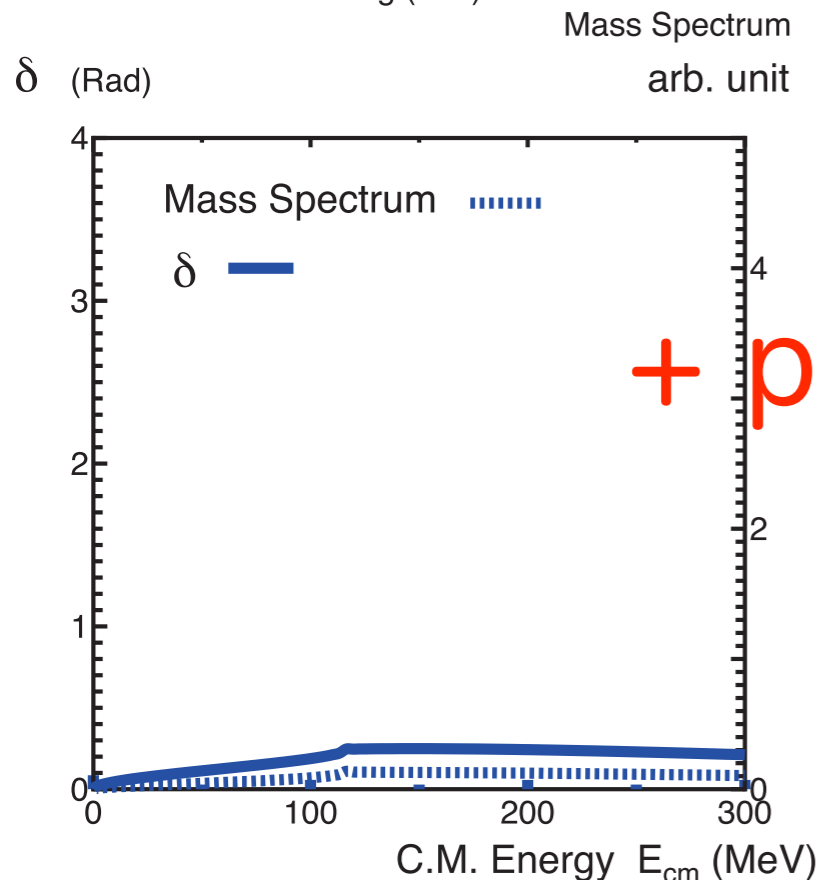
- $q^3$ -pole at  $\Sigma\pi + 130\text{MeV}$  ( $\sim 1460\text{ MeV}$ ) gives a resonance at  $1400\text{MeV}$ !

$\Sigma\pi + NK$

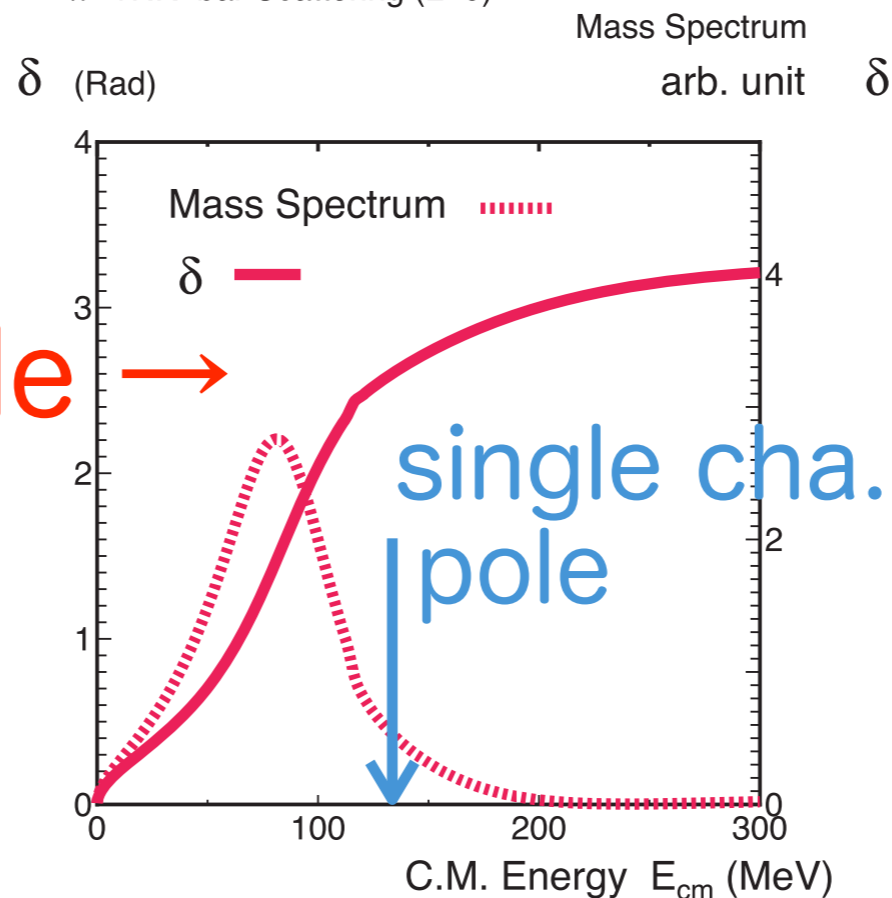
$\Sigma\pi + NK + \text{pole}$

$\Sigma\pi + NK$ -bar Scattering ( $L=0$ )

$\Sigma\pi + NK$ -bar Scattering ( $L=0$ )



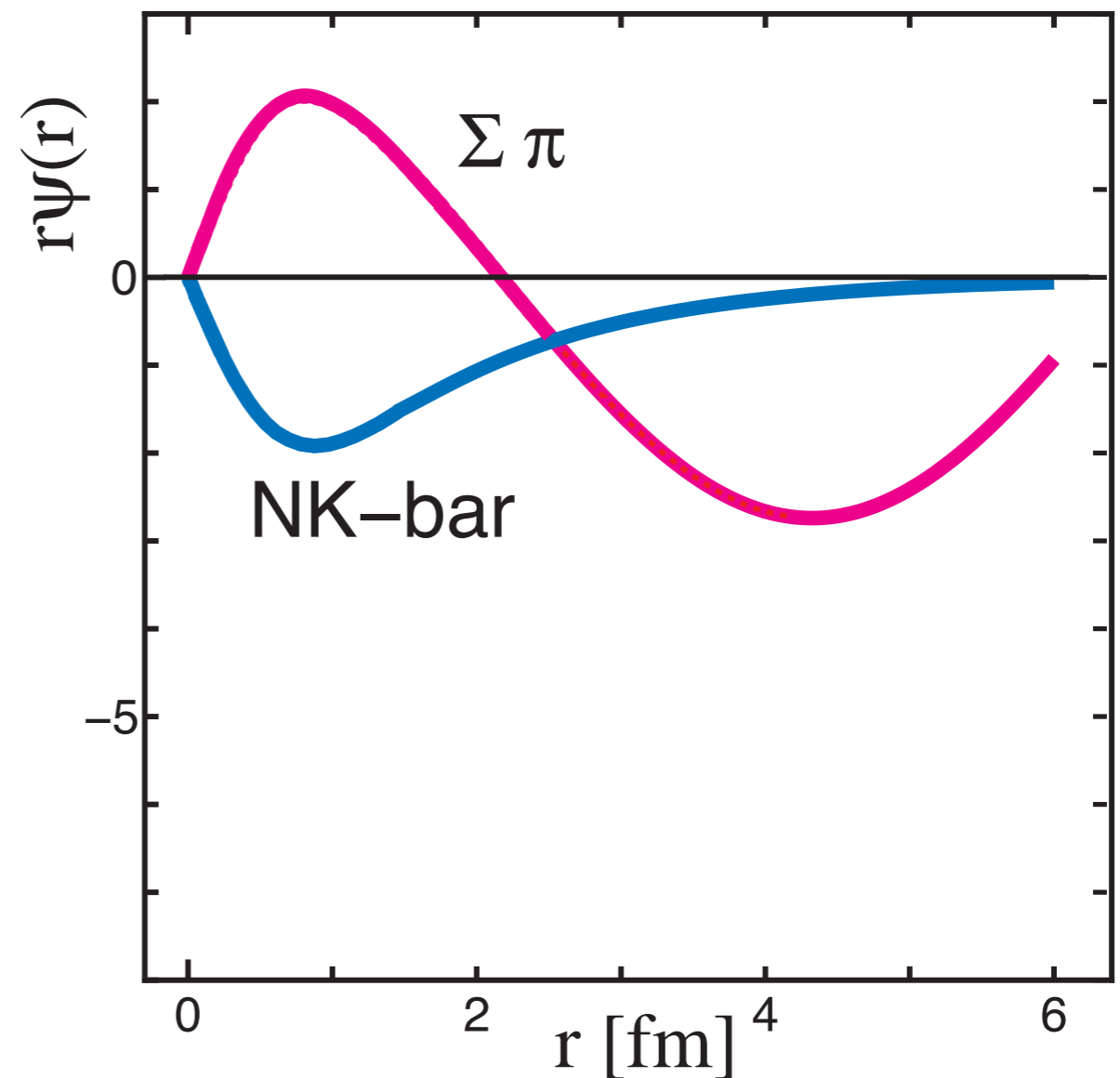
+ pole



# wave functions at resonance

- Contribution of the  $q^3$ -pole is large.

	$ \psi ^2$
NK	5.3
$q^3$ -pole	25.6

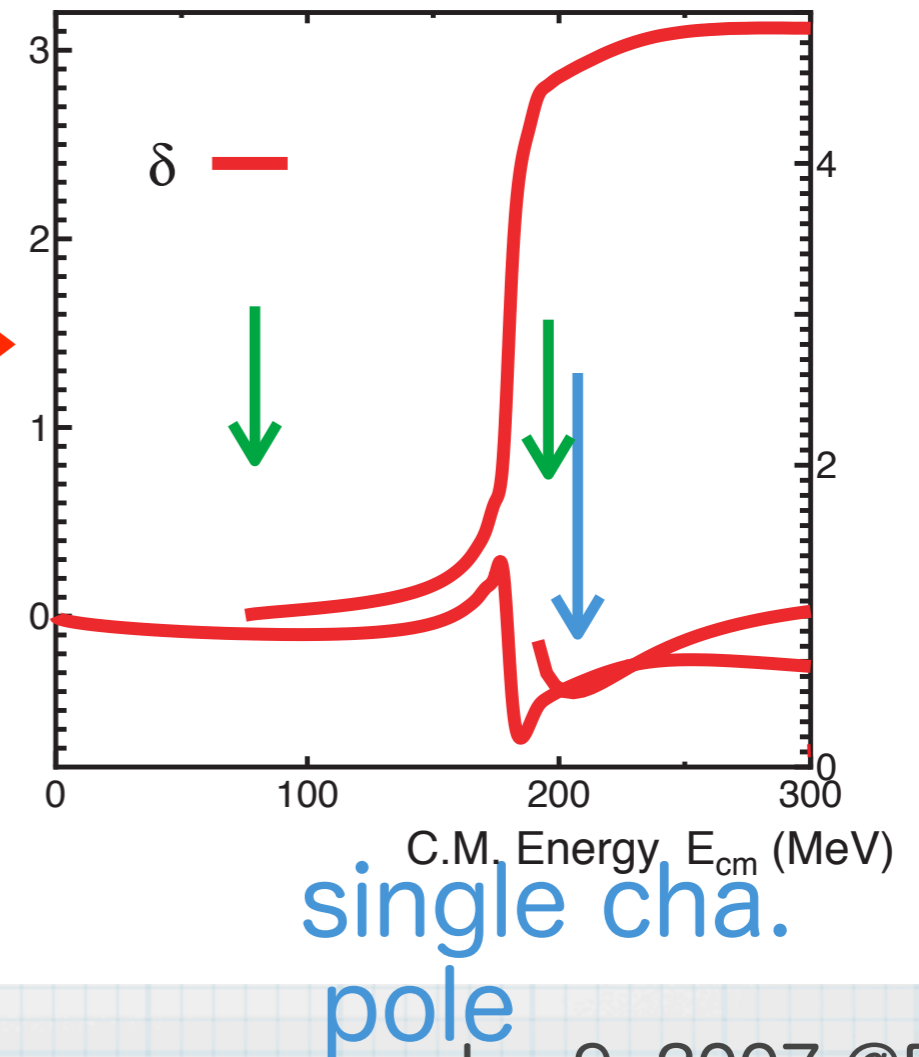
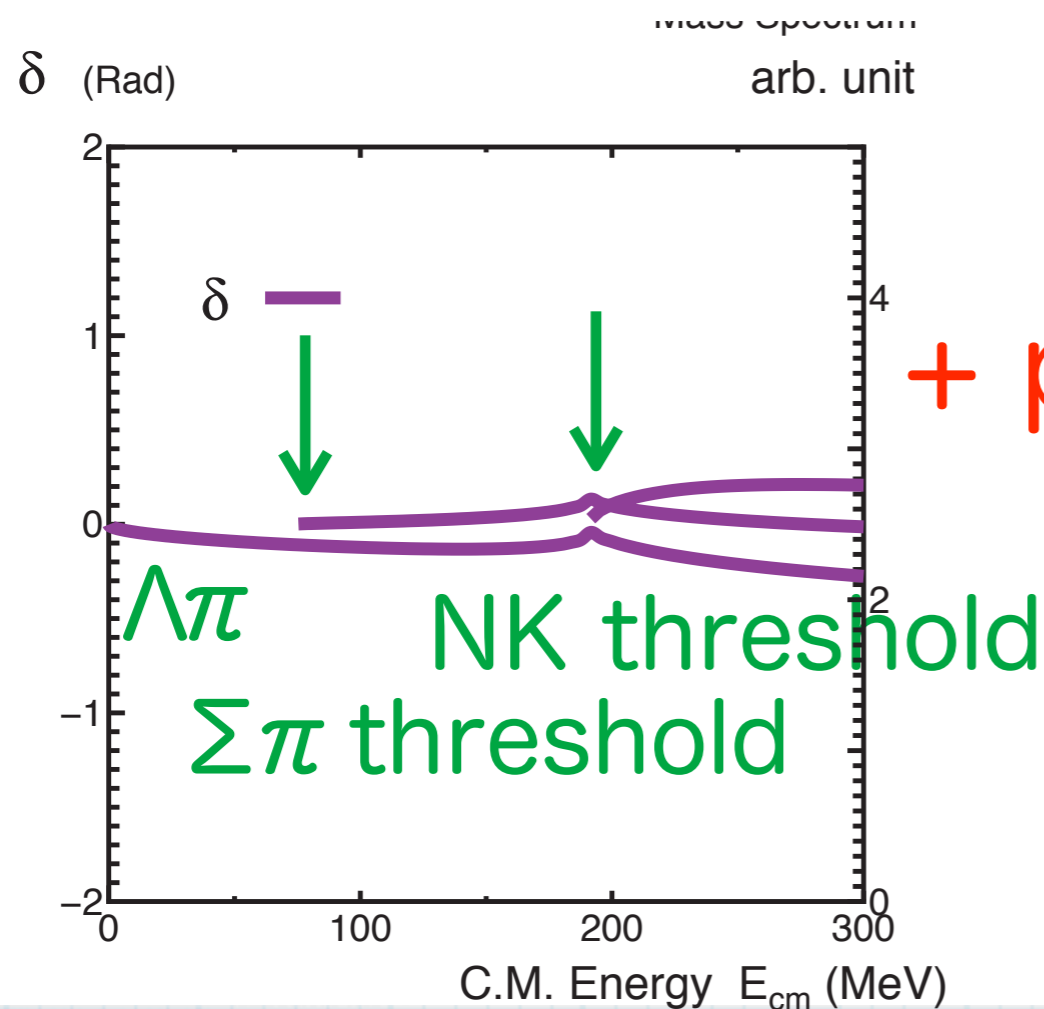


- Can this be observed...?



# $\Sigma^*$ (flavor octet)

- No peak is found around 1400MeV.
- mixing between  $q^4\bar{q}$  and  $q^3$  is small.
- The mass of the  $q^3$ -pole is heavy.



# Summary of parity -1 baryons

- $\Lambda(1405)$  and  $\Sigma^*$  are investigated as a  $(q^3 - q\bar{q}) + q^3$  pole system.
- Only  $\Lambda(1/2^-)$  has a resonance around 1400 MeV.
- The peak in  $\Sigma(1/2^-)$  is found at the higher energy.
- $\Lambda(3/2^-)$  is not calculated dynamically. But  $\Sigma^*\pi$  has smaller attraction than  $\Lambda(1/2^-)$ .
- But **what is a multiquark component** rather than the baryon-meson?

# X(3872)

- X(3872) found in  $B^\pm \rightarrow K^\pm X$

- $M(X) = 3871.7 \pm 0.6 \text{ MeV}$

- $\Gamma < 2.3 \text{ MeV}$

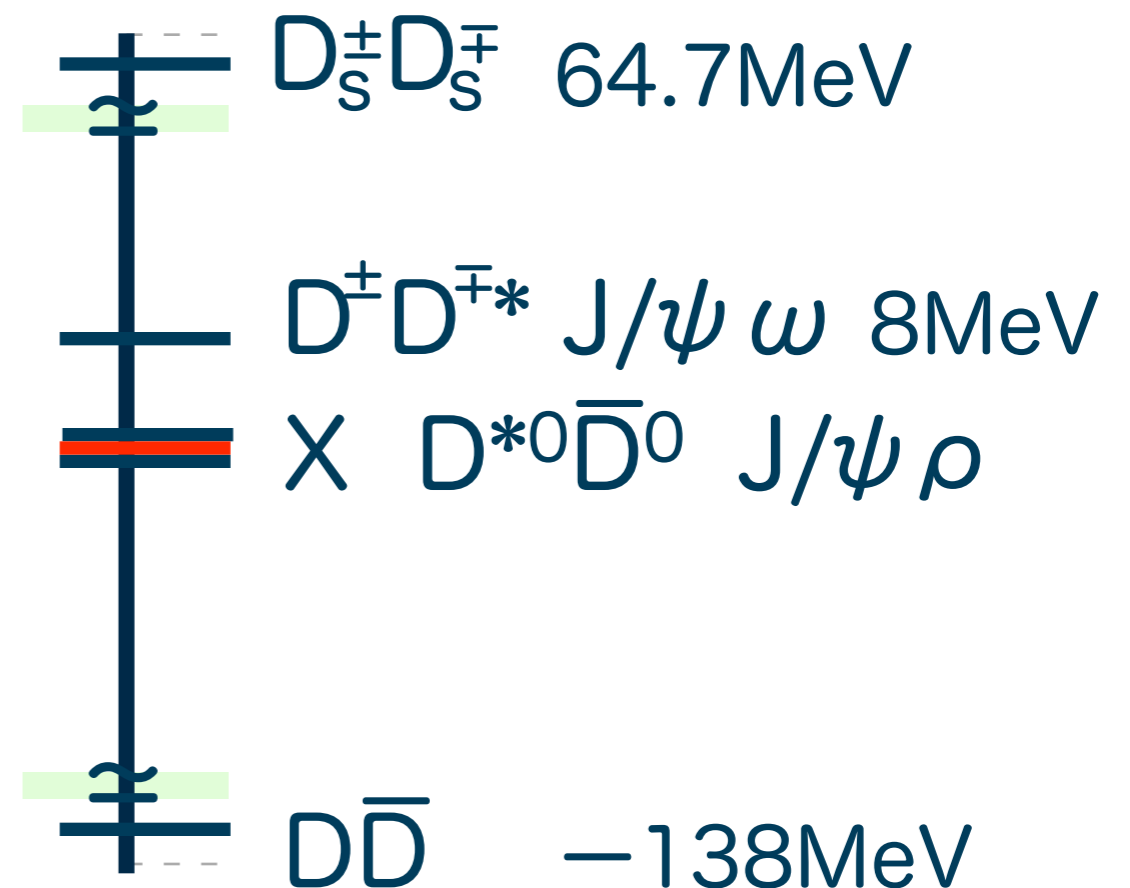
- Threshold

- $J/\psi \omega = 3879.5 \text{ MeV}$

- $D^\pm D^{*\mp} = 3879.1 \text{ MeV}$

- $J/\psi \rho = 3872.7 \text{ MeV}$

- $D^0 D^{*0} = 3871.3 \text{ MeV}$



# X(3872)

- X(3872) found in  $B^\pm \rightarrow K^\pm X$

- $M(X) = 3871.7 \pm 0.6 \text{ MeV}$

- $\Gamma < 2.3 \text{ MeV}$

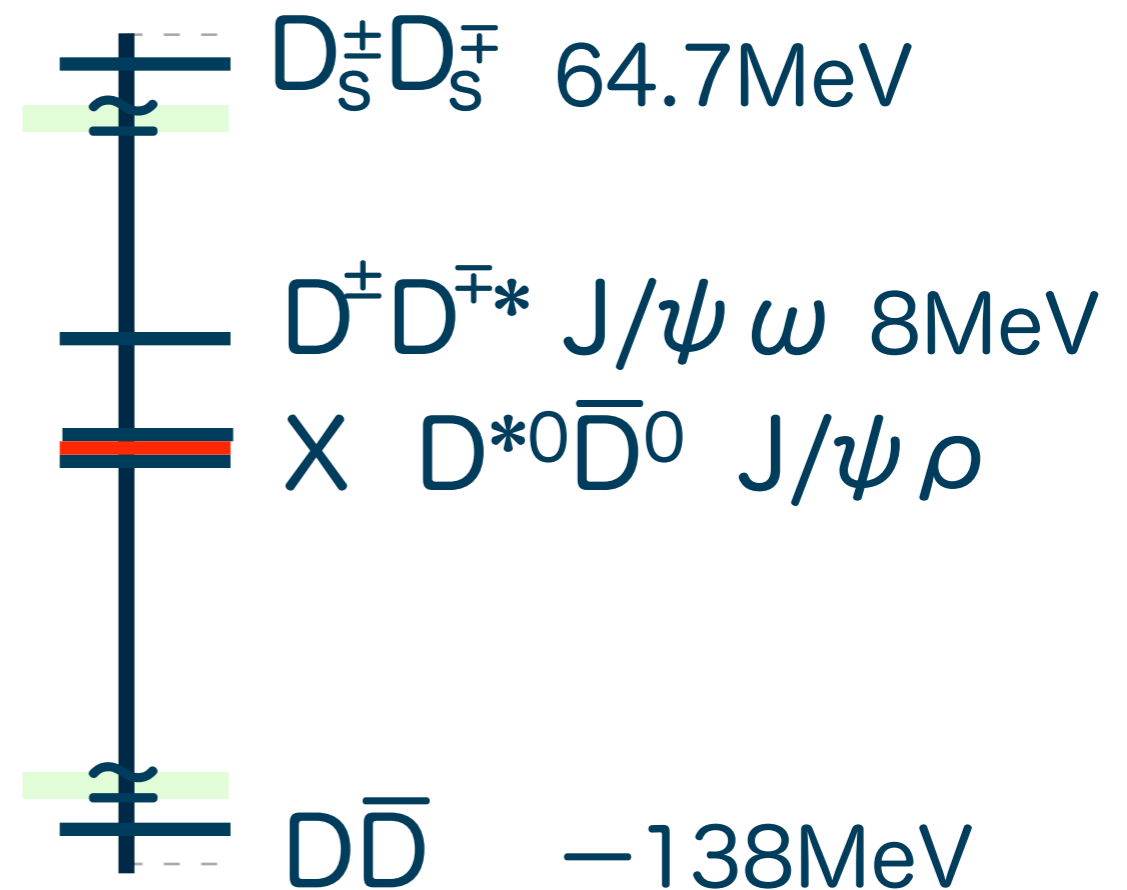
- Threshold

- $J/\psi \omega = 3879.5 \text{ MeV}$

- $D^\pm D^{*\mp} = 3879.1 \text{ MeV}$

- $J/\psi \rho = 3872.7 \text{ MeV}$

- $D^0 D^{*0} = 3871.3 \text{ MeV}$



# X(3872)

- X(3872) found in  $B^\pm \rightarrow K^\pm X$

- $M(X) = 3871.7 \pm 0.6 \text{ MeV}$

- $\Gamma < 2.3 \text{ MeV}$

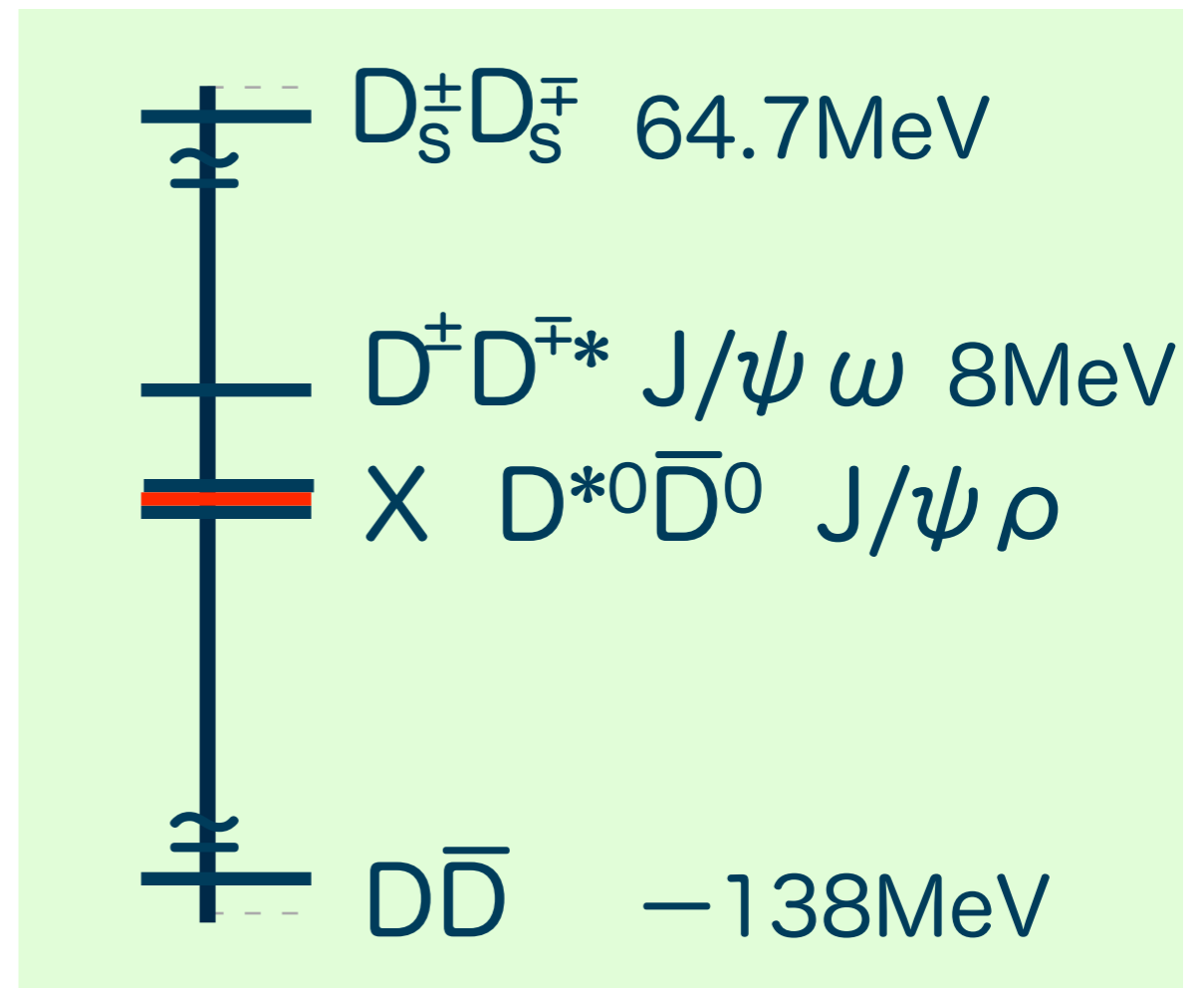
- Threshold

- $J/\psi \omega = 3879.5 \text{ MeV}$

- $D^\pm D^{*\mp} = 3879.1 \text{ MeV}$

- $J/\psi \rho = 3872.7 \text{ MeV}$

- $D^0 D^{*0} = 3871.3 \text{ MeV}$





# X(3872): $c\bar{c}$ or not $c\bar{c}$ ?

- X(3872) peak was found in  $\pi^+\pi^- J/\psi$  channel (Belle PRL91(2003)262001)
- narrow width  $< 2.3$  MeV. (Not decay to DD)
- Not  $c\bar{c}$  ?  $\pi\pi$  mass spectrum suggests that the peak is not a simple  $c\bar{c}$  state.  
(See, e.g. G.Bauer Int J Mod Phys A)
- $c\bar{c}g$  ? (Seth 05; Li 05)
- $D^0D^{*0}$  meson? (Swanson 04; Tornqvist 04)
- $q\bar{q}c\bar{c}$  ? (Maiani 05)

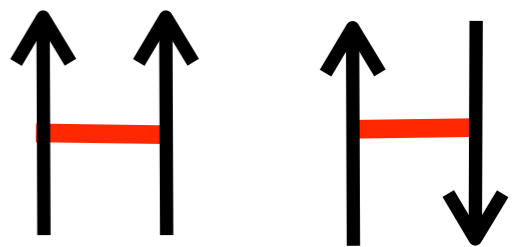
# X(3872): $c\bar{c}$ or not $c\bar{c}$ ?

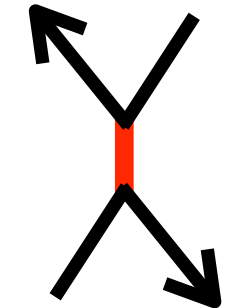
- X(3872) peak was found in  $\pi^+\pi^- J/\psi$  channel (Belle PRL91(2003)262001)
- narrow width < 2.3 MeV. (Not decay to DD)
- Not  $c\bar{c}$  ?  $\pi\pi$  mass spectrum suggests that the peak is not a simple  $c\bar{c}$  state.  
(See, e.g. G.Bauer Int J Mod Phys A)
- $c\bar{c}g$  ? (Seth 05; Li 05)
- $D^0D^{*0}$  meson? (Swanson 04; Tornqvist 04)
- $q\bar{q}c\bar{c}$  ? (Maiani 05)

# Hamiltonian for quarks

- $H = \text{Nonrela Kin} + \text{linear Conf} + \text{OGE} + \text{Ins} + \pi, \sigma \text{ exch}$

- **OGE**

$$V_{\text{ele}} = \sum_{i < j} -\pi\alpha_s \frac{\lambda_i \cdot \lambda_j}{4} \frac{1}{2} \left( \frac{1}{m_i^2} + \frac{1}{m_j^2} \right) \delta^3(\mathbf{r}_{ij})$$


$$V_{\text{CMI}} = \sum_{i < j} -\pi\alpha_s \frac{\lambda_i \cdot \lambda_j}{4} \sigma_i \cdot \sigma_j \frac{2\xi_i \xi_j}{3m_u^2} \delta^3(\mathbf{r}_{ij})$$


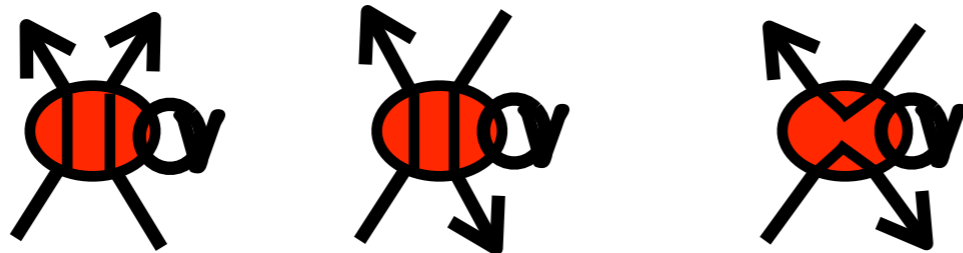
$$V_{\text{OGE}}^{(a)} = \sum_{i < j} \frac{1}{24} \left( \frac{16}{3} + \lambda_i \cdot \lambda_j \right) (3 + \sigma_i \cdot \sigma_j) \mathcal{P}_{ij} \pi\alpha_s \frac{1}{4mm'} \delta^3(\mathbf{r}_{ij})$$

# Hamiltonian for quarks

- **Ins** (affects only light quark pairs.)

$$V_{\text{INS}} = \sum_{i < j} \frac{V_0}{2} \xi_i \xi_j \left( 1 + \kappa \frac{3}{32} \lambda_i \cdot \lambda_j + \frac{9}{32} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j \right) \mathcal{P}'_{ij} \delta^3(\mathbf{r}_{ij})$$

$$V_{\text{INS}}^{(a)} = \sum_{i < j} -\frac{V_0}{2} \xi_i \xi_j \mathcal{P} \mathcal{P}'' \left( 1 - \frac{3}{32} \lambda_i \cdot \lambda_j + \frac{9}{32} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j \right) \delta^3(\mathbf{r}_{ij})$$



# Estimate by $(0s)^4$

- Effects of the interaction on  $q\bar{q}$  pairs  
 Rough sizes are obtained from  $N\Delta$ ,  
 and  $\eta'-\eta$  mass differences.

Color	Spin	Flavor	CMI	OgE-a	Ins	E[MeV]	State
1	0	1	-16	0	12	84	$\eta$
1	0	8	-16	0	-6	-327	$\pi$
1	1	1	16/3	0	0	63	$\omega$
1	1	8	16/3	0	0	63	$\rho$
8	0	1	2	0	3/4	41	
8	0	8	2	0	-3/8	15	
8	1	1	-2/3	9/2	9/4	97	
8	1	8	-2/3	0	-9/8	-34	In $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$



# Realistic Calc. - mesons

$$m_u = 313 \text{ MeV}$$

$$m_s = 593 \text{ MeV}$$

$$m_c = 1250 \text{ MeV}$$

$$a_{\text{conf}} = 172.4 \text{ MeV/fm}$$

$$\alpha_s = 0.73$$

$$V_{0,\text{ins}} = -143 \text{ MeV/fm}^3$$

( $p_{\text{III}} = 0.4$ )

$$\xi_{\text{cu}} = 0.586$$

$$\xi_{\text{cc}} = 0.198$$

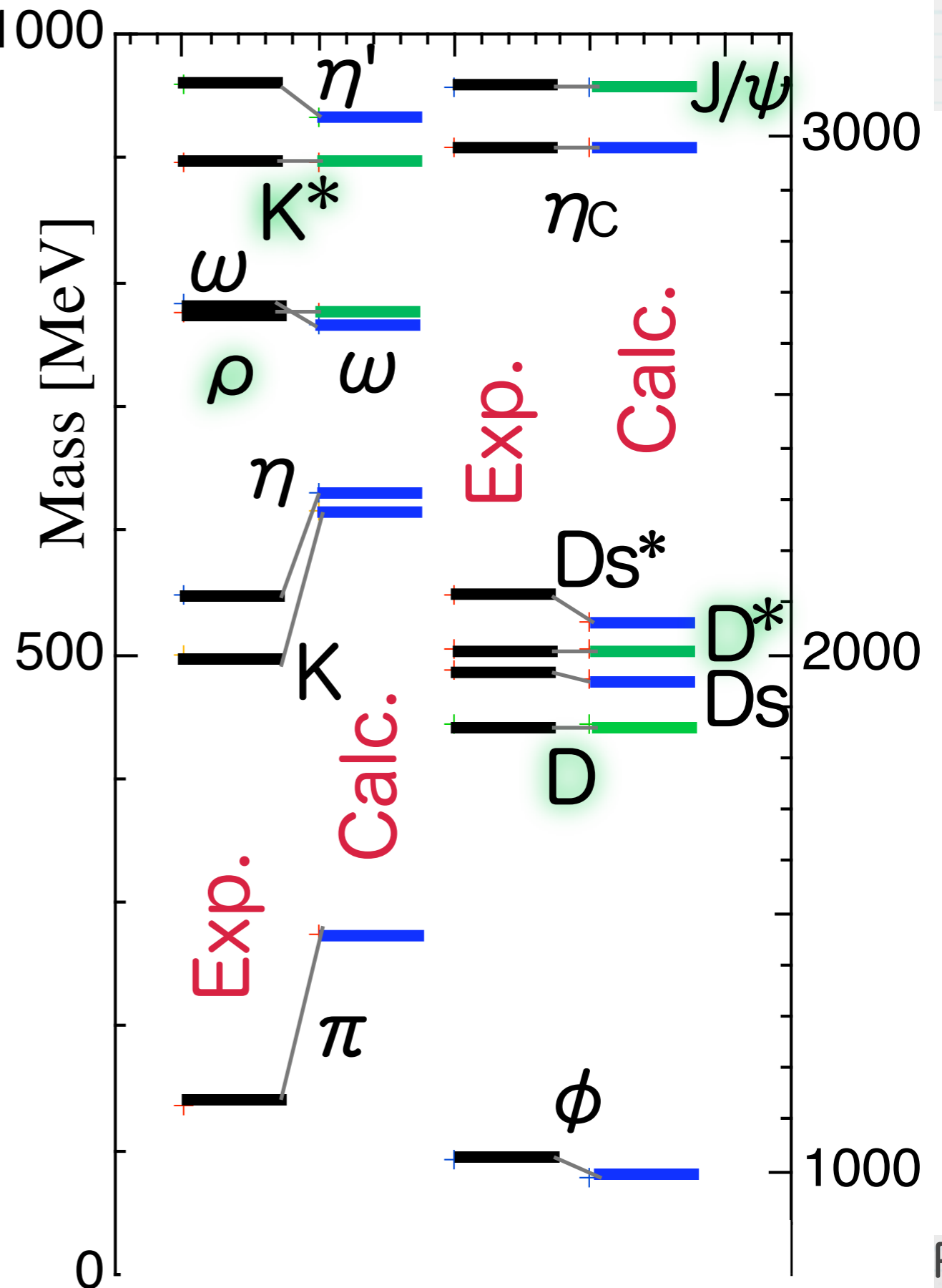
$$\Lambda_g = 3.3 \text{ fm}^{-1}$$

$$g_8^2/4\pi = 0.69$$

$$m_\sigma = 675 \text{ MeV}$$

$$\Lambda_\sigma = 5.3 \text{ fm}^{-1}$$

$$\Lambda_\pi = 1.1 \text{ fm}^{-1}$$



# Realistic Calc. - mesons

$$m_u = 313 \text{ MeV}$$

$$m_s = 593 \text{ MeV}$$

$$m_c = 1250 \text{ MeV}$$

$$a_{\text{conf}} = 172.4 \text{ MeV/fm}$$

$$\alpha_s = 0.73$$

$$V_{0,\text{ins}} = -143 \text{ MeV/fm}^3$$

$$(p_{\text{III}} = 0.4)$$

$$\xi_{\text{cu}} = 0.586$$

$$\xi_{\text{cc}} = 0.198$$

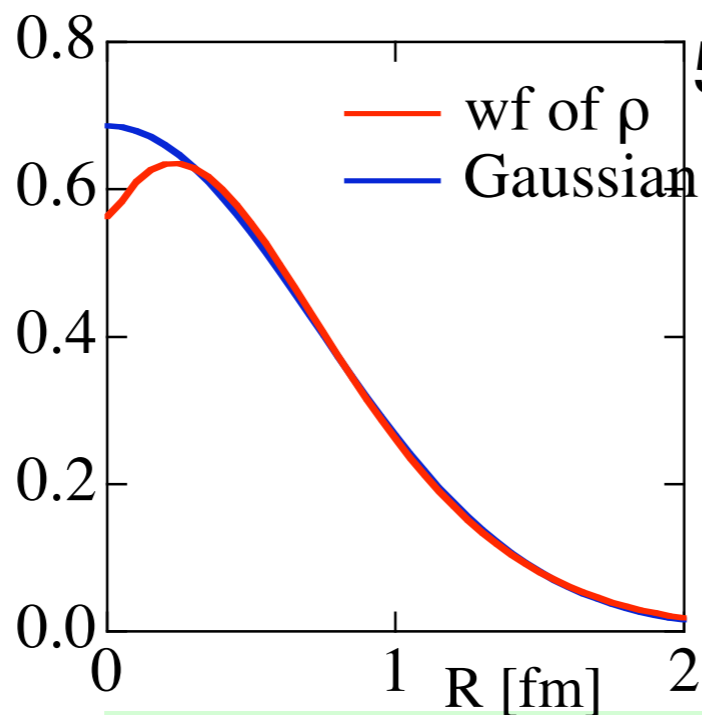
$$\Lambda_g = 3.3 \text{ fm}^{-1}$$

$$g_8^2/4\pi = 0.69$$

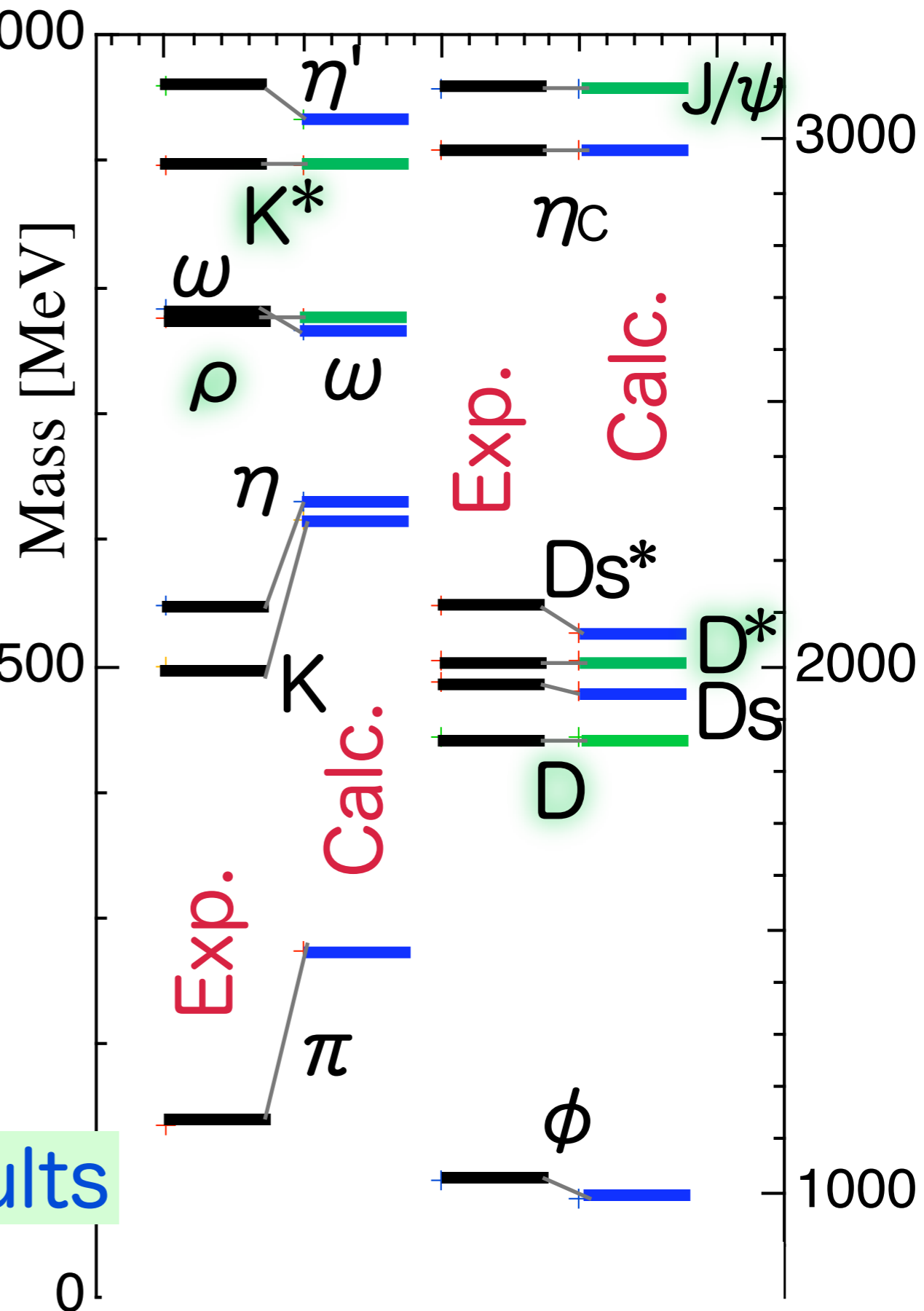
$$m_\sigma = 675 \text{ MeV}$$

$$\Lambda_\sigma = 5.3 \text{ fm}^{-1}$$

$$\Lambda_\pi = 1.1 \text{ fm}^{-1}$$



Solved results



# Realistic Calc. - $q\bar{q}c\bar{c}$

- Stochastic variational approach

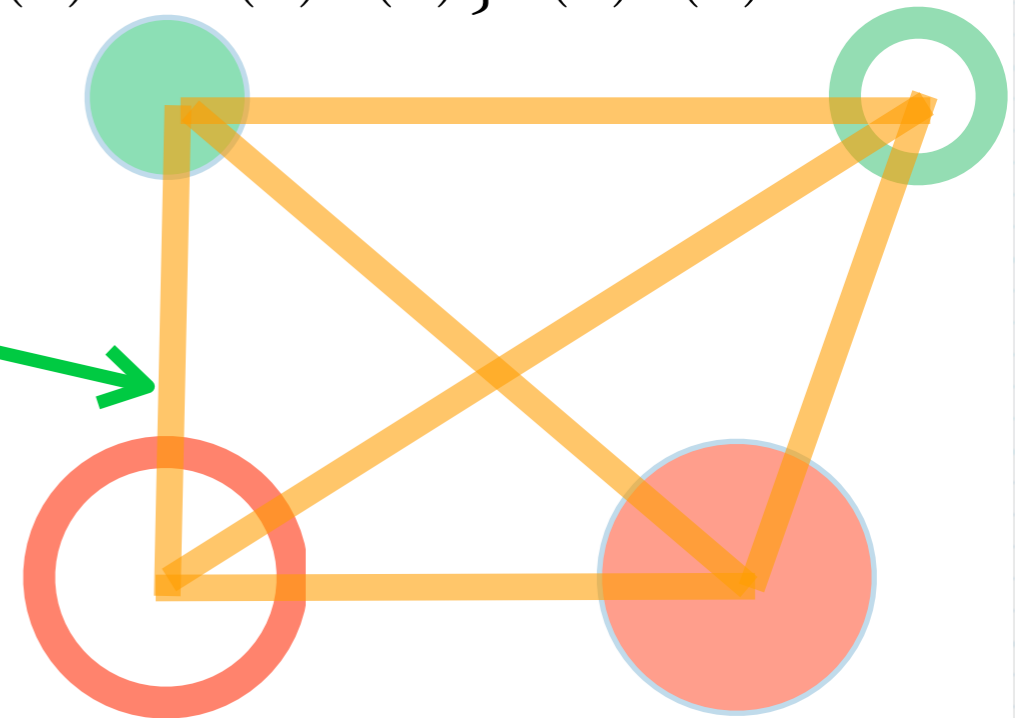
$$\Psi = \sum c_{k,m} \psi_m^c \psi^f \psi^\sigma \psi_k^{orb}$$

$$\psi_m^c = \psi^c(1)\psi^c(2)\psi^c(3)\psi^c(4), \quad \psi^c(1)\lambda^a\psi^c(2)\psi^c(3)\lambda^a\psi^c(4)$$

$$\psi^f = u(1)c(2)\bar{d}(3)\bar{c}(4), \quad \frac{1}{\sqrt{2}}\{u(1)\bar{u}(3) + d(1)\bar{d}(3)\}c(2)\bar{c}(4)$$

$$\psi_k^{orb} = \exp\left[-\sum_{i<j} \beta_{ij}^{(k)} r_{ij}^2\right]$$

$$\psi^\sigma = |(11)1\rangle \quad (J/\psi \text{ \& } \rho)$$



# Realistic Calc. - $q\bar{q}c\bar{c}$

- Binding Energy

$IJ^{PC}$	weaker meson-exch	stronger meson-exch
$11^{++}$ ( $J/\psi \rho$ )	5 MeV	26 MeV
$01^{++}$ ( $J/\psi \omega$ )	Not Bound	5 MeV

# Molecule AND diquarks ?

- Components and size

$J/\psi - \rho$  ( $1^{++}$ )  
BE=5.1 MeV

	$N$	rms	$R_{M_1}$	$R_{M_2}$	$R_{M_{12}}$	$\langle V_{CMI} \rangle$	$\langle V_{OgE}^{(a)} \rangle$	$\langle V_{Ins} + V_{Ins}^{(a)} \rangle$
$(J/\psi\rho)_{11}$	0.52	2.17	0.97	0.64	2.01	33	0	0
$(J/\psi\rho)_{88}$	0.48	1.42	1.43	1.24	0.48	-33	0	-5

c.f.  
meson size  
 $\rho$  0.89  
D 0.65  
D\* 0.73  
J/ $\psi$  0.51

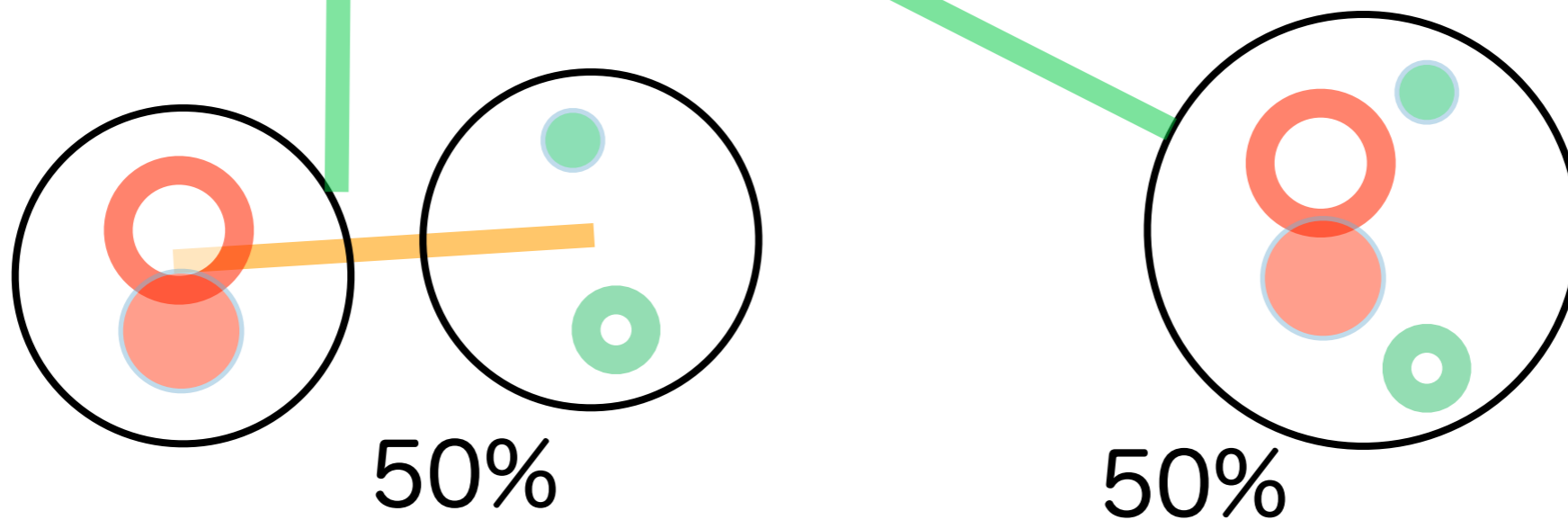


# Molecule AND diquarks ?

- Components and size

$J/\psi - \rho$  ( $1^{++}$ )  
BE=5.1 MeV

	$N$	rms	$R_{M_1}$	$R_{M_2}$	$R_{M_{12}}$	$\langle V_{\text{CMI}} \rangle$	$\langle V_{\text{OgE}}^{(a)} \rangle$	$\langle V_{\text{Ins}} + V_{\text{Ins}}^{(a)} \rangle$
$(J/\psi \rho)_{11}$	0.52	2.17	0.97	0.64	2.01	33	0	0
$(J/\psi \rho)_{88}$	0.48	1.42	1.43	1.24	0.48	-33	0	-5



c.f.  
meson size

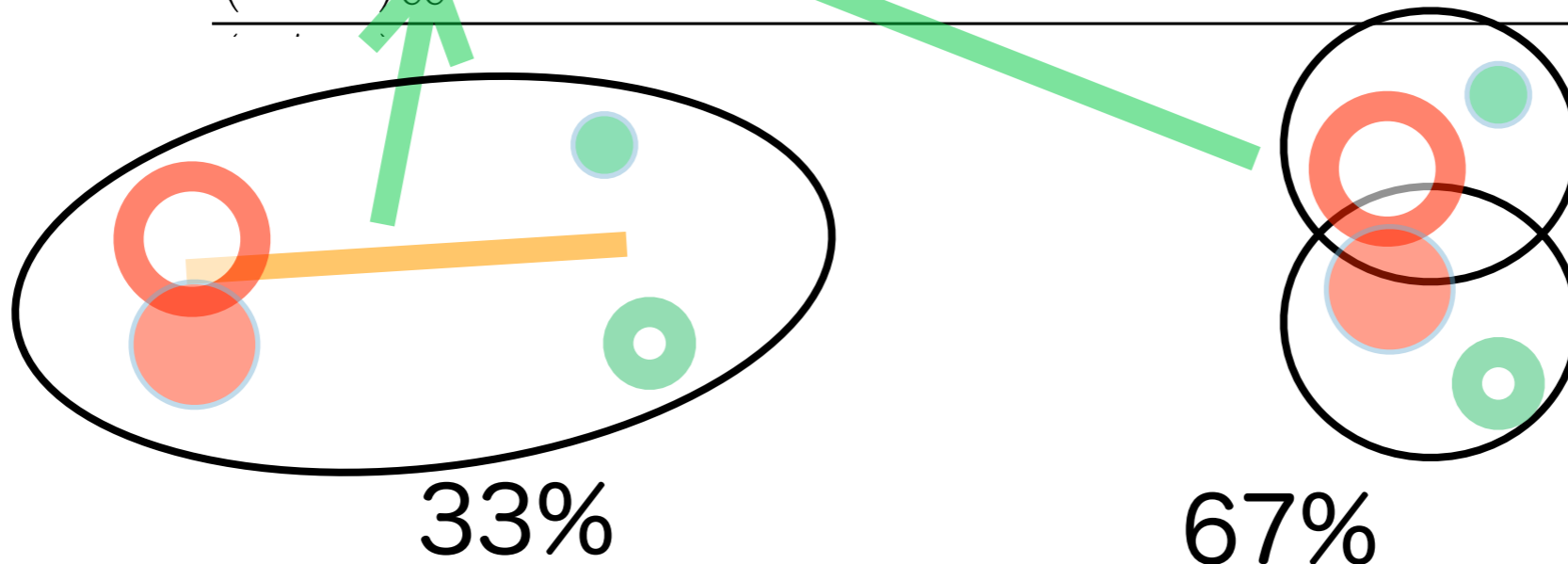
$\rho$	0.89
D	0.65
D*	0.73
J/ $\psi$	0.51

# Molecule AND diquarks ?

- Components and size

$J/\psi - \rho$  ( $1^{++}$ )  
BE=5.1 MeV

	$N$	rms	$R_{M_1}$	$R_{M_2}$	$R_{M_{12}}$	$\langle V_{\text{CMI}} \rangle$	$\langle V_{\text{OgE}}^{(a)} \rangle$	$\langle V_{\text{Ins}} + V_{\text{Ins}}^{(a)} \rangle$
$(J/\psi \rho)_{11}$	0.52	2.17	0.97	0.64	2.01	33	0	0
$(J/\psi \rho)_{88}$	0.48	1.42	1.43	1.24	0.48	-33	0	-5
$(DD^*)_{11}$	0.65	1.48	0.91	1.16	1.16	-41	0	-7
$(DD^*)_{88}$	0.35	2.39	2.33	0.52	0.52	21	0	-1



c.f.  
meson size  
 $\rho$  0.89  
D 0.65  
D\* 0.73  
J/ $\psi$  0.51

# Molecule AND diquarks ?

- Components and size

$J/\psi - \rho (1^{++})$   
BE=5.1 MeV

	$N$	rms	$R_{M_1}$	$R_{M_2}$	$R_{M_{12}}$	$\langle V_{\text{CMI}} \rangle$	$\langle V_{\text{OgE}}^{(a)} \rangle$	$\langle V_{\text{Ins}} + V_{\text{Ins}}^{(a)} \rangle$
$(J/\psi\rho)_{11}$	0.52	2.17	0.97	0.64	2.01	33	0	0
$(J/\psi\rho)_{88}$	0.48	1.42	1.43	1.24	0.48	-33	0	-5
$(DD^*)_{11}$	0.65	1.48	0.91	1.16	1.16	-41	0	-7
$(DD^*)_{88}$	0.35	2.39	2.33	0.52	0.52	21	0	-1

c.f.

meson size

$\rho$  0.89

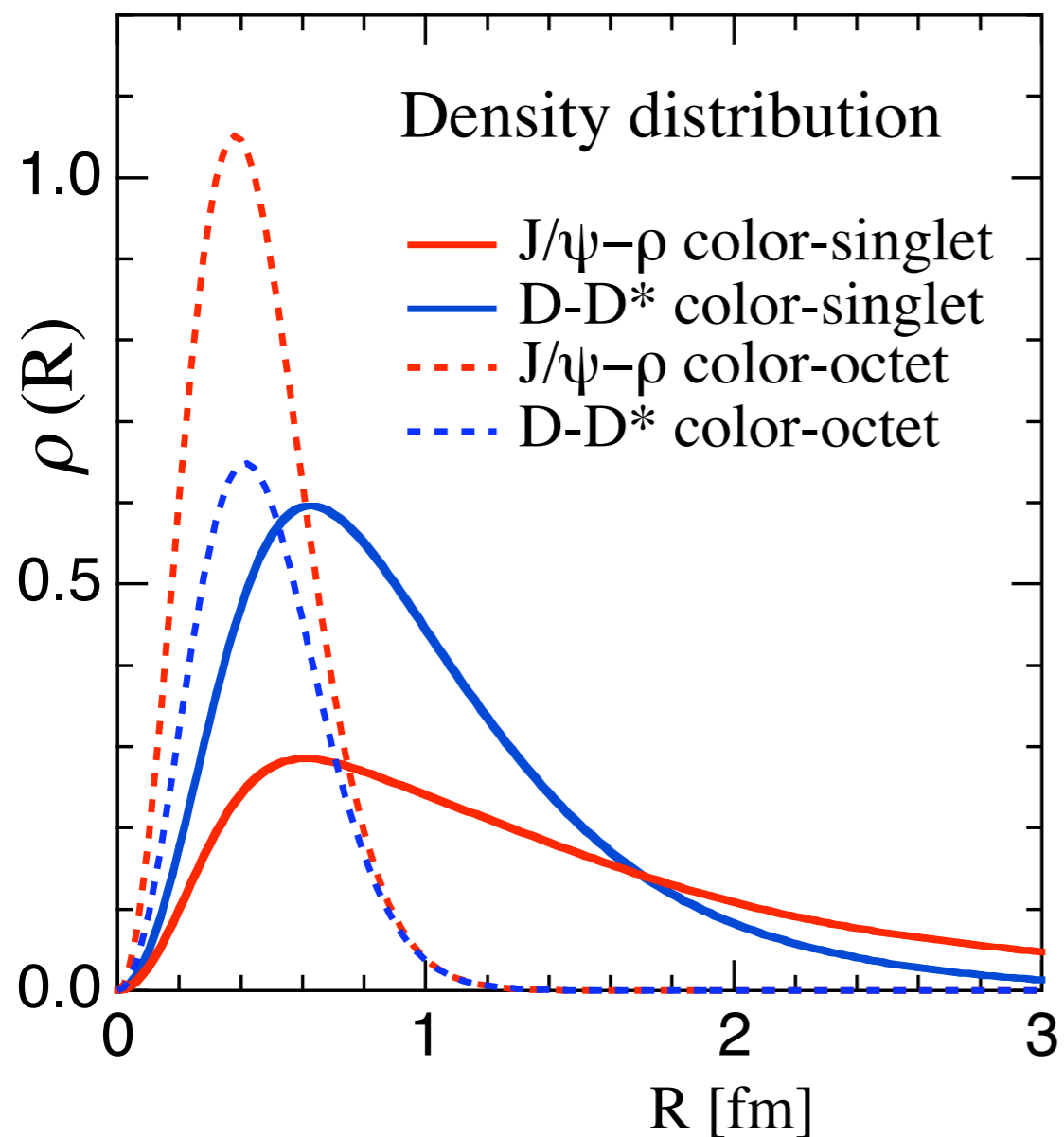
D 0.65

D\* 0.73

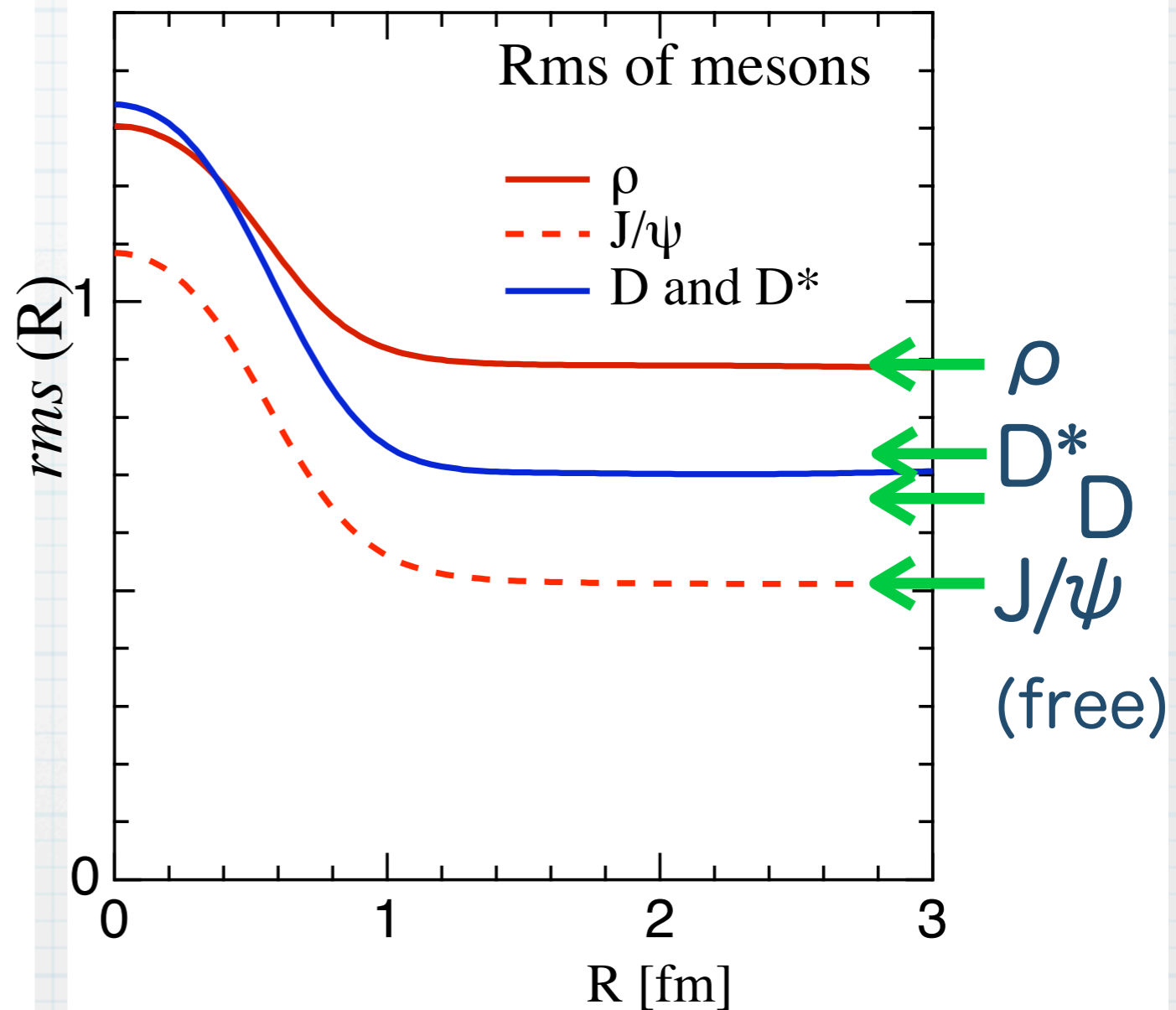
J/ $\psi$  0.51

# Density distri & rms

●  $\langle \delta (R_{mm'} - X) \rangle$

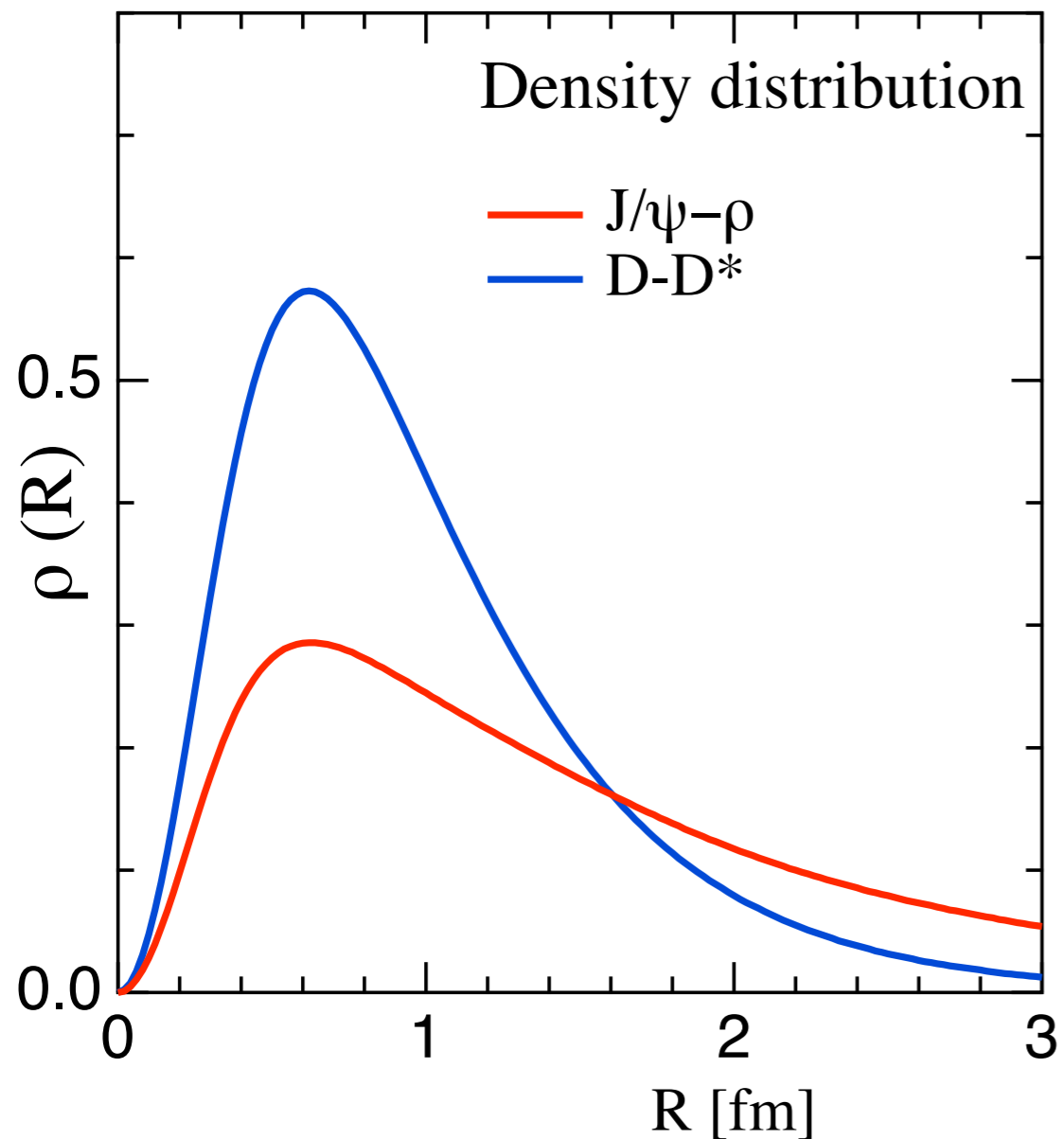


$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$

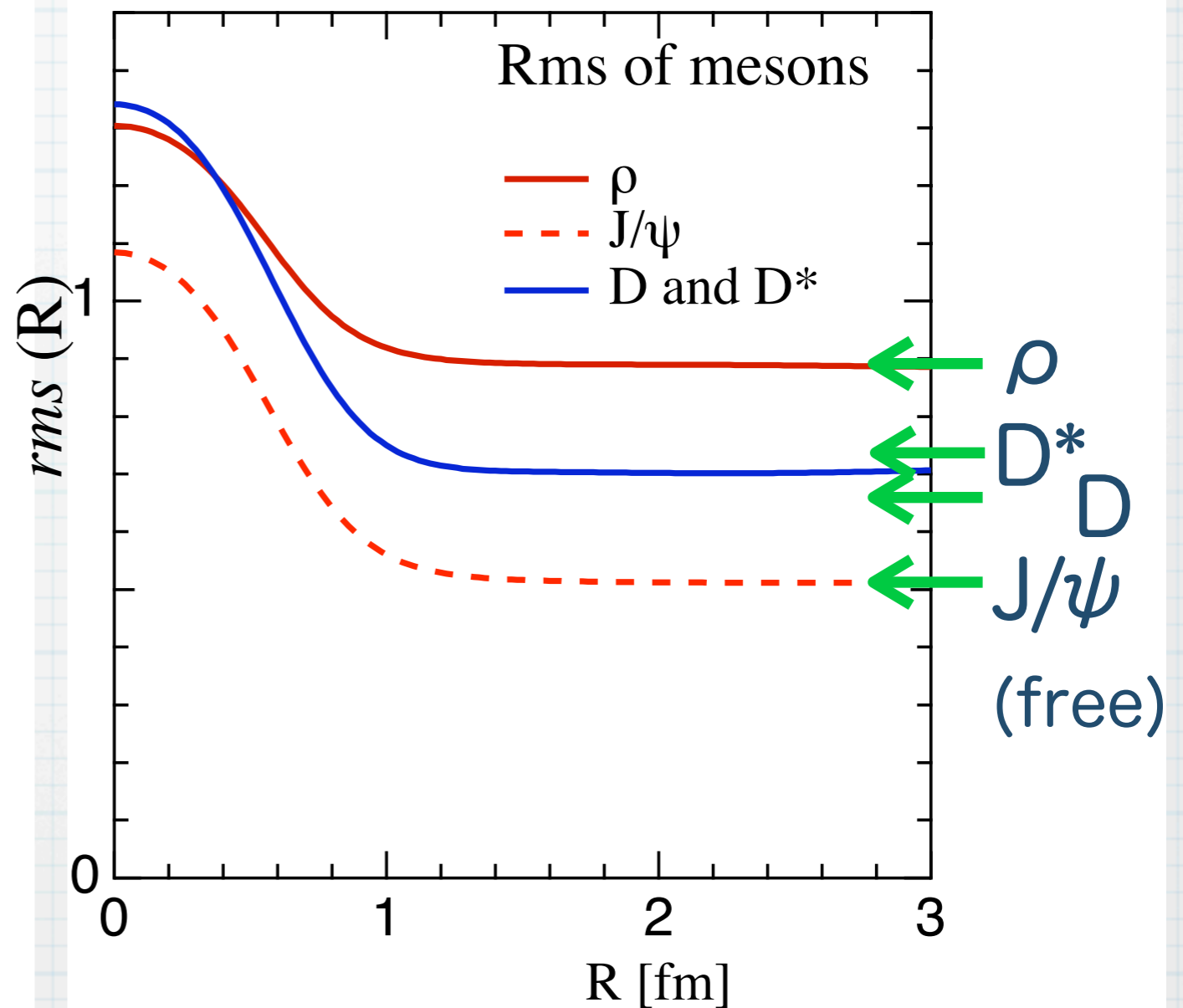


# Density distri & rms

•  $\langle \delta (R_{mm'} - X) \rangle$

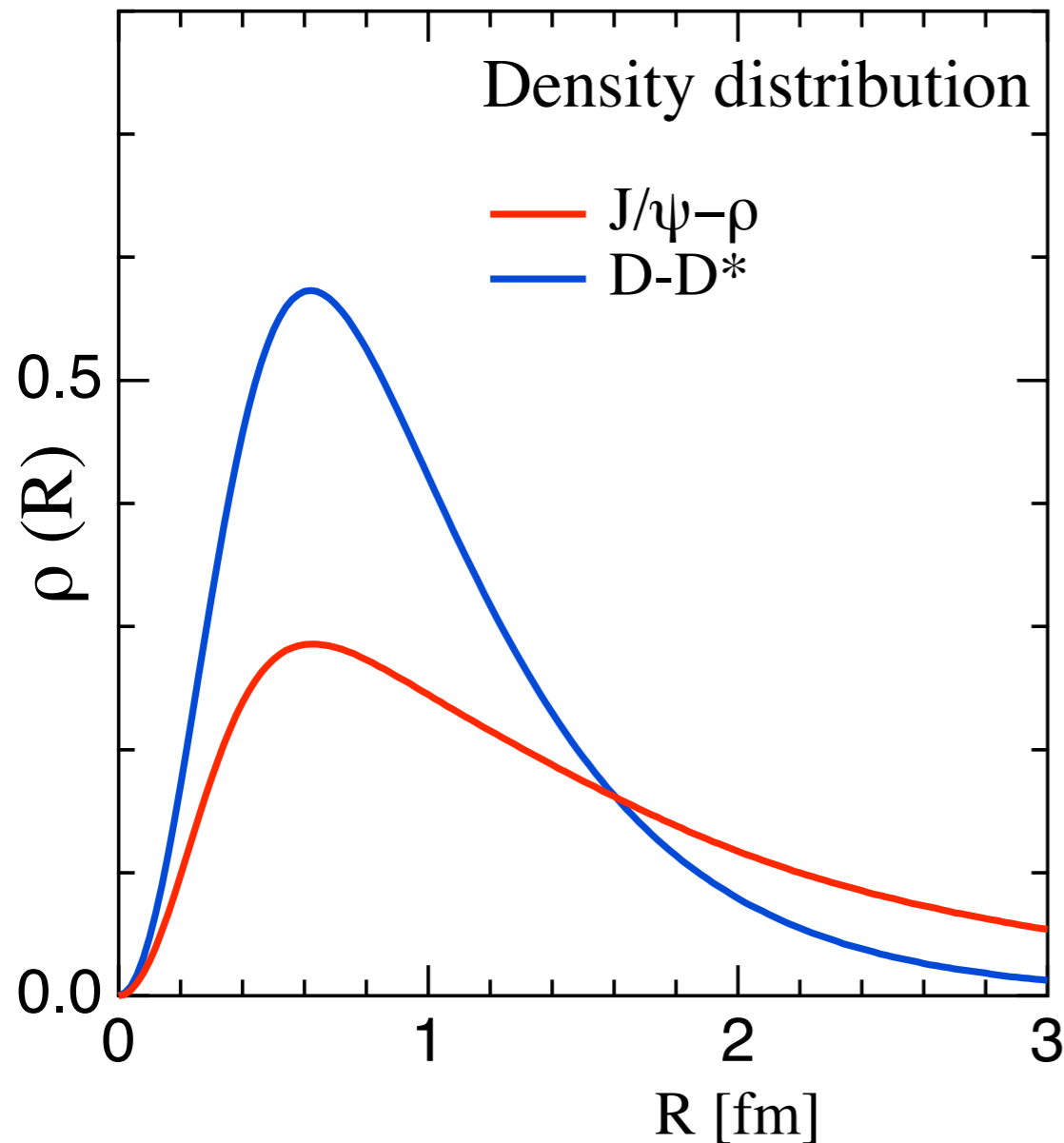


$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$

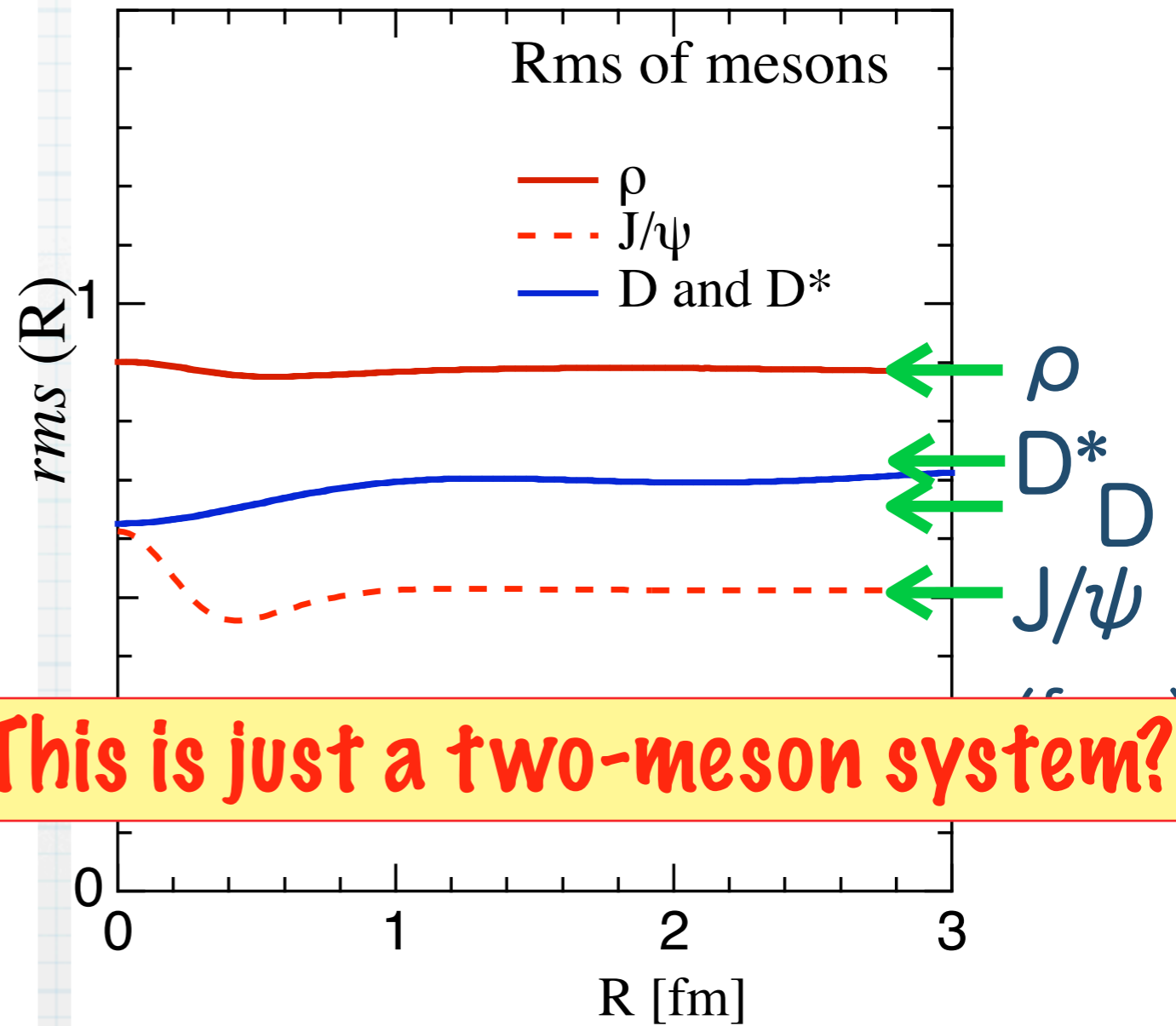


# Density distri & rms

•  $\langle \delta (R_{mm'} - X) \rangle$



$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$

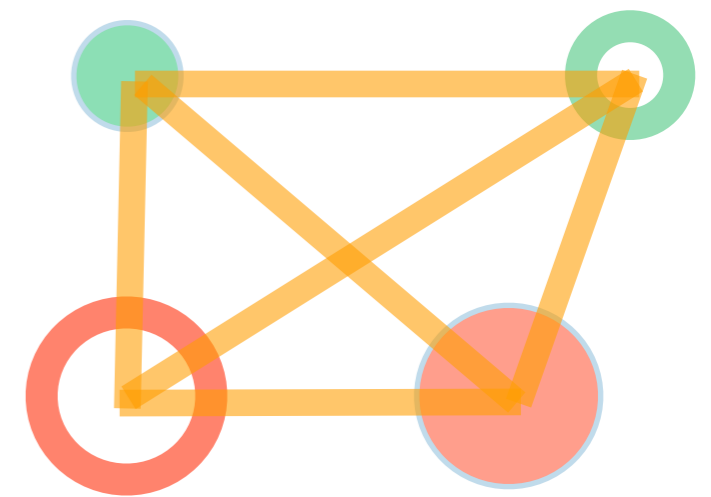
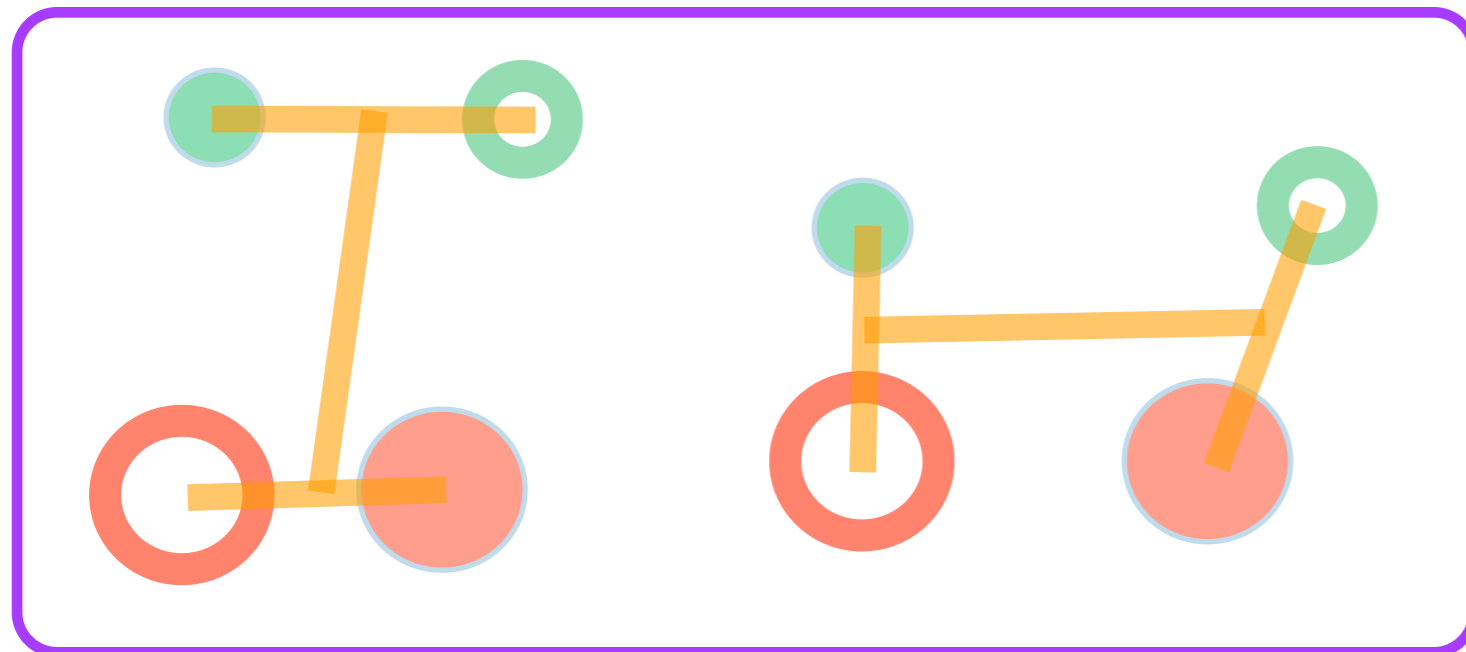


**This is just a two-meson system?**



# Effects of multiquark components

- When only correlations between  $u\bar{u}$  &  $c\bar{c}$  or  $u\bar{c}$  &  $c\bar{u}$  are included, what happens?



No correlations among more than 3 quarks  
→ two-meson-like configuration

# Effects of multiquark components

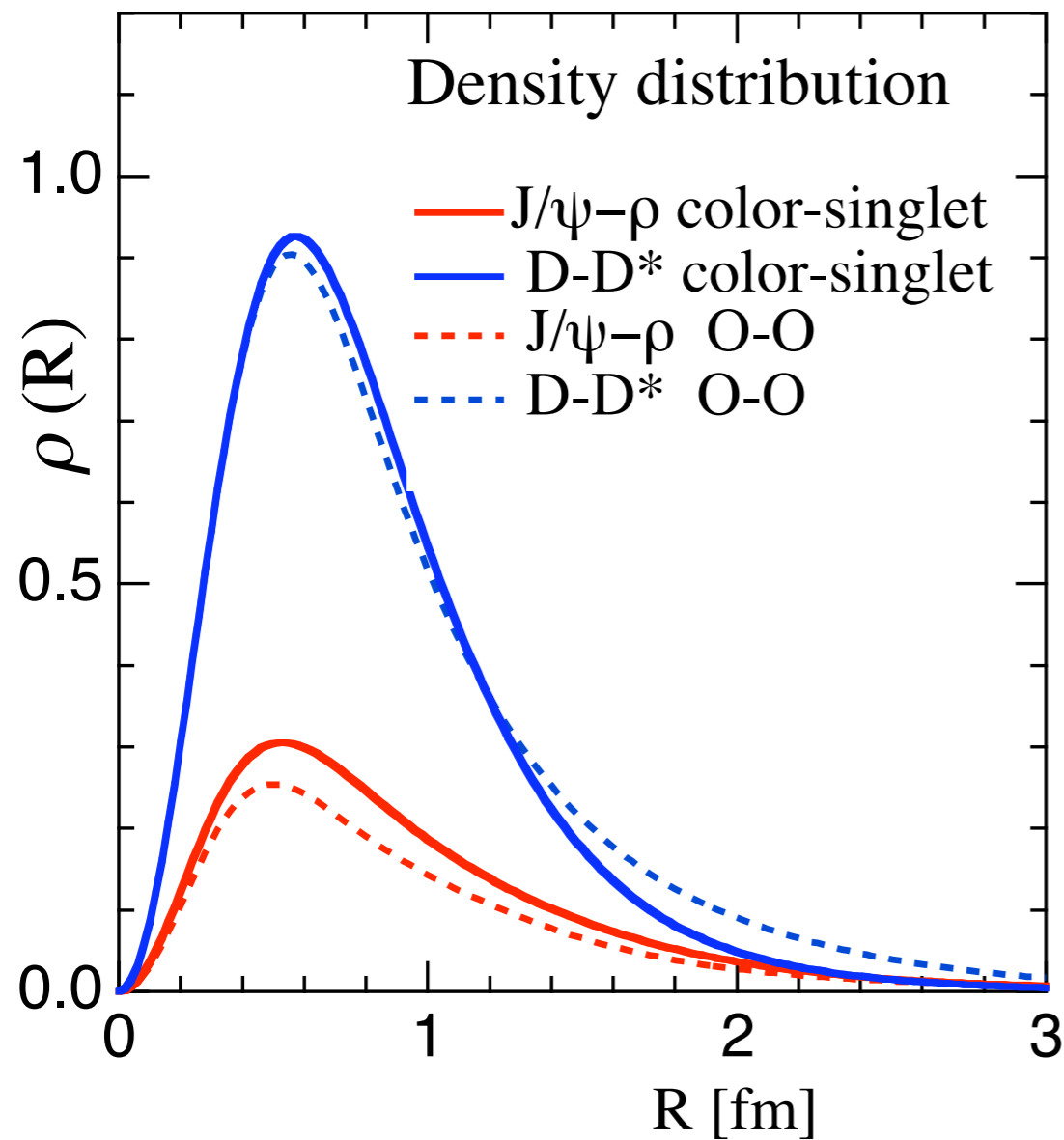
- Binding Energy

$IJ^{PC}$	weaker meson-exch	stronger meson-exch	$J/\psi \rho$	$DD^*$
$11^{++} (J/\psi \rho)$	5 MeV	26 MeV	0.33	0.85
$\bigcirc - \bigcirc$ config	Not Bound	9 MeV	0.26	0.89

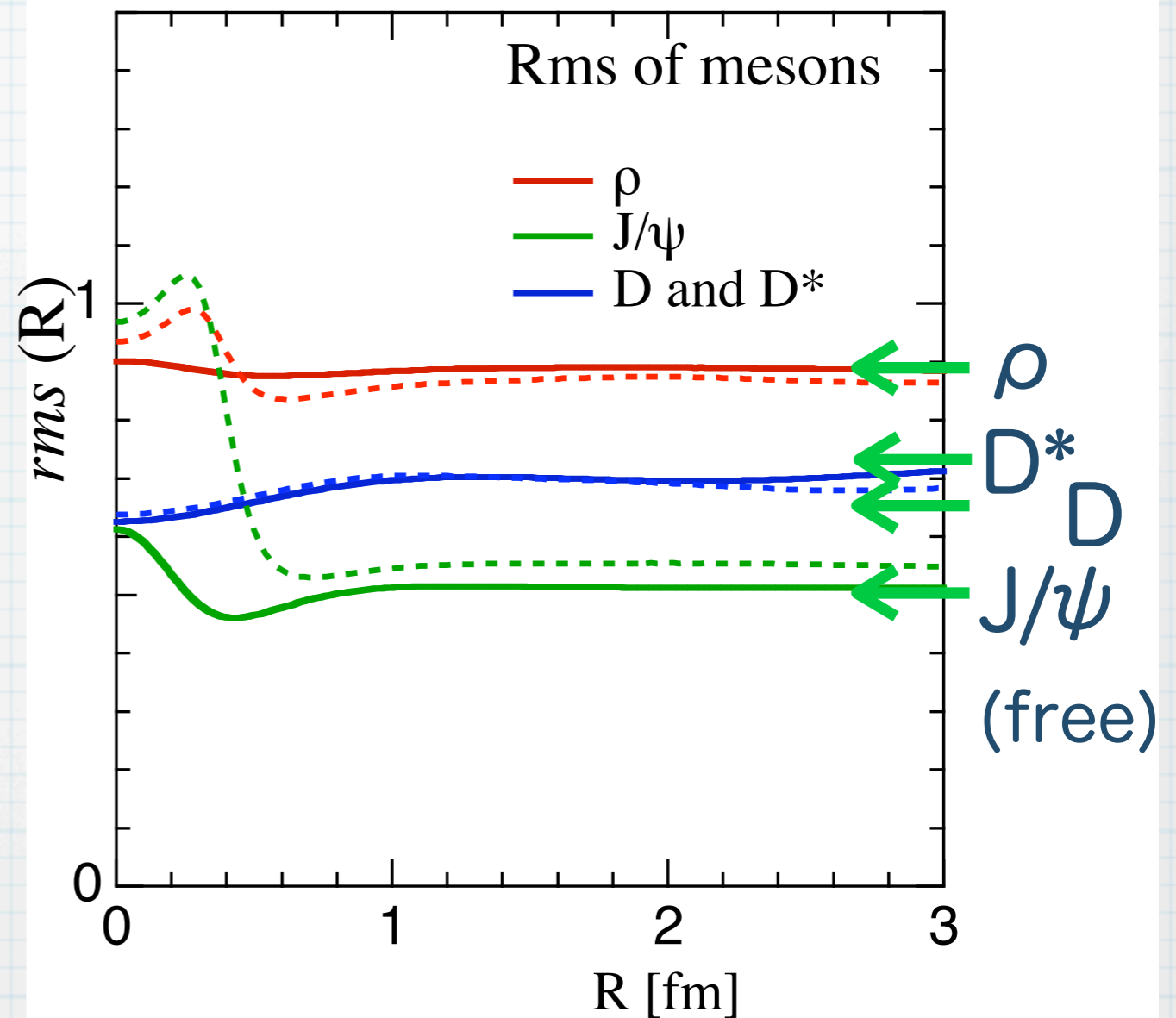
17 MeV difference: effects from correlations among more than 3 quarks

# Density distri & rms

•  $\langle \delta (R_{mm'} - X) \rangle$

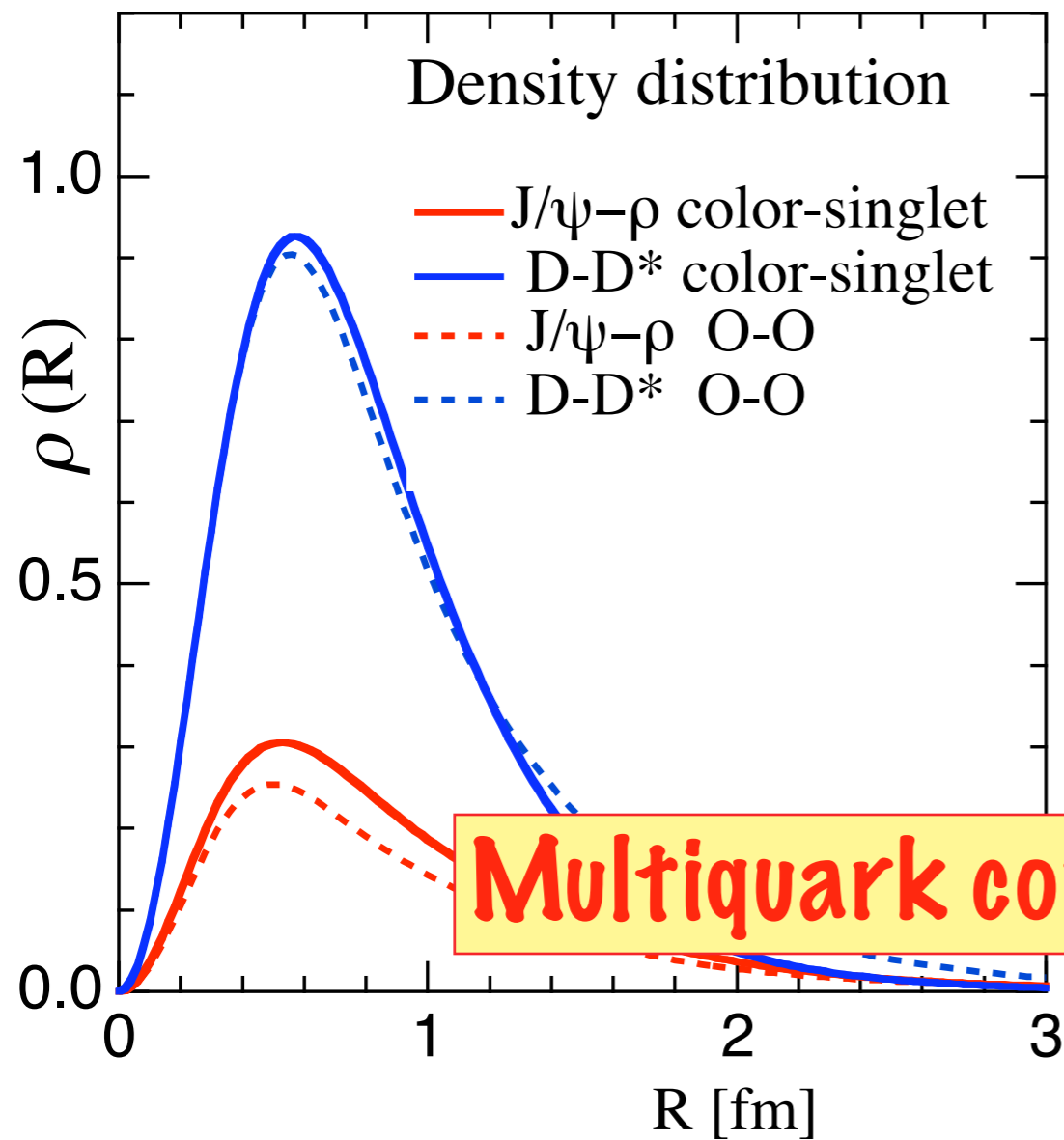


$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$

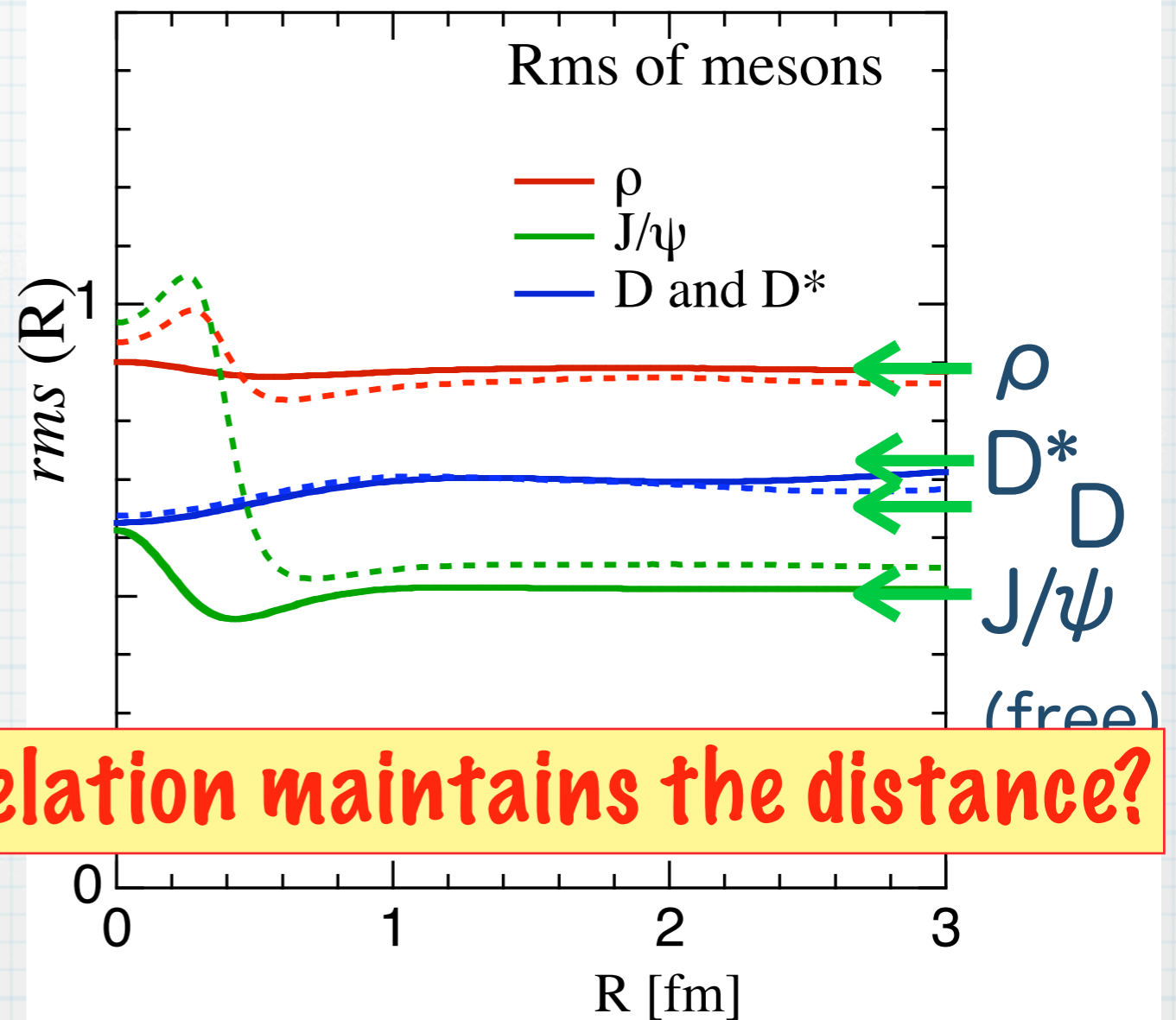


# Density distri & rms

•  $\langle \delta (R_{mm'} - X) \rangle$



$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$



**Multiquark correlation maintains the distance?**

# Summary

- $\Lambda(1405)$  is investigated as a  $(q^3 - q\bar{q}) + q^3$  pole system.
  - Only  $\Lambda(1/2^-)$  has a resonance around 1400 MeV.
- $X(3872)$  is investigated by assuming  $q\bar{q}c\bar{c}$  system.
  - $T=1$   $J^{PC}=1^{++}$  seems to become a two-meson molecule ( $J/\psi - \rho$  and  $DD^*$ ) with a sizable diquark component.

# Outlook

- ‘Multiquark component’ may be defined as multiquark correlation in the hadrons.
- It is necessary to introduce ‘Multiquark component’ ?
  - not yet investigated in the negative-parity baryon resonances.
  - Sizable effect is found in the bound state X(3872).
- LEPS2 → Baryons, light scalar mesons, reactions,...