Structures of $^{11}\text{Li}$ with tensor and pairing correlations

1. Mechanism of breaking of magic number N=8 and halo formation.
2. Charge Radii.
3. Coulomb breakup strength in $^9\text{Li}+n+n$.

Collaborators
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- Hiroshi Toki, RCNP, Osaka Univ.
- Kiyomi Ikeda, RIKEN

Informal Seminar of Theory Group @ RCNP (2006. 10.20)
Development of radioactive beams provides us with new phenomena of unstable nuclei such as the discovery of a neutron halo structure.

I. Tanihata et al.  
Nuclear Radii

|  |  |  |  |  |
|---|---|---|---|
| Z | N | S_n / S_{2n} (MeV) |  |
| 6He | 2 | 4 | 0.98 | 4He+n+n |
| 11Li | 3 | 8 | 0.3 | 11Li+n+n |
| 11Be | 4 | 7 | 0.5 | 10Be+n |
| 14Be | 4 | 10 | 1.3 | 12Be+n+n |

Stable nuclei --- 8 MeV/A

I. Tanihata et. al
PLB206(1988)592
Neutron halo / skin structure.

- Large matter radius deviated from empirical $r_0 A^{1/3}$ rule.
- Small momentum distribution of halo / skin part.
- "Core+ valence neutrons" picture — small $S_n / S_{2n}$. 

[Diagram showing stable, skin, and halo structures with density profiles for protons and neutrons]
Characteristics of $^{11}\text{Li}$

- $S_{2n} = 0.31$ MeV
- $R_m = 3.12 \pm 0.16 / 3.53 \pm 0.06$ fm  ($^9\text{Li}: 2.32 \pm 0.02$ fm)

**Halo structure**

**Borromean system**
- No bound state in any two-body subsystem

Breaking of magic number N=8
- Simon et al. (exp, PRL83)  $(1s)^2 \sim 50\%$.
- Mechanism is unclear
11Li with coupled 9Li+n+n model

• System is solved based on the RGM equation

\[ H(11Li) = H(9Li) + H_{nn} \]

\[ \Phi(11Li) = A \left\{ \sum_{i=1}^{N} \psi_i(9Li) \cdot \chi_i(nn) \right\} \]

\[ \sum_{i=1}^{N} \langle \psi_j(9Li) | H(11Li) - E | A \{ \psi_i(9Li) \cdot \chi_i(nn) \} \rangle = 0 \]

\[ \psi_i(9Li) : \text{shell model type configuration (0s+0p+sd)} \]

\[ \text{Up to 2p2h} \]

• Orthogonality Condition Model (OCM) is applied.

\[ \sum_{i=1}^{N} \left[ h_{ij}(9Li) + (T_1 + T_2 + V_{c1} + V_{c2} + V_{12} + \Lambda_{1,i} + \Lambda_{2,i}) \cdot \delta_{ij} \right] \chi_j(nn) = E \chi_i(nn) \]

\[ h_{ij}(9Li) = \langle \psi_i | H(9Li) | \psi_j \rangle : \text{Internal Hamiltonian for } 9Li \]

\[ \Lambda_i = \lambda \sum_{\alpha \in 9Li} | \phi_\alpha \rangle \langle \phi_\alpha | : \text{Projection operator to remove the Pauli-forbidden states} \]

\[ \{ \phi_\alpha \} \text{ from the relative motion between } 9Li-n \]
Expected effects of pairing and tensor correlations in $^{11}\text{Li}$

Pairing-blocking:
### $^4$He with configuration mixing ($l_{\text{max}}=5$)

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$-28.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle V_{\text{tensor}} \rangle$</td>
<td>$-51.0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$(0s_{1/2})^4$</th>
<th>$85.0%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0s_{1/2})^2_{JT}(0p_{1/2})^2_{JT}$</td>
<td>$5.0$</td>
</tr>
<tr>
<td>$JT=10$</td>
<td></td>
</tr>
<tr>
<td>$JT=01$</td>
<td>$0.3$</td>
</tr>
<tr>
<td>$(0s_{1/2})^2_{10}(1s_{1/2})(0d_{3/2})_{10}$</td>
<td>$2.4$</td>
</tr>
<tr>
<td>$(0s_{1/2})^2_{10}(0p_{3/2})(0f_{5/2})_{10}$</td>
<td>$2.0$</td>
</tr>
<tr>
<td>$(0s_{1/2})^2_{10}(0p_{1/2})(0p_{3/2})_{10}$</td>
<td>$0.9$</td>
</tr>
<tr>
<td>$P[D]$</td>
<td>$9.6$</td>
</tr>
</tbody>
</table>

$\langle T \rangle = 71.2\text{ MeV}$

$\langle V_{\text{central}} \rangle = -48.6\text{ MeV}$

C.m. excitation = 0.6 MeV

Three cases are selectively mixed:
- $0^-$ pion nature.
- Deuteron correlation $(J,T)=(1,0)$
- $\Delta L = \Delta S = 2$ property of $V_{\text{tensor}}$

4 Gaussians instead of HO
\(^9\)Li with tensor and pairing correlations

- Configuration mixing with H.O. basis function

- 0s+0p+1s0d within 2p2h excitations.

- Length parameters \( \{b_\alpha\} \) such as \( b_{0s}, b_{0p1/2}, b_{0p3/2}, \ldots \) are determined independently and variationally.

- Describe high momentum component from \( V_{\text{tensor}} \)
  (cf. CPPHF by Sugimoto et al,(NPA740) / Akaishi (NPA738))
Hamiltonian and variational equations for $^9$Li

$$H(^9\text{Li}) = \sum_{i=1}^{A} t_i - T_G + \sum_{i<j}^{A} v_{ij}, \quad v_{ij} : \text{central+tensor+LS+Coulomb}$$

$$\Phi(^9\text{Li}) = \sum_{k} C_k \psi_k \quad \psi_k : \text{shell model type configuration}$$

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0 \quad \Rightarrow \quad \frac{\partial \langle H - E \rangle}{\partial b_\alpha} = 0, \quad \frac{\partial \langle H - E \rangle}{\partial C_k} = 0$$

- Effective interaction : Akaishi force (NPA738)
  - G-matrix from AV8’ with $k_Q = 2.8 \text{ fm}^{-1}$
  - Long and intermediate ranges of $V_{\text{tensor}}$ survive.
  - Adjust $V_{\text{central}}$ to reproduce B.E. and radii of $^9\text{Li}$
Energy surface for $\{b\}$ in $^9\text{Li}$

$\langle V_{\text{Tensor}} \rangle$

$V_{\text{tensor}}$ is optimized with spatially shrunk H.O. basis.

Pairing correlation of p-shell neutrons $b_{0p_{1/2}}; b_{0p_{3/2}}$

Tensor correlation with $(0s_{1/2})^{-2}(0p_{1/2})^{2}$ excitation of p-n pair (T=0).
Superposition of two minima in $^{9}$Li

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$-44.3$</th>
<th>(exp. : $-45.3$ MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle V_{\text{tensor}} \rangle$</td>
<td>$-31.9$</td>
<td></td>
</tr>
<tr>
<td>$R_m$ (fm)</td>
<td>$2.31$</td>
<td>(exp. : $2.32 \pm 0.02$)</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$0p0h$</td>
<td>$78.5%$</td>
<td></td>
</tr>
<tr>
<td>$(0p_{3/2})^{-2}<em>1(0p</em>{1/2})^2_0$</td>
<td>$8.8$</td>
<td>Pairing correlation of n-n pair</td>
</tr>
<tr>
<td>$(0s_{1/2})^2_{JT}(0p_{1/2})^2_{JT}$</td>
<td>$6.8$</td>
<td>Tensor correlation of p-n pair</td>
</tr>
<tr>
<td>$JT=10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$JT=01$</td>
<td>$0.2$</td>
<td></td>
</tr>
<tr>
<td>$(0s_{1/2})^2_{10}(1s_{1/2})(0d_{3/2})_{10}$</td>
<td>$1.9$</td>
<td>$0^{-}$ coupling of $0s_{1/2}-0p_{1/2}$</td>
</tr>
<tr>
<td>$(0s_{1/2})^{-2}<em>{10}(0d</em>{3/2})^2_{10}$</td>
<td>$1.2$</td>
<td>• pion nature of $V_{\text{tensor}}$</td>
</tr>
</tbody>
</table>
Hamiltonian for $^{11}\text{Li}$

- $V_{9\text{Li}-n}$: folding potential
  - Same strength for s- and p-waves
  - Adjust to reproduce $S_{2n}=0.31$ MeV

- $V_{n-n}$: Argonne potential (AV8')

- Gaussian Expansion Method to describe $nn$ wave function

$^{11}\text{Li G.S. properties}$ ($S_{2n}=0.31$ MeV)

<table>
<thead>
<tr>
<th>Energy Difference ($E(s^2)-E(p^2)$) [MeV]</th>
<th>Inert Core</th>
<th>Pairing</th>
<th>Tensor + Pairing</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pairing correlation couples $(0p)^2$ and $(1s)^2$ for last $2n$. 

$P(s^2)$ in $^{11}\text{Li} [\%]$
2n correlation density in $^{11}$Li

2n density in $^{11}$Li

Talk by Y. Kikuchi (9/21PM)

\[
\langle R_{\text{ch}}^2(^{11}\text{Li}) \rangle = \langle R_{p}^2(^{11}\text{Li}) \rangle + \langle R_{\text{proton}}^2 \rangle + \frac{N}{Z} \langle R_{\text{neutron}}^2 \rangle \\
\langle R_{p}^2(^{11}\text{Li}) \rangle = \langle R_{p}^2(^{9}\text{Li}) \rangle + \left(\frac{2}{11}\right)^2 \langle R_{\text{C-2n}}^2 \rangle
\]

**Charge Radii**

\[R_{\text{C-2n}} = \begin{array}{cccc}
4.67 & 3.74 & 5.36 & 5.69 \\
\end{array}\text{[fm]}]

Expt. (Sanchez et al., PRL96(2006))
Coulomb breakup strength of $^{11}\text{Li}$

No three-body resonance

$^{11}\text{Li} \rightarrow ^9\text{Li} + n + n$

E1 strength by using the Green’s function method
+ Complex scaling method
+ Equivalent photon method
(TM et al., PRC63(2001))

- Energy resolution is considered with $\sqrt{E} = 0.17$ MeV.
Correlations in the final states of $^{11}\text{Li}$ breakup

E1 of $^{11}\text{Li}$

![Graph showing correlations in the final states of $^{11}\text{Li}$ breakup.](image)

- Full cal.
- $V_{nn}=0$
- $V_{cn}=0$
- $V_{nn}=V_{cn}=0$ (PW)
Summary

• Explicit effects of the tensor and pairing correlations in $^{11}\text{Li}$.

• $^{9}\text{Li}$ with tensor optimization
  – Spatial shrinkage of particle states
  – Specific 2p2h excitations, $0^-$ coupling for $0s_{1/2}-0p_{1/2}$.

• $^{11}\text{Li}$
  – Pauli-blocking naturally explains breaking of magicity.
  – Charge radii, Coulomb breakup strength
Characteristics of $^{10}$Li

- $0p_{1/2}$ resonance:
  \[(0p_{3/2})_{\pi}(0p_{1/2})_{\nu} \Rightarrow 1^+, 2^+\]

- $1s_{1/2}$ virtual state:
  \[(0p_{3/2})_{\pi}(1s_{1/2})_{\nu} \Rightarrow 1^-, 2^-\]

  scattering length : $-10 \sim -20$ fm


Models

1. Inert $^9$Li + n (single channel) with folding potential
2. Coupled $^9$Li + n with pairing correlation of $^9$Li

Models

- Exp. Bohlen et al., Z.Phys. A344('93)381
$^{11}$Li in a naive 3-body model

- 50% of $(1s)^2$ in G.S.
- Only s-wave potential is deep.

Three Dipole-Resonances $\rightarrow 3/2^+, 1/2^+, 5/2^+$

- Strength has two peaks: $3/2^+$ and $1/2^+, 5/2^+$ due to spin of $^9$Li$(3/2^-)$.
- $(1s_{1/2})(0p_{1/2}) = 0^- / 1^-$, $(0^-)(3/2^-) = 3/2^+$, $(1^-)(3/2^-) = 1/2^+, 3/2^+, 5/2^+$.

Motivation of this study

We would like to understand the roles of the tensor force ($V_{\text{tensor}}$) on the nuclear structure by describing tensor correlation (TC) explicitly in the model space.

- Properties of TC in $^4$He has been investigated with the shell model type approach. (PTP113(2005))
- Spectroscopy of neutron-rich nuclei with cluster model.
  - LS splitting in $^5$He and $^6$He.
  - Breaking of magic number N=8 in $^{11}$Li.
Energy surface of $^9$Li for length parameters of HO

- (a) shows $b_{0p1/2} \sim b_{0s} \times 0.5$
- (b) shows $b_{0p1/2} \sim b_{0p3/2}$

Pairing correlation of neutrons with $(0p_{3/2})^{-2}(0p_{1/2})^2$ excitation.

Two minima, (a), (b) with a common $b_{0p3/2}$ value.
Tensor-optimized shell model for $^4$He

- Configuration mixing within 2p2h excitations (PTP113, nucl-th/0607059)
- Length parameters $\{b_\alpha\}$ such as $b_{0s}, b_{0p1/2}, b_{0p3/2}, \ldots$ are determined independently and variationally.
  - Describe high momentum component from $V_{\text{tensor}}$
    (cf. CPPHF by Sugimoto et al., (NPA740) / Akaishi (NPA738))
- Extension from the previous work
  - Add high-L orbits beyond p-shell to describe $\Delta L = \Delta S = 2$ of $S_{12}$ in $V_{\text{tensor}}$ and check the convergence
    $0s + 0p \Rightarrow 0s + 0p + sd + f + K$
$^4$He with adding high-L orbits

Length parameters

<table>
<thead>
<tr>
<th>Orbit</th>
<th>$b_{nj}/b_{0s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0p_{1/2}$</td>
<td>0.65</td>
</tr>
<tr>
<td>$0p_{3/2}$</td>
<td>0.58</td>
</tr>
<tr>
<td>$1s_{1/2}$</td>
<td>0.63</td>
</tr>
<tr>
<td>$0d_{3/2}$</td>
<td>0.58</td>
</tr>
<tr>
<td>$0d_{5/2}$</td>
<td>0.53</td>
</tr>
<tr>
<td>$0f_{5/2}$</td>
<td>0.66</td>
</tr>
<tr>
<td>$0f_{7/2}$</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Solutions shows a good convergence

Higher shell effect: $16\hbar\omega$
Gaussian expansion for $^4$He

$(lj)^2$ with HO $\Rightarrow \sum_{q,q'} a_q a_{q'} (lj_q)(lj_{q'})$

$lj_q$ : Gaussian with length $b_q$

- We determine $\{a_q\}$ variationally.
- Solutions converge with 4 Gaussians
- Gaussian with narrow lengths are important

\[ b_q / b_{0s} \approx 0.5 : 0.7 \]

Gaussian expansion of particle states instead of HO for the quantitative description of the radial shrinkage.