Dispersion Matching of Stable and Radioactive Beams

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Outline

- Dispersion matching in a nutshell
- Brief summary of long history of dispersion matching
- Dispersion matching at stable beam facilities
- Dispersion matching at RI facilities
Why do we dispersion match beam lines and spectrometers?

- Resolution better than energy spread of accelerator, limited by resolving power of spectrometer $D/(M*2x_0)$
- Reconstruction of scattering angle $\Theta_{\text{target}} (\Theta_{fp})$ in dispersive plane ($x$); non-dispersive plane, angle $\phi(y)$, out-of-focus mode

What ion-optical parameters on target need to be “matched” to the spectrometer?

- Spacial Dispersion $b_{16}$, for resolution
- Angular dispersion $b_{26}$, for $\Theta_{\text{target}} (\Theta_{fp})$ reconstruction
- Focus on target $b_{12}=0$, for $k = dp/(d\Theta*p) = 0$
Spacial and Angular Dispersion Matching
A Cartoon to Remember

Achromatic Beam on Target

Dispersive Beam on Target

Angular dispersion on Target

Figure 2.2: Schematic ion trajectories under different matching conditions of a beam line

Great diagnostic for beam momentum distribution

\[ b_{16} = -\frac{s_{16}}{s_{11}}(1 + s_{11}s_{26}K - s_{21}s_{16}K) \frac{C}{T} \]

\[ b_{26} = (s_{21}s_{16} - s_{11}s_{26})C \]
Defining a RAY

Code TRANSPORT:
$(x, \Theta, y, \Phi, l, dp/p)$
$(1, 2, 3, 4, 5, 6)$

Convenient “easy to use” program for beam lines with paraxial beams

Not defined in the figure are:

$dp/p = \text{rel. momentum}$
$l = \text{beam pulse length}$

All parameters are relative to “central ray”

Code: COSY Infinity:
$(x, a, y, b, l, \delta_K, \delta_m, \delta_z)$

Needed for complex ion-optical systems including several
charge states
different masses
velocities (e.g. Wien Filter)
higher order corrections

Not defined in the figure are:

$\delta_K = dK/K = \text{rel. energy}$
$\delta_m = dm/m = \text{rel. mass}$
$\delta_z = dq/q = \text{rel. charge change}$
a = $p_x/p_0$
b = $p_y/p_0$
All parameters are relative to “central ray” properties

Note: Notations in the Literature are not consistent!
Transport of a ray

6x6 Matrix representing optic element (first order)

Note: We are not building “random” optical elements. Many matrix elements = 0 because of symmetries, e.g. mid-plane symmetry.
Transport of a ray through a system of beam line elements

$\begin{align*}
  x_n &= R_n R_{n-1} \cdots R_0 x_0 \\
\end{align*}$

Complete system is represented by

**one** Matrix $R_{\text{system}} = R_n R_{n-1} \cdots R_0$
Dispersion Matching

- High resolution experiments
- Secondary beam (large dp/p)

Fig. 1. Schematic layout of the incident particle 1 and the outgoing particle 2 relative to the beam and spectrometer.
Solution of first order Transport and Complete Matching

For best Resolution in the focal plane, minimize the coefficients of all terms in the expression of $x_{f.p.}$.

For best Angle Resolution
Minimize Coefficients of $\delta_0$ in expression of $Y_{f.p.}$.

Note: Also the beam focus $b_{12}$ on target is important ($b_{12} = 0$ for kinem. $k = 0$)

Complete Matching

Hendrie, Dispersion Matching

$b_{16} = \frac{-D}{M} \cdot \frac{C}{T}$

$D = s_{16} = $ Spectrometer dispersion

$M = s_{11} = $ Spectrometer magnification
Spacial and Angular Dispersion Matching

Solutions for $b_{16}$ and $b_{26}$ under conditions that both $\delta_0$-coefficients $= 0$ in (1) and (2)

\[
\begin{align*}
    s_{11} b_{16} T + s_{12} b_{26} + s_{16} C &= 0 \\
    s_{21} b_{16} T + s_{22} b_{26} + s_{26} C &= 0
\end{align*}
\]

Solutions:

\[
\begin{align*}
    b_{16} &= -\frac{s_{16}}{s_{11}} (1 + s_{11} s_{26} K - s_{21} s_{16} K) \frac{C}{T} \tag{19} \text{ Spacial Dispersion Matching} \\
    b_{26} &= (s_{21} s_{16} - s_{11} s_{26}) C \tag{20} \text{ Angular Dispersion Matching} \\
    b_{12} &= -\frac{s_{12} b_{22}}{s_{11} T} = \frac{s_{16} b_{22} K}{s_{11} T} \tag{21} \text{ Focusing Condition}
\end{align*}
\]
Brief History of Dispersion matching

- 1956 Early spectrometers, MIT, ND (Browne-Buechner), effects on resolution
- 1974 D.L. Hendrie, - D*C/(M*T), target functions T,C, k defined and discussed
- 1978 Big Karl, disp. matched BL, ion-optics, insufficient diag., S. Martin, K. Brown
- 1986 K600, IUCF, Disp. Matching incl. angular dispersion, improved diagnostics, k>0 matching, 0 deg measurements, angle reconstruction.
- 2000 Grand Raiden, development WS incl. all known effects and diagnostics, k=0 disp. matching. Resolv. Power limit of about p/dp =37000 at 300 – 400 MeV (p,p’)
- Grand Raiden unique (one on this planet) high Resol. facility to study (GT fine structure with 20- 30 keV at 140 MeV/u, Yoshi Fujita, (K600 E(3He) ~ 70Mev/u)
- 2008 K600, iThembaLABS (Ricky Smit, R. Neveling): Successful Int’l initiative (Japan (Hiro Fujita, Yoshi Fujita), Germany (P. von Neumann-Cosel, USA(GB) to implement dispersion matching incl. 0 deg measurements.
- 2006 T. Kawabata design of Matching for RI beam at BigRIPS/SHARAQ system.
- > 2015 Future developments of High Energy Spectrometers at RI beam facilities, e.g. FAIR, LEBS, H. Geissel, H. Weick, J. Winfield; FRIB, HRS, Remco, GB.
BIG KARL Spectrometer (Juelich, KFZ)

Bending radius $\rho_0 = 1.98 \text{ m}$
$B_{\text{max}} = 1.7 \text{ T}$
Gap = 6cm
Weight = ~ 50 tons (D1)
~ ~ 70 tons (D2)

Resolv. power: $p/\Delta p = 0 - 20600$
Dispersion = -2.0 to 26 cm/%
Magnification $M_x = 0.63 - 1.26$
Magnification $M_y = 25.4 - 1.94$
Large range: $E_{\text{min}}/E_{\text{max}} = 1.14$
Solid angle: < 12.5 msr

Fig. 9. Arrangement of the magnetic elements of the QQDDQ spectrometer BIG KARL. The central ray (optical axis) is shown as dashed curve. The outermost rays with the extreme radial distances are drawn as full lines. Four channels in the inner yokes allow NMR probes to be moved into the gaps of the dipoles for radial field measurements. The multipole element between Q1 and Q2 allows the correlation of vertical aberration.
Fig. 4. Spectra of $^{58}$Ni(p,$\alpha$) measured for different dispersions $D = 26$, 16, 6.3, 3, 1.5, 0.25 and 0.25 cm/%, The spectrograph was optimized for $D = 16$ cm/%,

Fig. 19. High resolution spectrum of the (p, d) neutron pick up reaction on $^{109}$Ag at 25 MeV incident energy and a solid angle of 1.2 msr. The resolution was 4 keV.
Dispersion matched beam line WS to the high resolution spectrometer Grand Raiden

\[ D = S_{16} = 17 \text{ cm/\%} = 17 \text{ m} \]

\[ M = S_{11} \sim - 0.45 \]

Dispersion on target:
\[ B_{16} = D/M = - 37 \text{ m} \]

Resolving power:
\[ 2x_0 = 1 \text{ mm} \]
\[ R = p/\Delta p = 37000 \]
Momentum and Angular Resolution

Spacial & Angular Dispersion Matching & Focus Condition allows

Energy Resolution: $E/\Delta E = 23000$, $p/\Delta p = 40000$, despite beam spread: $E/\Delta E = 1700 - 2500$

Angular resolution: $\Delta Y_{\text{scatt}} = \sqrt{(\Delta Y_{\text{hor}}^2 + \Delta \Phi^2)} = 4 - 8$ msr

At angles close to beam (e.g. 0 deg) vert. angle component is needed $\rightarrow$ Overfocus mode, small target dimension, because $(y|y)$ is large, Limitation: multiple scattering in detector

Grand Raiden High Resolution Spectrometer

Max. Magn. Rigidity: 5.1 Tm
Bending Radius $\rho_0$: 3.0 m
Solid Angle: 3 msr
Resolv. Power $p/dp$ 37000

Beam Line/Spectrometer fully matched

Dipole for in-plane spin component
Faraday cup for $({}^3\text{He},t)$
$B\rho(t) \sim 2*B\rho({}^3\text{He})$

Magnetic Spectrometer
Target Point
Focusing Q section
Grand-analyzer Focus
Q-lens for Angular Dispersion Matching
Intermediate Focus
Pre-analyzer Focus
Pre-analyzer section
Source Point (SP)

IUCF K600!
Diagnostic of Dispersion Matching
of beam line & spectrometer using a double strip target & multi slit

Fig. 4. Scatterplots of horizontal position $x_{fp}$ versus angle $\theta_{fp}$ and projections measured in the focal plane of the K600 using the “multi-slit system”. For details, see text.
Data suggest: Use $y_{fp}$ not $\Phi_{fp}$ to calibrate angle!

**New ion-optical mode run for R=300 cm**

- **\( \theta_{fp} - y_{fp} \) plot**
  - Projection onto $y_{fp}$ axis
    - $\Delta \phi_{tgt} = 3 - 5$ mrad ($\sigma$)

- **\( \theta_{fp} - \Phi_{fp} \) plot**
  - Projection onto $\Phi_{fp}$ axis
    - $\Delta \phi_{tgt} = 11 - 14$ mrad ($\sigma$)

Calibrated!
Scattering Angle reconstructed from focal plane measurements using complete dispersion matching techniques.

$L=0$ Angular Distributions

$\Phi(\text{target})$

$\Theta(\text{target})$

$E(^3\text{He}) = 420 \text{ MeV}$

Figure 4.4: Spectrum of $^{58}\text{Ni}(^3\text{He},^t)$ reaction. The lateral and angular dispersion matching technique and over-focus mode were applied in this experiment for high energy and scattering angle resolution. Energy resolution of about 30 keV (FWHM) was realized.

Figure 4.5: Example of angle dependence in the $^{58}\text{Ni}(^3\text{He},^t)$ spectra near 0°. Three spectra are shown for the angle ranges 0-0.8° (left), 0.8-1.4° (middle) and over 1.4° (right), respectively. The 3.54 MeV state show clearly different angular distribution from the adjacent 1+ states which are dominated at forward angle.
• QM8U
  → Control lateral dispersion

• QM9S
  → Control angular dispersion

• Lateral and angular dispersions can be controlled independently

• References
  Y. Fujita at al., NIMB 126(1997)274
  H. Fujita et al., NIMA 469(2001)55
  T. Wakasa et al., NIMA 482(2002)79

Dispersion matching for K = 0 with faint beam
High Resolution Spectrometers
Momentum Analysis

- Momentum Resolving Power

\[ R_p = \frac{(x|dp)}{(x'|x) * 2x_0} \]

\( (x|dp) = M_{16} = \text{Momentum (p) dispersion} \)

\( (x'|x) = M_{11} = \text{Magnification} \)

\( 2x_0 = \text{Target spot size} \)

- Momentum Resolution:

\[ R_p^{HO} = \frac{(x|dp)}{x_{HO}} \]

\[ x_{HO} = (x'|x) * 2x_0 + \text{Higher Orders} \]

- For High Resolution using Spectrometers (no physical separation) consider the following

- Momentum resolving power \( R_p \) has to meet the design goal (e.g. Grand Raiden: 37000, SHARAQ: 15000 for \( 2x_0 = 1 \) mm), given by science requirements.

- If beam momentum spread \( \delta p/p > 1/ R_p \) need Dispersion Matching or Beam Tracking, count rate limit \( \sim 10^6 \) p/sec, not suitable for high intensity stable beams.

- RI beam with \( \delta p/p \sim 1-3 \% \) dispersion matched beam \((-S_{16}/S_{11})\) on target too large (50 – 100 cm). Therefore, SHARAQ has several modes (achromatic, high resol. achromatic, dispersive)

- RI beams, high energies, 100 – 300 MeV/A, tracking detectors in beam line (BigRips, SHARAQ)

- Within limits (multiple-scattering in focal plane (FP) detectors) HO can be corrected using standard FP detectors \((x,x',y,y')\).
Dispersion matching modes

- Beam momentum spread $p/dp < \text{Resolving power } R_p$: Full resolution without dispersion matching, beam line achromatic mode sufficient.

- Beam momentum spread $p/dp \sim (1-10)* R_p$: Full resolution requires dispersion matching, e.g. Grand Raiden: 300 MeV p: beam $\sim 150$ keV, resolution 13 keV, 400 MeV p: beam $\sim 150$ keV, resol. 17 keV

- Secondary Radioactive Beam (RI): Beam momentum spread $p/dp > 10* R_p$: Dispersion matching with full beam is possible but typically dispersed beam on target impractically large, e.g. SHARAQ: $> 10$ cm). Mitigation: Intermediate modes with reduced beam momentum spread/intensity or reduced resolution.
SHARAQ: Pioneering spectrometer in high resolution
Dispersion Matching with RI beam.

BigRIPS: T. Kubo
Ion-optical design: T. Kawabata
Spectrometer: H. Sakai, T. Uesaka

Future projects under design:
FAIR, GSI: LEBS (Low energy buncher spectrometer)
FRIB, MSU: HRS (High rigidity spectrometer)
FIGURE 3. Dispersion-matched beam-line for the SHARAQ spectrometer. The RI beams produced at F0 are transported to the SHARAQ target through the BigRIPS beam-line and the SHARAQ beam-line. The dispersive beam transport starts from the achromatic focus at F3.
4. Check and calibration of F4–FH10 detectors
Pair of Drift Chambers at Location H10
SHARAQ, Modes of Operation

- Beam line requires special design for high-resolution spectrometer measurements with RI-beams.

- To achieve the high-resolution measurement with SHARAQ both dispersion matching and beam tracking methods are used.

- Depending on experiments, the following modes are available

<table>
<thead>
<tr>
<th>Mode</th>
<th>Dispersive</th>
<th>Achromatic (high resolution)</th>
<th>Achromatic (large acceptance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution</td>
<td>$\Delta p/p = 1/15000$ at target</td>
<td>$\Delta p/p = 1/7500$ Tracking at F6</td>
<td>$\Delta p/p = 1/1500$ Tracking at F5</td>
</tr>
<tr>
<td>Acceptance</td>
<td>$\Delta p/p = +/- 0.3%$</td>
<td>$\Delta p/p = +/- 0.3%$</td>
<td>$\Delta p/p = +/- 2%$</td>
</tr>
<tr>
<td></td>
<td>$\Delta \theta_x = +/- 10$ mr, $\Delta \theta_y = +/- 30$ mr</td>
<td>$\Delta \theta_x = +/- 10$ mr, $\Delta \theta_y = +/- 30$ mr</td>
<td>$\Delta \theta_x = +/- 20$ mr, $\Delta \theta_y = +/- 20$ mr</td>
</tr>
<tr>
<td>Hor. Target spot</td>
<td>$\sim 100$ mm</td>
<td>$\sim 30$ mm</td>
<td>$\sim 30$ mm</td>
</tr>
</tbody>
</table>
Matching condition

\[ b_{16} = -14.76 \text{ m} \quad b_{26} = 4.79 \text{ rad} \]

\[ b_{12} = 0, \text{ for } k = \frac{dp}{d\theta/p} = 0 \]

T. Kawabata et.al.
NIM B 266 (2008).

FIGURE 4. Calculated optics of the dispersive beam transport from F3 to the SHARAK target.
“High resolution” achromatic mode

\[ \Delta \theta_x = \pm 10 \text{ mr}, \quad \Delta \theta_y = \pm 30 \text{ mr}, \]
\[ \Delta x = \pm 3 \text{ mm}, \quad \Delta y = \pm 3 \text{ mm}, \]
\[ \Delta P = \pm 0.3 \% \]

Beam size < a few cm

\[ \langle x' | x \rangle = 1.56 \quad \langle x' | \theta \rangle = 0.00 \quad \langle x' | \delta \rangle = 0.00 \]
\[ \langle \theta' | x \rangle = 0.00 \quad \langle \theta' | \theta \rangle = 0.64 \quad \langle \theta' | \delta \rangle = 0.36 \]
\[ \langle y' | y \rangle = 1.36 \quad \langle y' | \phi \rangle = 0.00 \]
\[ \langle \phi' | y \rangle = 0.00 \quad \langle \phi' | \phi \rangle = 0.74 \]

Momentum acceptance is \( \pm 0.3 \% \), keeping \( \Delta p/p \) of \( \sim 1/7500 \). (F6)
“Large acceptance” achromatic mode

\[ \Delta \theta_x = \pm 20 \text{ mr, } \Delta \theta_y = \pm 20 \text{ mr, } \]
\[ \Delta x = \pm 3 \text{ mm, } \Delta y = \pm 3 \text{ mm, } \]
\[ \Delta P = \pm 2 \text{ %} \]

Momentum acceptance can be increased up to \( \pm 2 \text{ %} \).

Beam transport is different from dispersion matching mode.

same as the standard BigRIPS transport up to F5.

resolution \( \Delta p/p \sim 1/1500 \) (at F5)
Dispersion Matching of Beam Line and SHARAQ Spectrometer

250 MeV/u $^{14}$N beam, approx. 1000 events/s
Resolution

Resolution: approx. 3.5 MeV

Resolution: approx. 0.65 MeV

Resolv. Power: 0.43 MeV
End
FIGURE 2. The SHARAQ spectrometer.

TABLE 1. Specifications of the SHARAQ spectrometer.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum rigidity</td>
<td>6.8 Tm</td>
</tr>
<tr>
<td>(c.f. 9 Tm in BigRIPS)</td>
<td></td>
</tr>
<tr>
<td>Momentum resolution</td>
<td>$p/\delta p \sim 1.5 \times 10^4$</td>
</tr>
<tr>
<td>Angular resolution</td>
<td>$\sim 1$ mrad</td>
</tr>
<tr>
<td>Momentum acceptance</td>
<td>$\pm 1%$</td>
</tr>
<tr>
<td>Angular acceptance</td>
<td>$&gt; \text{a few mrad}$</td>
</tr>
<tr>
<td>Rotating angle</td>
<td>from $-2$ deg. to $+15$ deg.</td>
</tr>
</tbody>
</table>