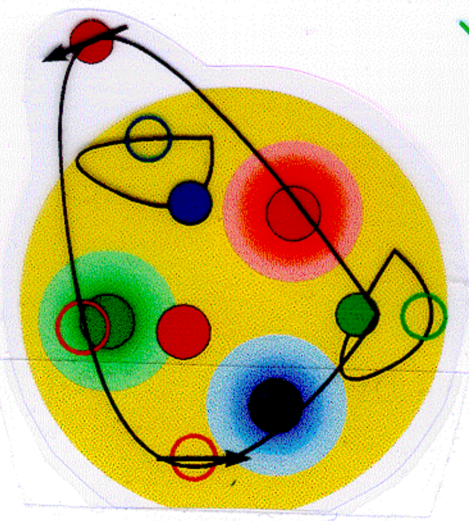
D. Drechsel

Nov 03

THE GDH SUM RULE AND THE NUCLEON'S SPIN STRUCTURE IN THE RESONANCE REGION

- anomalous magnetic moment



GDH sum rule

- form factors
  size and shape

- excitation spectrum
  pion continuum
  resonances
  Regge , DIS
  polarizabilities

BC sum rule

$$\boxed{\int_{\nu_0}^{\infty} \frac{\sigma_{3/2}(\nu)-\sigma_{1/2}(\nu)}{\nu}\, d\nu = \frac{\pi e^2 \kappa^2}{2M^2}}$$

$$\left.\begin{array}{l}\text{helicity difference}\\ \text{of photoabsorption}\end{array}\right\} \longleftrightarrow \left\{\begin{array}{l}\text{anomalous}\\ \text{magn. moment } \kappa\end{array}\right.$$

ANOMALOUS MAGNETIC MOMENTS

(Untersuchungen zur Molekularstrahlmethode aus dem Institut für physikalische Chemie der Hamburgischen Universität. Nr. 24.)

Über die magnetische Ablenkung von Wasserstoffmolekülen und das magnetische Moment des Protons. I.

Von R. Frisch und O. Stern in Hamburg.

Mit 12 Abbildungen. (Eingegangen am 27. Mai 1933.)

Strahlen aus Wasserstoffmolekülen wurden nach der Methode von Gerlach und Stern magnetisch abgelenkt und so ihr magnetisches Moment bestimmt. Die Messungen an Parawasserstoff ergaben das von der Rotation des Moleküls herrührende magnetische Moment zu etwa 1 Kernmagneton ($^1/_{1840}$ Bohrmagneton) pro Rotationsquant. Die Messungen an Orthowasserstoff ergaben das magnetische Moment des Protons zu 2 bis 3 Kernmagnetonen (nicht 1 Kernmagneton, wie bisher vermutet wurde).

- 1933 R. Frisch & O. Stern, Zs. f. Phys. 85

$$\mu_p = (2-3)\frac{e\hbar}{2m_p c}$$

- 1938 Fröhlich, Heitler, Kemmer pion cloud ?
  1946 Pauli, 1950-54 Nakabaysi et al.

$$\kappa_p = 0.45 + 5.09 \pm \ldots \neq 1.79$$

$$\kappa_n = \underset{\text{1-loop}}{-3.39} - \underset{\text{2-loop}}{2.65} \pm \ldots \neq -1.91$$

- 1960's Lipkin, Morpurgo & Becchi

  $SU(3)_{color} \otimes SU(6)_{spin-flavor}$ quarks!

  $$\kappa_p / \kappa_n = -1 \approx 0.937 \text{ (exp't)}$$

- 1980's SLAC, EMC ... 1990's SMC, SLAC, ...

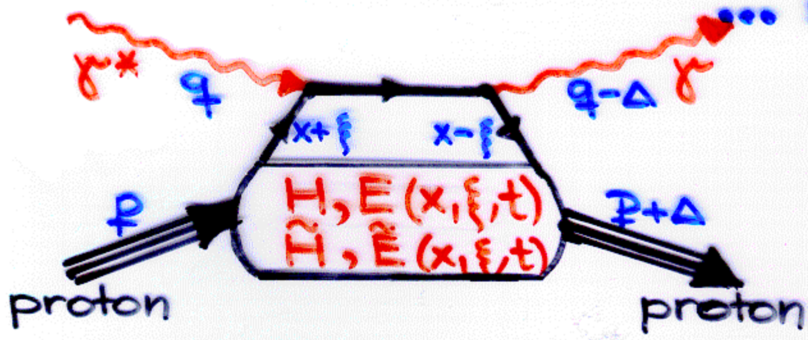
  $$\Gamma_1^p = 0.118 \pm 0.004 \pm 0.007 \neq 0.185 \text{ (Ellis-Jaffe)}$$

  only $(30 \pm 7)\%$ of spin carried by quarks!

- semi-inclusive reactions, e.g. DVCS

  2000's HERMES, COMPASS, JLAB

  ... EIC, ELFE



GPD's separate individual contributions to nucleon spin: quark, orbital, gluon

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

MAGNETIC MOMENTS FROM QCD

- plethora of "QCD inspired" models
  new developm'ts → D.O. Riska "Relativistic kinematics"
- chiral effective field theory / ChPT

$$\kappa_V = \kappa_V^0 - \frac{g_A^2 m_\pi M_N}{4\pi f_\pi^2} + f\left(m_\pi^2, \log(m_\pi/\ldots), \underbrace{M_\Delta - M_N}_{\text{"SSE"}}\right)$$

& LEC's

$$\kappa_S = \kappa_S^0 - 8 E_2 M_N m_\pi^2 + \ldots$$

counterterms
LEC's to be fixed to exp't

NO absolute prediction

BUT functional dependence on m_π

- lattice

  e.g. M. Göckeler et al., QCDSF Collaboration
  hep-lat/0303019

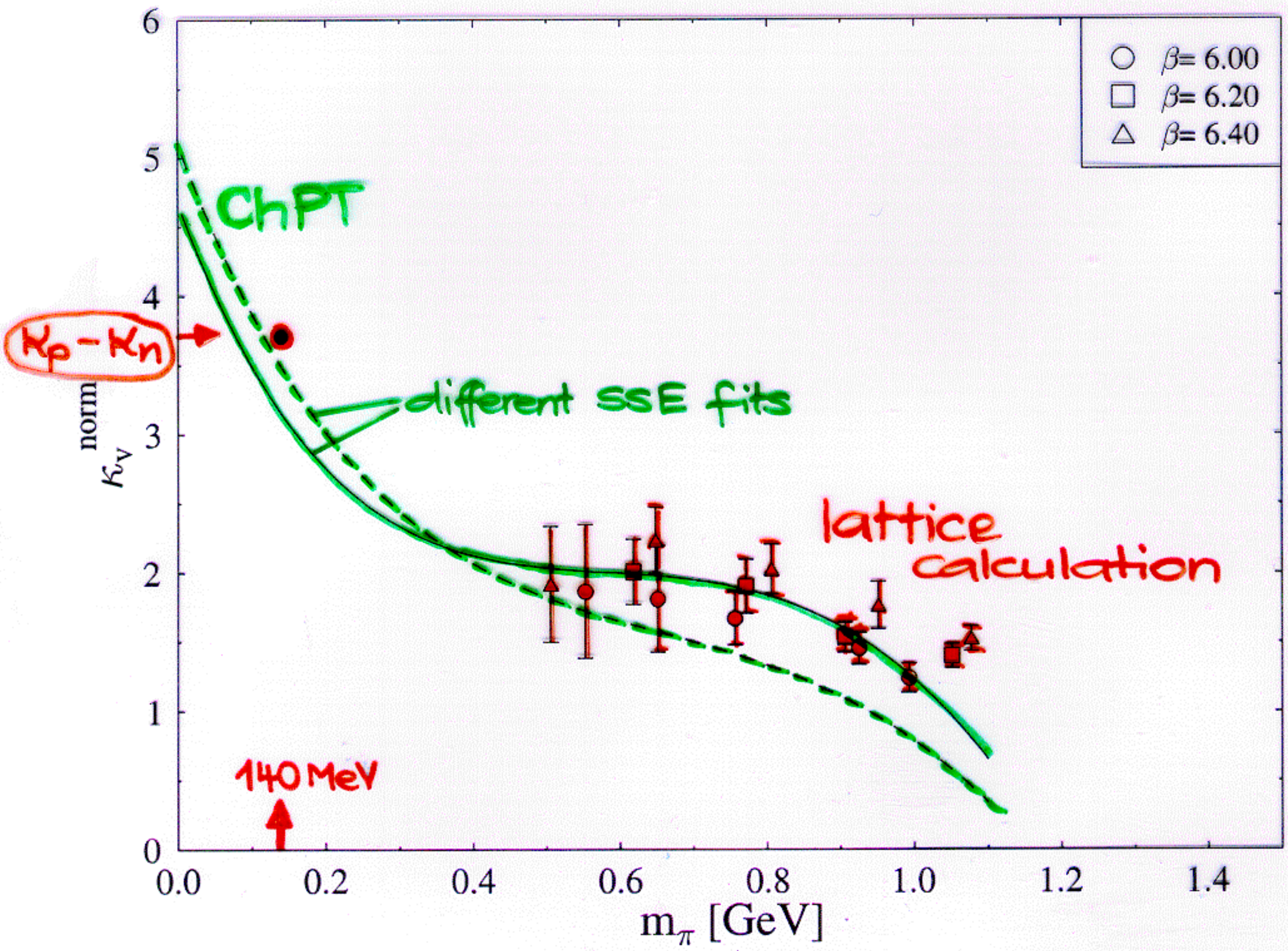
  e.m. form factors in quenched lattice QCD
  (non-perturbatively improved Wilson loops)
  extrapolation to m_π (exp't) by ChPT

  Figs.: (4) μ_V (linear fit) $= 3.7 \neq 4.71$ (exp't)
  (8) μ_V (SSE) quite different extrapolation to physical pion mass!
  (13) μ^p/μ^n (chiral extrapolation)

- "pion mass" $\gtrsim 500$ MeV still too large to see chiral effects on lattice

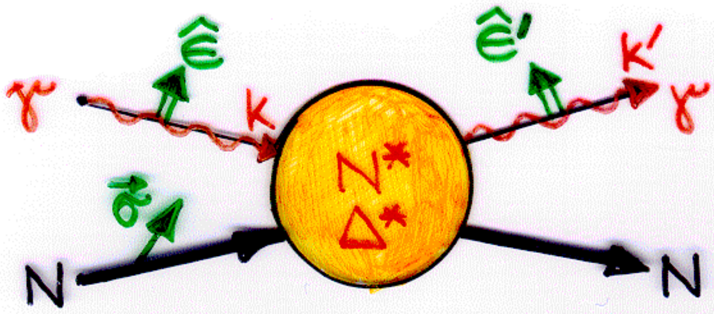
- extension of ChPT to such masses introduces more LEC's

ISOVECTOR MAGN. MOMENT



M. Göckeler et al., hep-lat/0303019
Th. Hemmert (ChPT aspects)

Compton Scattering

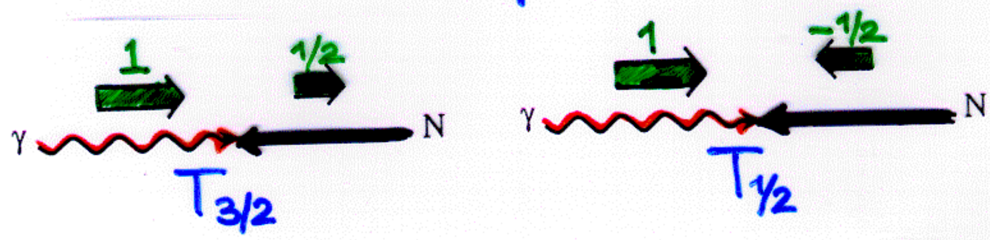


Crossing symmetry $(k \leftrightarrow -k', \epsilon \leftrightarrow \epsilon')$

photon lab energy $\nu = k_0^{lab}$

- **Forward Compton Amplitude**

$$T(\nu) = \hat{\epsilon}'\cdot\hat{\epsilon}\, f(\nu) + i\vec{\sigma}\cdot(\hat{\epsilon}'\times\hat{\epsilon})\, g(\nu)$$



$$\rightarrow f(\nu) = \tfrac{1}{2}(T_{1/2} + T_{3/2}), \quad g(\nu) = \tfrac{1}{2}(T_{1/2} - T_{3/2})$$

- **Low Energy Theorem**

Low, Gell-Mann, Goldberger (1954)

$$4\pi f(\nu) = -\boxed{\frac{e^2}{m}} + \boxed{4\pi(\alpha+\beta)}\,\nu^2 + [\nu^4] \qquad \text{even}$$

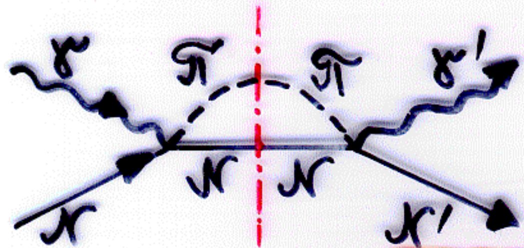
$$4\pi g(\nu) = -\boxed{\frac{e^2\kappa^2}{2m^2}}\,\nu + \boxed{4\pi\gamma_0}\,\nu^3 + [\nu^5] \qquad \text{odd}$$

global properties (e, κ, m) — internal properties (polarizabilities)

- LET = Xing + gauge inv. + Lorentz inv.

DISPERSION RELATIONS

- causality /analyticity, unitarity/opt. theorem, crossing



$$\mathrm{Im}\, T_{\gamma\gamma'} \sim |T_{\gamma\pi}|^2 \sim \sigma_{\gamma\pi}$$

"optical theorem"

$$4\pi f(\nu) = -\boxed{\frac{e^2}{m}} + \boxed{\frac{1}{\pi}\int_{\nu_0}^{\infty} \frac{\sigma_{1/2}+\sigma_{3/2}}{\nu'^2}\, d\nu'} \cdot \nu^2 + [\nu^4]$$

$$4\pi\, g(\nu) = \boxed{\frac{1}{\pi}\int_{\nu_0}^{\infty} \frac{\sigma_{1/2}-\sigma_{3/2}}{\nu'}\, d\nu'} \cdot \nu + \boxed{\frac{1}{\pi}\int_{\nu_0}^{\infty} \frac{\sigma_{1/2}-\sigma_{3/2}}{\nu'^3}\, d\nu'} \cdot \nu^3 + [\nu^5]$$

- LET + DR = SUM RULES

Baldin (1960)

$$\boxed{\alpha + \beta = \frac{1}{4\pi^2}\int_{\nu_0}^{\infty} d\nu\, \frac{\sigma_{1/2}(\nu)+\sigma_{3/2}(\nu)}{\nu^2}}$$

GDH (1966)

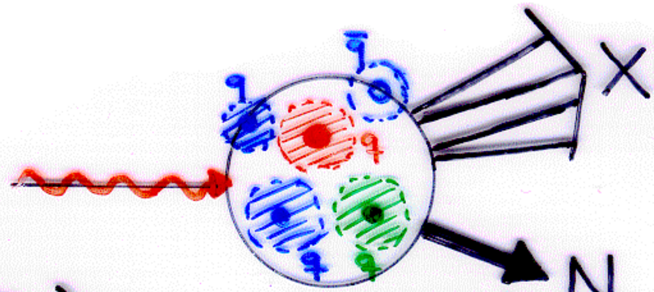
$$\boxed{-\frac{\pi e^2 \kappa^2}{2m^2} = \int_{\nu_0}^{\infty} d\nu\, \frac{\sigma_{1/2}(\nu)-\sigma_{3/2}(\nu)}{\nu}}$$

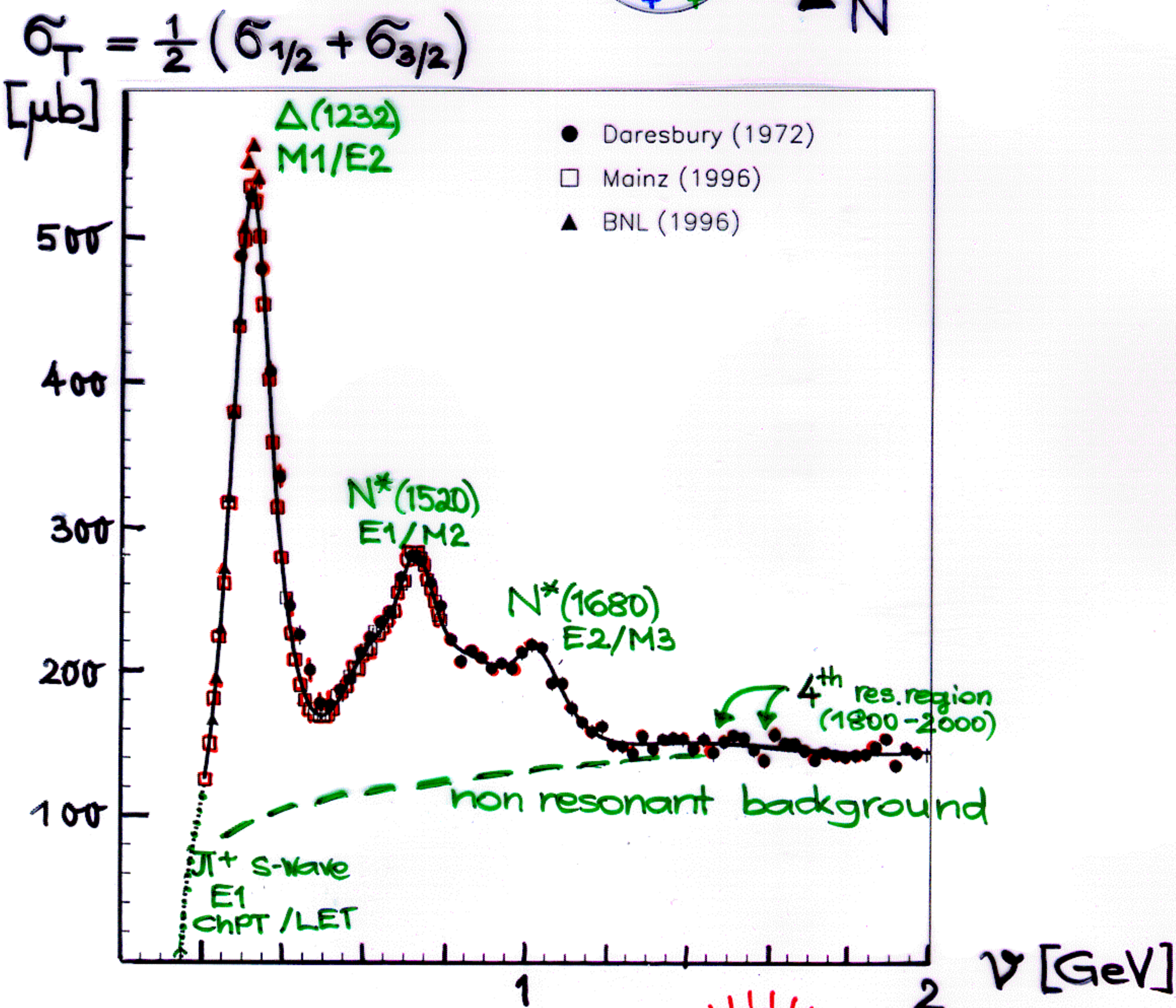
FSP (1954)
Gell-Mann/Goldberger/Thirring

$$\boxed{\gamma_0 = \frac{1}{4\pi^2}\int_{\nu_0}^{\infty} d\nu\, \frac{\sigma_{1/2}(\nu)-\sigma_{3/2}(\nu)}{\nu^3}}$$

- Compton scattering: incoming photon deforms the system, outgoing photon analyzes system in the presence of e.m. field
- In general 6 structure functions (2 scalar + 4 vector) forward ($\Theta = 0$): scalar $f \to \alpha+\beta$, vector $g \to \gamma_0$

TOTAL PHOTOABSORPTION



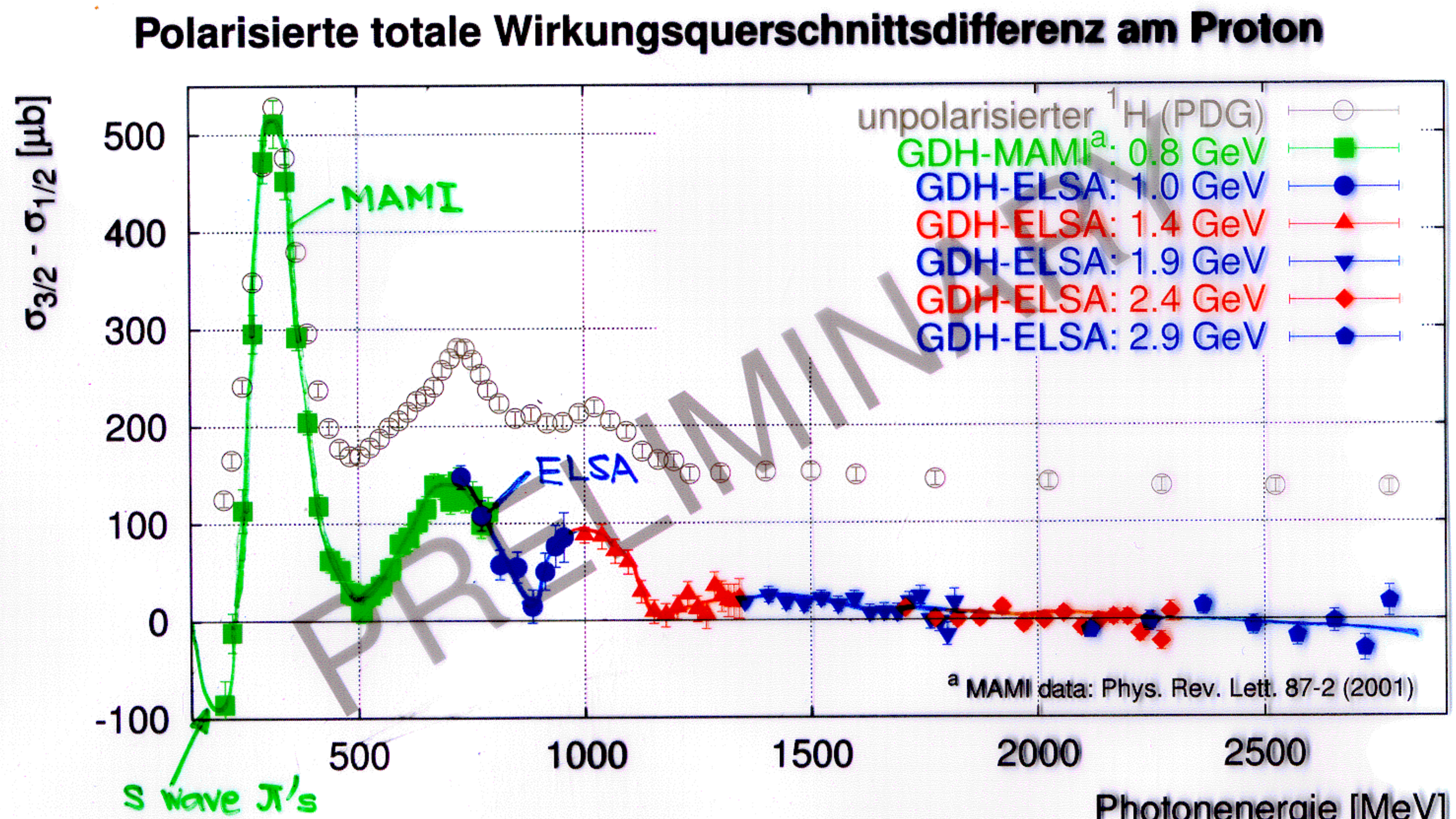


$$-\frac{e^2}{m} \neq \frac{2}{\pi}\int_{\nu_0}^{\infty} \sigma_T(\nu)\, d\nu$$

$$\alpha+\beta = \frac{1}{2\pi^2}\int_{\nu_0}^{\infty} \frac{\sigma_T(\nu)}{\nu^2}\, d\nu$$

Baldin ✓

$\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)$ [μb]



GDH FOR PROTON

$$\int_{\nu_0}^{\infty} \frac{\sigma^p_{3/2} - \sigma^p_{1/2}}{\nu} d\nu \overset{?}{=} 204\,\mu b$$

	[μb]	
threshold region (150 – 200)	-27.5 ± 3.0	LET / ChPT DR, MAID, SAID
1^{st} and 2^{nd} res. (200 – 800)	$+226 \pm 5 \pm 12$	GDH @ MAMI
2^{nd} res. – 4^{th} res. (800 – 2800)	$+28.5 \pm 2.1 \pm 1.2$	GDH @ ELSA
$\nu_0 \leq \nu \leq 2.8$ GeV	227 ± 14	
? Regge asymptotics	-13.5 ± 5	Bianchi, Thomas (1999) Simula (2002) Helbing (2003)

$$I^p_{GDH} = \underline{213.5 \pm 15}\ \mu b\ ✓$$

- SLAC E159 $\nu = 5 - 40$ GeV check Regge!
- JLab "quasireal", a few GeV @ small Q^2

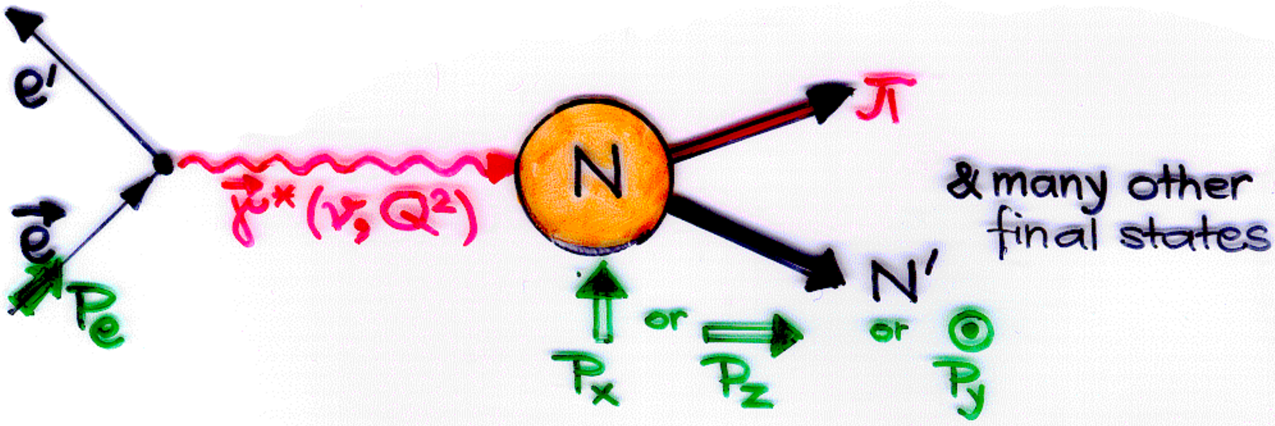
NEUTRON PUZZLE

1π component : $p \to 180$, $n \to 140$ -40
2π " about the same
Regge asymptotics : sign change $+25$

$$I^n_{GDH} \lesssim 200\,\mu b < 233\,\mu b \text{ (sum rule)}$$

- data from GDH @ ELSA + MAMI forthcoming
- neutron in d, ^3He : mixing of nuclear and nucleonic excitations. Further studies at LEGS, TUNL : study helicity structure below π threshold final state interactions effect multipole analysis ?! coherent π production, Δ induced nucleon break-up

VIRTUAL PHOTONS



$$\frac{d^5\sigma}{d\Omega_{e'}\, dE_{e'}\, d\Omega_\pi} = \Gamma_V \frac{d\sigma^V}{d\Omega_\pi} \;, \quad \Gamma_V = \frac{\alpha_{em}}{2\pi^2}\frac{E'}{E}\frac{K}{Q^2}\frac{1}{1-\epsilon}$$

$$\frac{d\sigma^V}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \varepsilon \frac{d\sigma_L}{d\Omega_\pi} + \varepsilon \frac{d\sigma_{TT}}{d\Omega_\pi}\cos 2\phi_\pi + \sqrt{2\varepsilon(\varepsilon+1)}\,\frac{d\sigma_{LT}}{d\Omega_\pi}\cos\phi_\pi$$

+ 14 other structures involving $\vec{P}_e$ and $\vec{P}_N$

- super "Rosenbluth plot" by varying $\varepsilon, \phi_\pi, \vec{P}_e, \vec{P}_N$
- $\frac{d\sigma_i}{d\Omega_\pi} = f_i(\nu, Q^2, \theta_\pi) \rightarrow$ angular distribution yields multipoles $\mathcal{M}^I_{\ell\pm}(\nu, Q^2)$
- $\sum\limits_{\text{channels}} \int d\Omega_\pi \ldots \rightarrow$ inclusive (e,e')

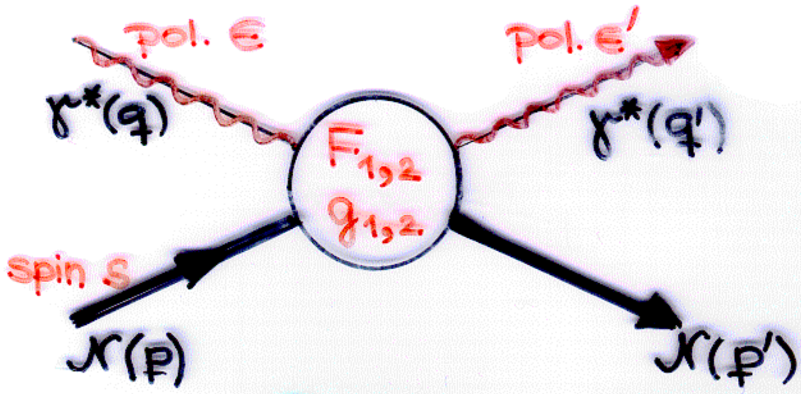
$$\frac{d^3\sigma}{d\Omega_{e'}\, dE_{e'}} = \Gamma_V \Big(\sigma_T + \varepsilon\sigma_L + hP_z\sqrt{1-\varepsilon^2}\,\sigma'_{TT} + hP_x\sqrt{2\varepsilon(1-\varepsilon)}\,\sigma'_{LT} \Big)$$

$$\boxed{\{\sigma_T, \sigma_L, \sigma'_{TT}, \sigma'_{LT}\} \Leftrightarrow \{F_1, F_2, g_1, g_2\}}$$

partial c.s. $\qquad$ nucleon structure functions

$\sigma_i(\nu, Q^2) \qquad x = \frac{Q^2}{2m\nu} \qquad g_i(x, Q^2) \xrightarrow[Q^2, \nu \to \infty]{} g_i(x)$

VVCS



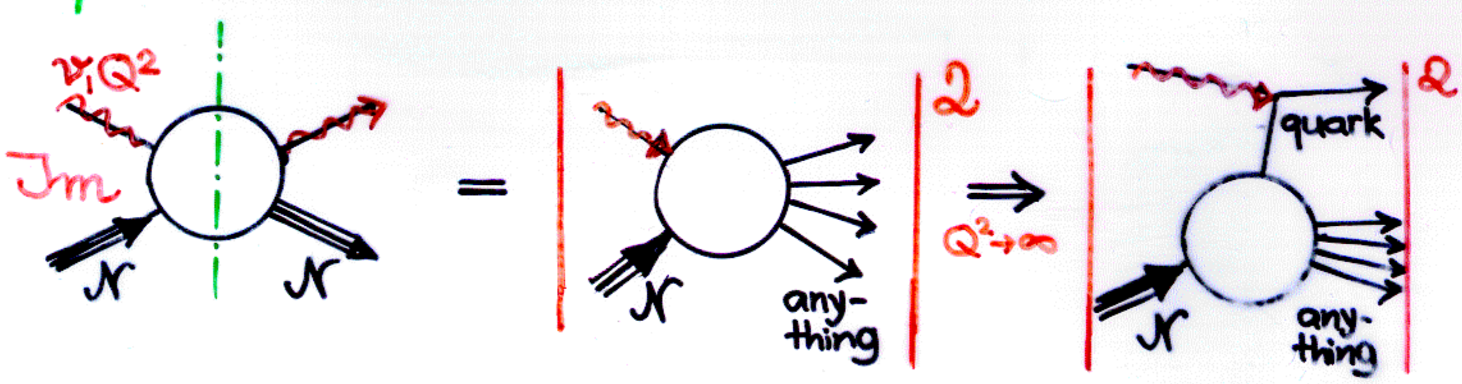
generalized GDH-integrals & polariz. to be analyzed in framework of double VCS: X. Ji and collab.

forward scattering

$$T(\nu, Q^2, \theta = 0) = \epsilon'^{*}_{\mu} \epsilon_{\nu}$$

$$\otimes \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{1}{p \cdot q}(p^\mu - \ldots)(p^\nu - \ldots) T_2 + \frac{i}{M} \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha \left(s^\beta S_1 + \frac{p \cdot q\, s^\beta - s \cdot q\, p^\beta}{M^2} S_2\right) \right\}$$

optical theorem

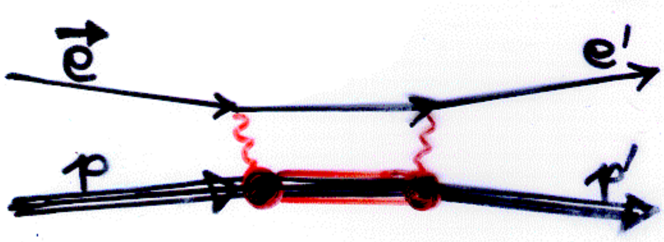


$$\mathrm{Im}\, T_1 = \frac{K}{4\pi}\sigma_T(\nu, Q^2) = \frac{e^2}{4M} F_1(x, Q^2)$$

$$\mathrm{Im}\, T_2 = \frac{\nu Q^2 K}{M(\nu^2 + Q^2) 4\pi}(\sigma_T + \sigma_L) = \frac{e^2}{4M} F_2$$

$$\mathrm{Im}\, S_1 = \frac{\nu M K}{(\nu^2 + Q^2) 4\pi}\left(\sigma'_{TT} + \frac{Q}{\nu}\sigma'_{LT}\right) = \frac{e^2}{4M}\frac{M}{\nu} g_1$$

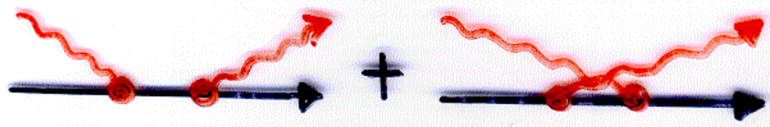
$$\mathrm{Im}\, S_2 = -\frac{M^2 K}{(\nu^2 + Q^2) 4\pi}\left(\sigma'_{TT} - \frac{\nu}{Q}\sigma'_{LT}\right) = \frac{e^2}{4M}\frac{M^2}{\nu^2} g_2$$



single spin asymmetry
e.m. forbidden to 1st order
parity allowed

DR FOR VVCS

$$\mathrm{Re}\, S_1(\nu, Q^2) = \mathrm{Re}\, S_1^{pole}(\nu, Q^2) + \frac{2}{\pi} \mathcal{P} \int_{\nu_0}^{\infty} \frac{\nu' \,\mathrm{Im}\, S_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'$$


+ excited intermed. states

$$S_1^{Born}(\nu, Q^2) = -\frac{\alpha_{em}}{2M} F_P^2(Q^2) - \frac{\alpha_{em}}{2M} \frac{Q^2}{\nu^2 - \nu_B^2 + i\varepsilon} F_D(Q^2)\left(F_D(Q^2) + F_P(Q^2)\right)$$

regular, real, $\Rightarrow \kappa^2$ for real γ

2 poles (s & u channel)
complex
vanishes for real γ

- Born term (pole term) is not a continuous function of Q^2: $\lim\limits_{\nu \to 0} \lim\limits_{Q^2 \to 0} \neq \lim\limits_{Q^2 \to 0} \lim\limits_{\nu \to 0}$ [X. Ji et al.]
- inelastic contribution is continuous $f(Q^2)$
  $\rightarrow$ generalization of GDH & related integrals
- continuation of quark structure functions to small Q^2, $g_1(x) \longrightarrow g_1(x, Q^2)$, requires to include the elastic contribution.
  For $Q^2 \to 0$, the elastic contributions dominate.

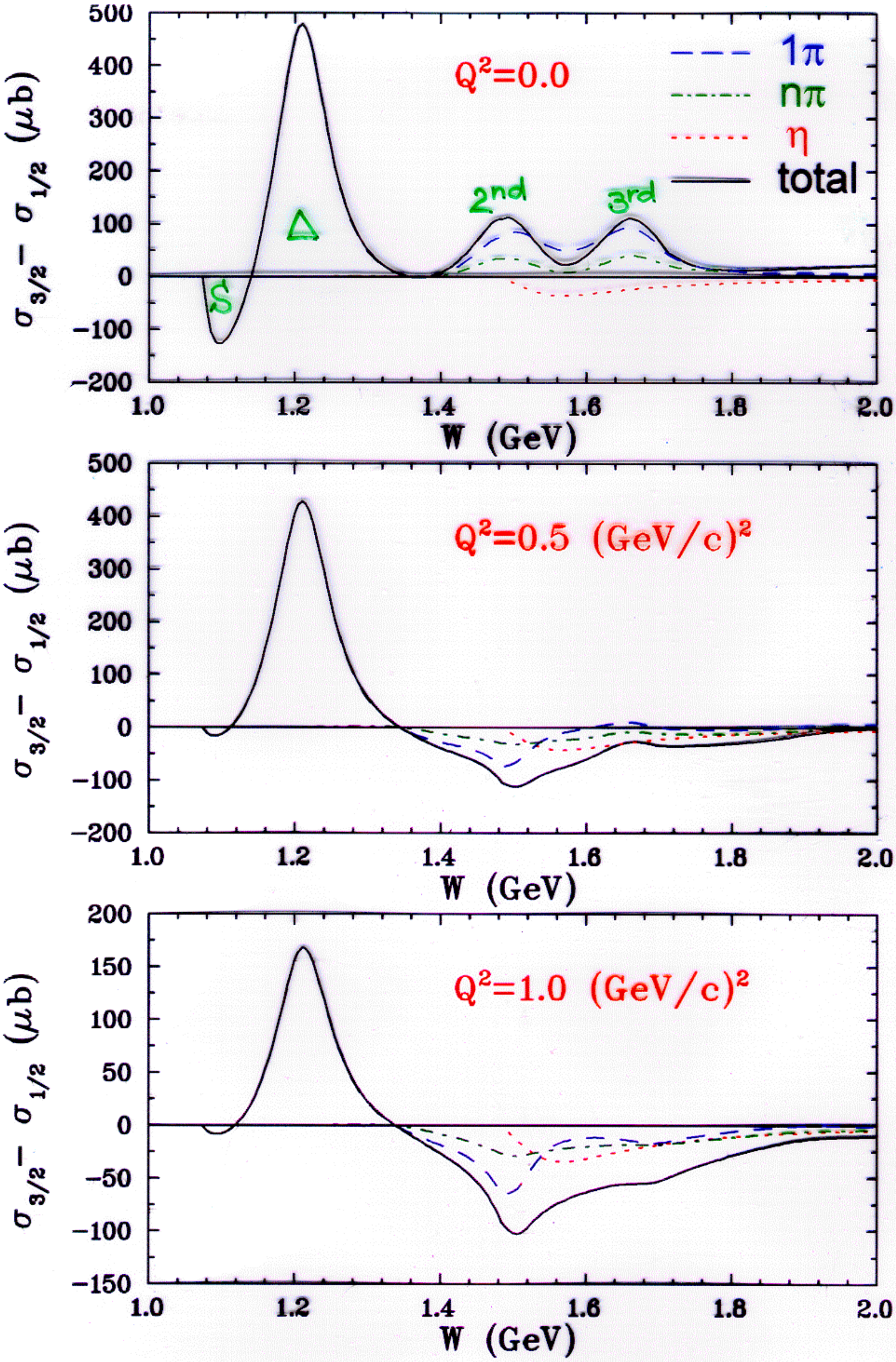
$$\Gamma_1^{el} = \int g_1^{el}(x, Q^2)\, dx = \frac{G_M}{2} \frac{G_E + \tau G_M}{1 + \tau} = \begin{cases} \frac{e_N \mu_N}{2}, & Q^2 \to 0 \\ \sim Q^{-10}, & Q^2 \to \infty \end{cases}$$

$$\Gamma_1^{inel} = \frac{Q^2}{8\pi^2 \alpha_{em}} \int_{\nu_0}^{\infty} \frac{K(\nu, Q^2)}{\nu^2 + Q^2} \left\{ \sigma_{TT}^{inel} + \frac{Q}{\nu} \sigma_{LT}^{inel} \right\} d\nu$$

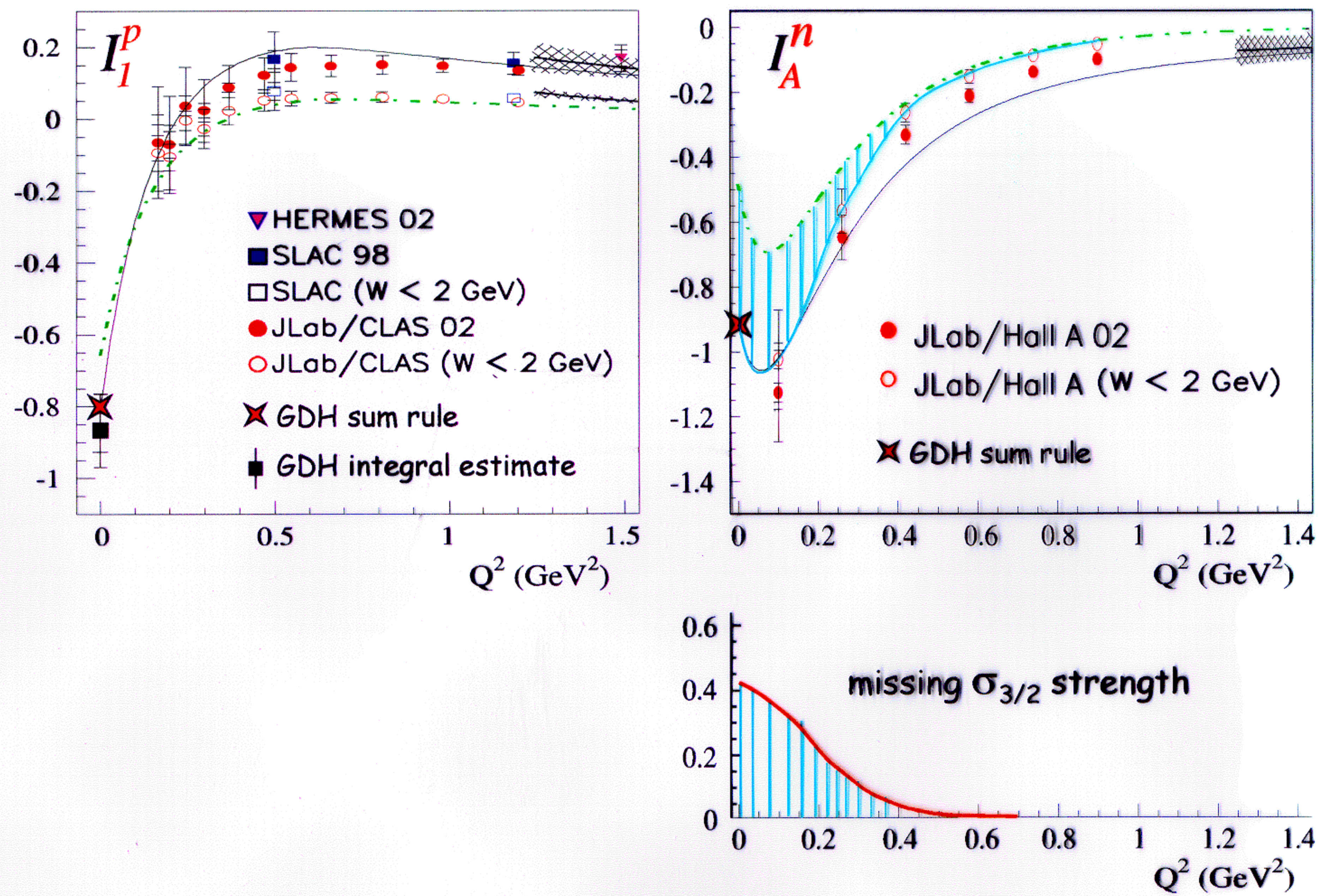
$$= \begin{cases} 0.118 \pm 0.004 \pm 0.007, & Q^2 = 5\,\mathrm{GeV}^2 \ (\mathrm{E155}) \\ \frac{Q^2}{8\pi^2 \alpha_{em}} \cdot 204\, \mu b, & Q^2 \to 0 \ (\mathrm{GDH}) \end{cases}$$

good resolution and understanding of radiative tail necessary at large ν, small Q^2

PROTON : $\sigma_{3/2} - \sigma_{1/2}$



generalized GDH sum rules for proton and neutron



• BURKHARDT - COTTINGHAM $I_2(Q^2)$

S_2 odd in ν, unsubtracted DR

ν | $$\mathrm{Re}\, S_2(\nu, Q^2) = \frac{2\nu}{\pi} \int_0^\infty \frac{\mathrm{Im}\, S_2(\nu', Q^2)}{\nu'^2 - \nu^2}\, d\nu' = \text{el.} + \text{inel.}$$

• if $S_2 \to \nu^{\alpha_2}$, $\alpha_2 < -1$ for large ν,
also DR for νS_2 (even in ν) to be questioned!

-1 | $$\mathrm{Re}\left[\nu S_2(\nu, Q^2)\right] = \frac{2}{\pi} \int_0^\infty \frac{\nu'^2\, \mathrm{Im}\, S_2(\nu', Q^2)}{\nu'^2 - \nu^2}\, d\nu'$$

ADD

$$0 = \frac{2}{\pi} \int \frac{\nu^2 - \nu'^2}{\nu'^2 - \nu^2}\, \mathrm{Im}\, S_2\, d\nu'$$

SUPERCONVERGENCE

$$0 = \int_0^\infty \mathrm{Im}\, S_2(\nu', Q^2)\, d\nu'$$

$$\to 0 = \int_0^1 g_2(x, Q^2)\, dx = \text{el.} + \text{inel.}$$

$$\to \boxed{I_2 = \frac{2M^2}{Q^2} \int_0^{x_0} g_2(x, Q^2)\, dx = \frac{1}{4} F_P(Q^2)\, G_M(Q^2)}$$

BC SUM RULE

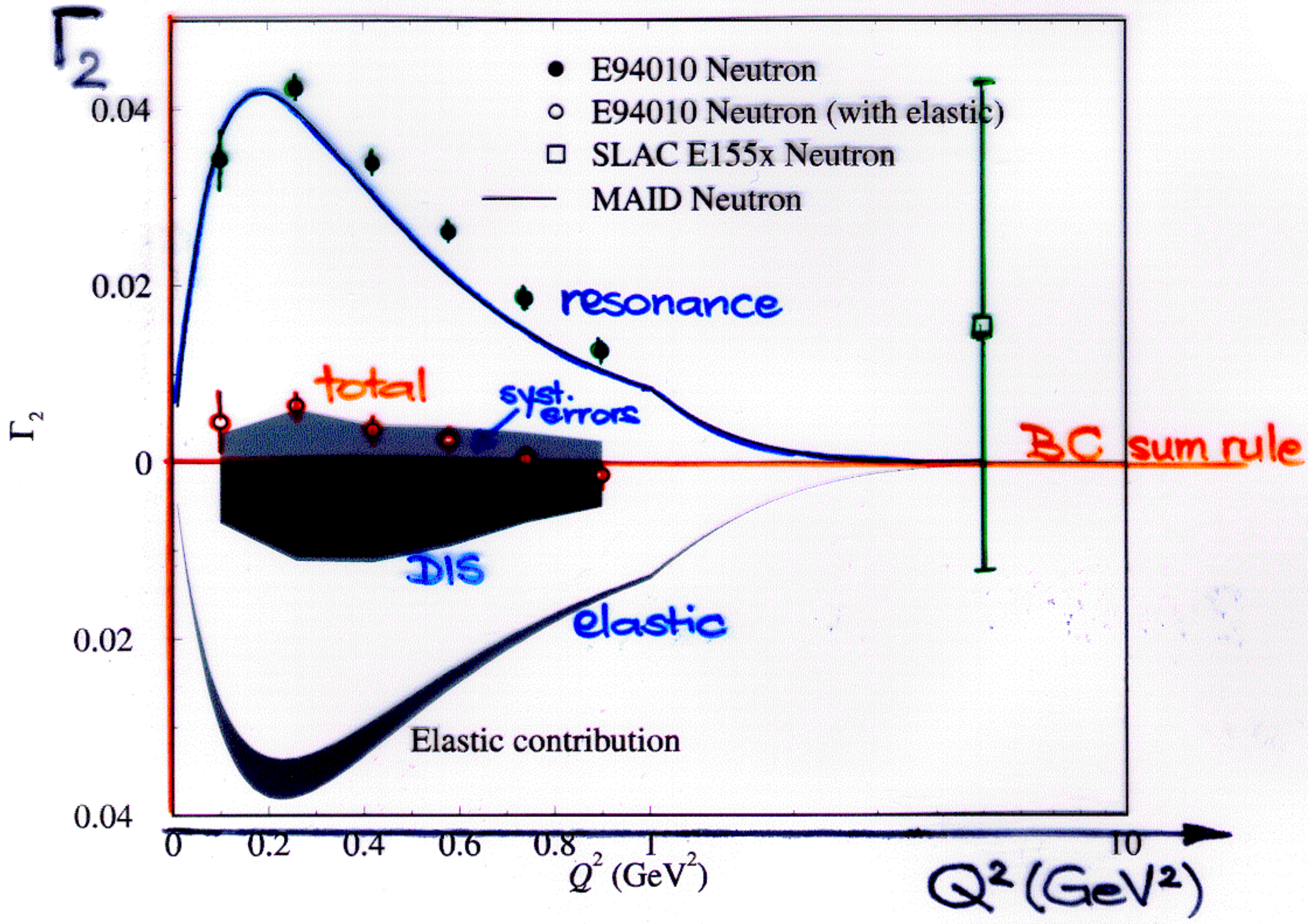
• $I_2 \to \frac{1}{4} \kappa_N \mu_N$ if $Q^2 \to 0$ and BC holds

• $I_2 \to Q^{-10}$ if $Q^2 \to \infty$ and BC holds

$\to \mathcal{O}(m_N^4 / Q^4)$ if $Q^2 \to \infty$ (higher twist)

$$\Gamma_2^n(\text{inel.}) = \int_0^{x_0} g_2(x, Q^2)\,dx$$

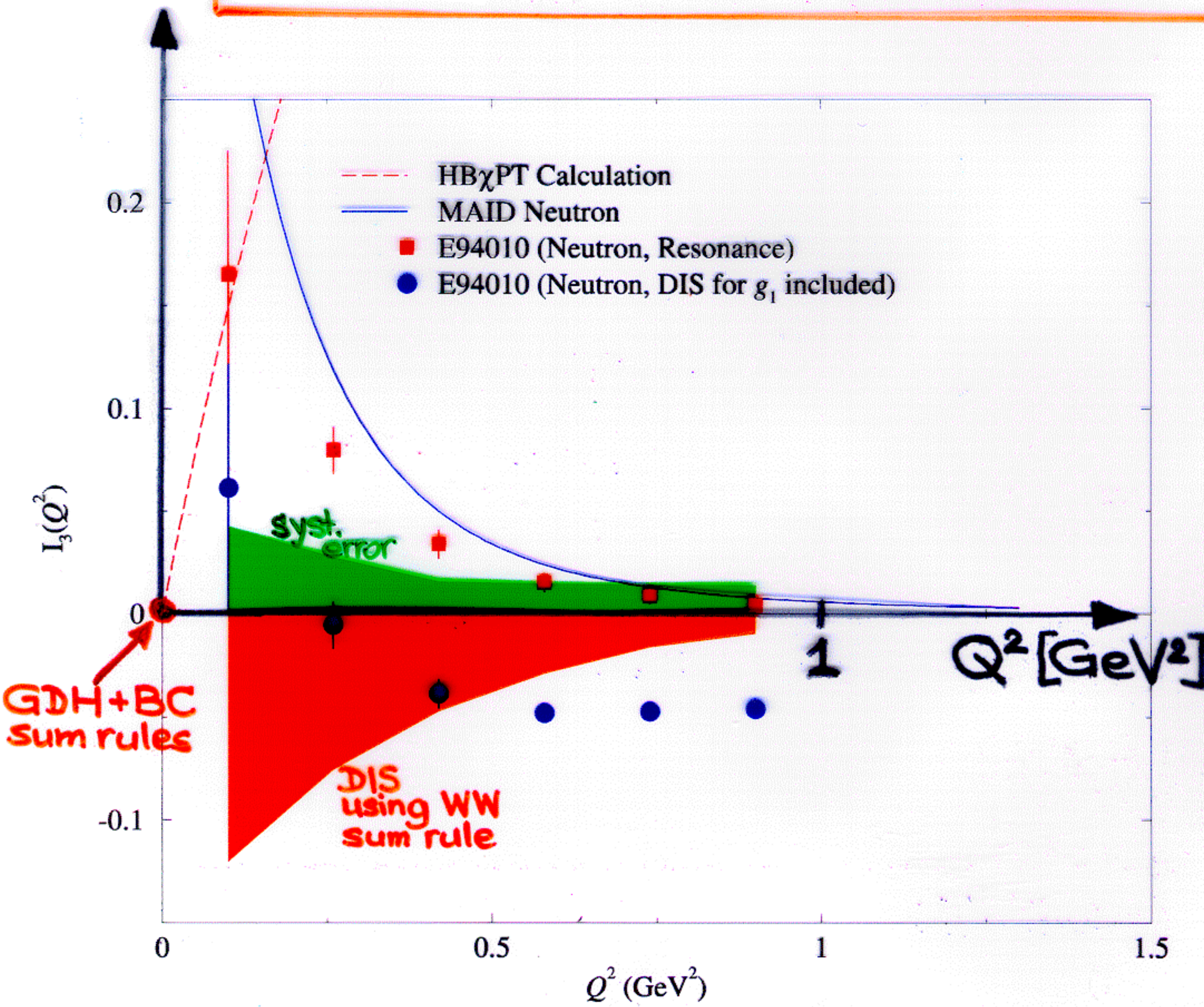
$$\Gamma_2^n(\text{el.}) = -\frac{Q^2}{8M^2} F_P(Q^2)\left(F_D(Q^2) + F_P(Q^2)\right)$$



[from: Z.-E. Meziani et al., JLab proposal (2003)]

- BC sum rule valid within exp'tal errors

$$I_3^n(Q^2) = \frac{2M^2}{Q^2}\int_0^{x_0}(g_1+g_2)\,dx = \dots \int_{\nu_0}^{\infty} \frac{K}{\nu}\,\frac{\sigma_{LT}}{Q}\,d\nu$$

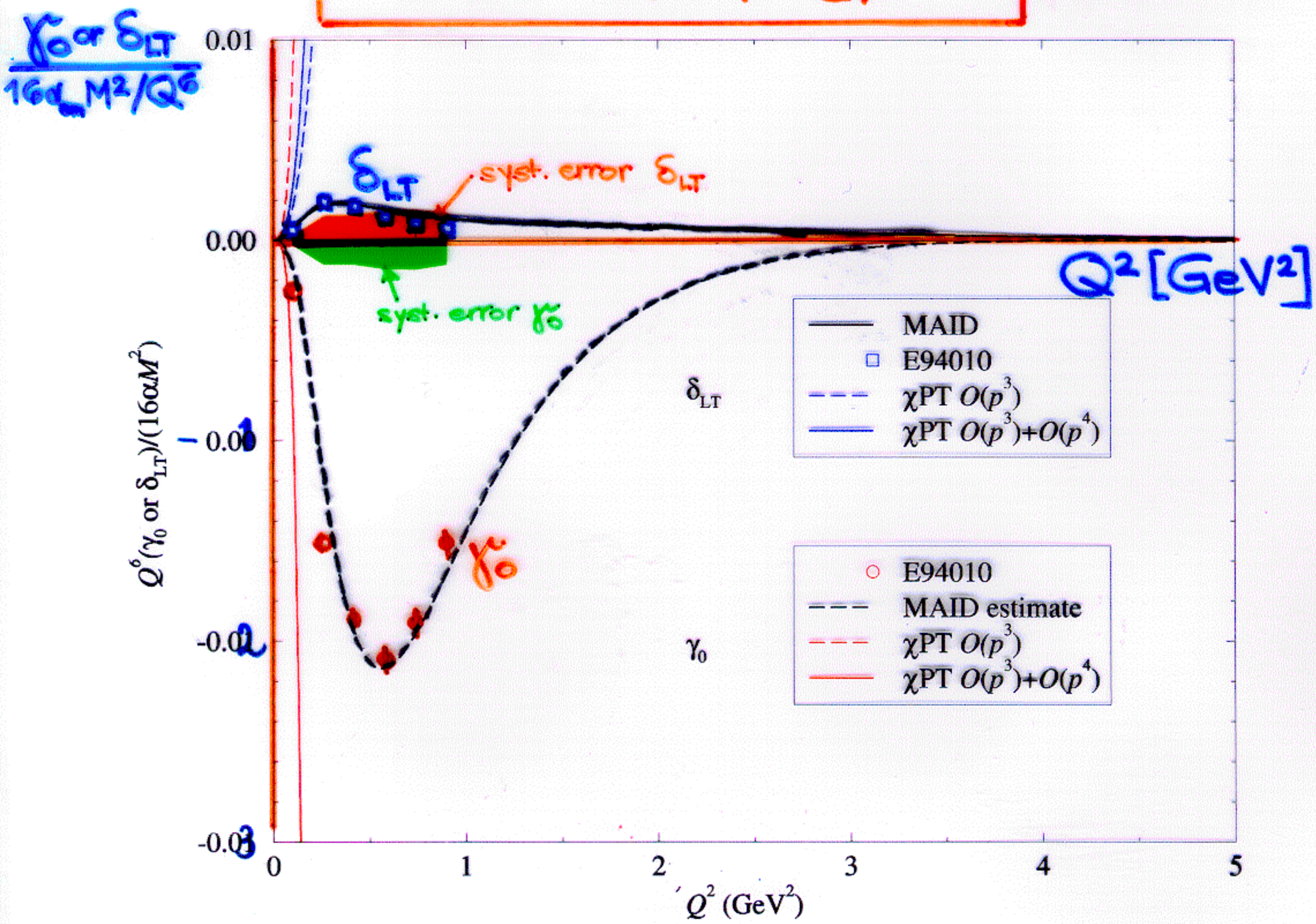


S. Choi et al. (JLab E94010) priv. comm.

→ S. Choi et al., JLab E9010 : moments and polarizabilities of the neutron

$$\boxed{\begin{aligned} \gamma_0^{n}(Q^2) &= \frac{1}{2\pi^2}\int_{\nu_0}^{\infty} \frac{K}{\nu}\,\frac{\sigma_{TT}}{\nu^3}\,d\nu \\ \delta_{LT}^{n}(Q^2) &= \frac{1}{2\pi^2}\int \frac{K}{\nu}\,\frac{\sigma_{LT}}{Q\nu^2}\,d\nu \end{aligned}}$$

PRELIMIN. DATA!



- forward spin polarizability

large cancellations

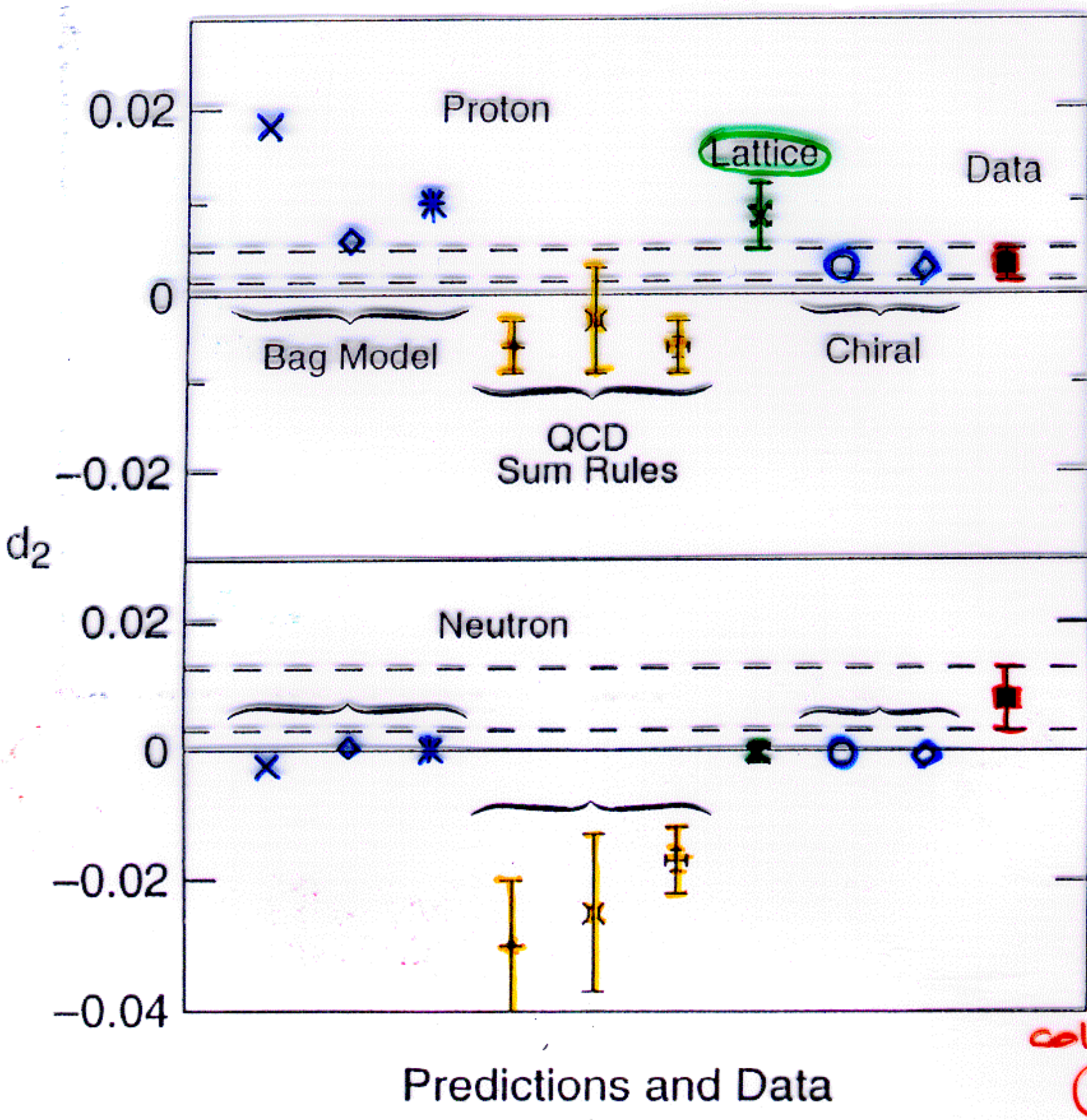
$$\gamma_0 = \cdots \int \frac{d\nu}{\nu^3} \cdots \{ |E_{0+}|^2 - |M_{1+}|^2 + 6 E_{1+}^* M_{1+} \cdots \}$$

- long.-transverse polarizability

small

$$\delta_{LT} \text{ or } \delta_0 = \cdots \int \frac{d\nu}{\nu^3} \cdots \{ L_{0+}^* E_{0+} + 2 L_{1+}^* M_{1+} + \cdots \}$$

$d_2(Q^2 \to \infty)$

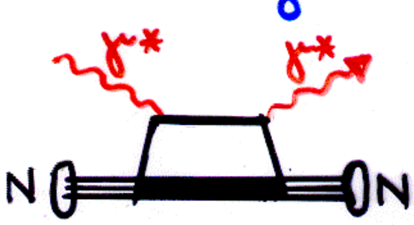


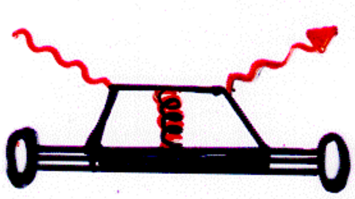
color e.m. resp.
(χ_E, χ_B)

twist-3 twist-4

- **Twist Expansion**

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2)\,dx = \overset{\text{leading}}{\tfrac{1}{2} a_0} + \frac{M^2}{9Q^2}\Big(\overset{\text{target mass}}{a_2} + 4\overset{\text{twist-3}}{d_2} + 4\overset{\text{twist-4}}{f_2}\Big) + \mathcal{O}\Big(\frac{M^4}{Q^4}\Big)$$



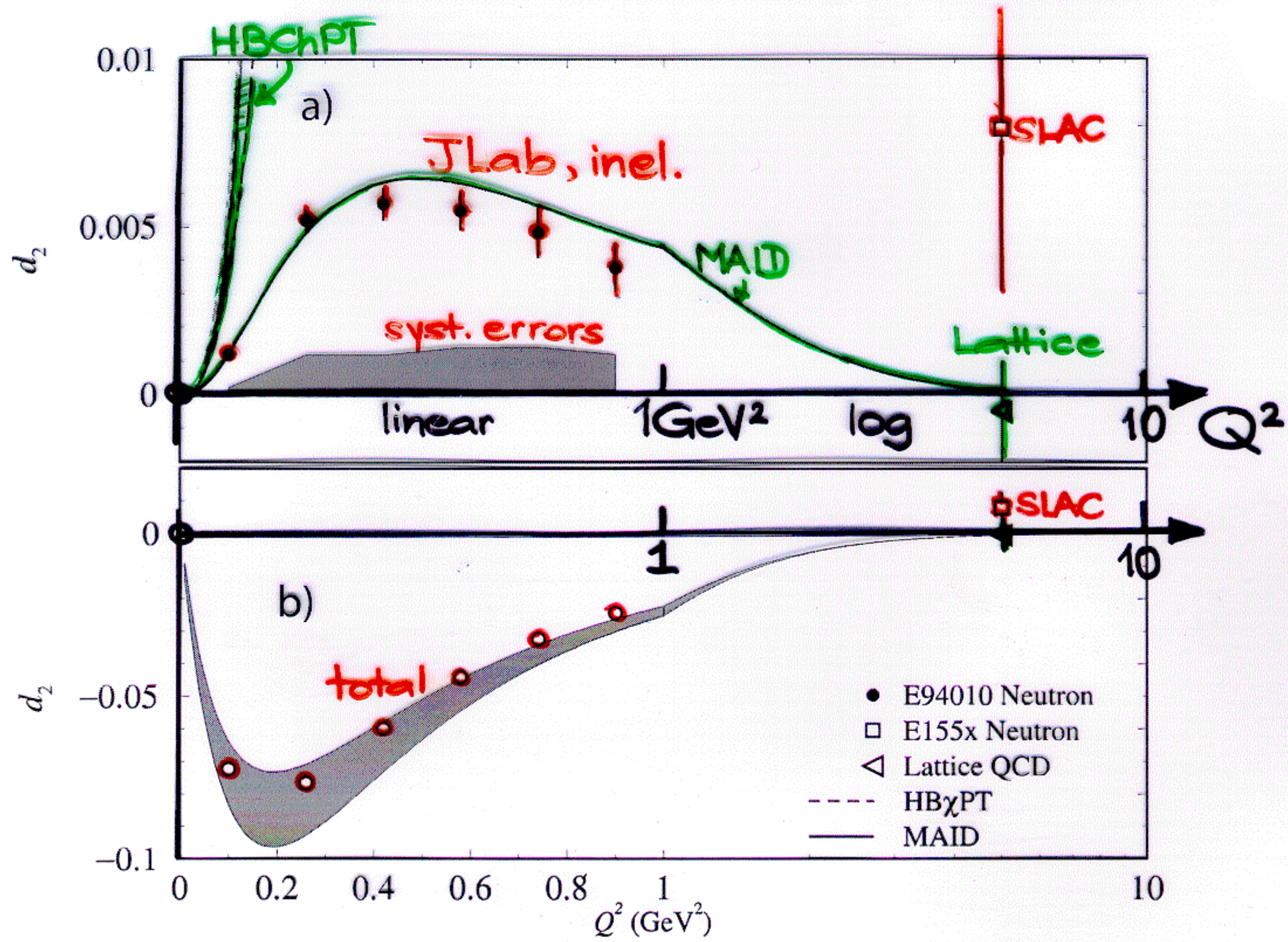


leading twist:
incoherent VVCS
on constituents

higher twists:
constituents interact during VVCS
correlations as precursor of
coherent processes = resonances

$$\boxed{d_2^n(Q^2) = \int_0^1 x^2 \left(2g_1^n + 3g_2^n\right) dx}$$

TWIST-THREE



[from Z.-E. Meziani, JLab proposal 2003]

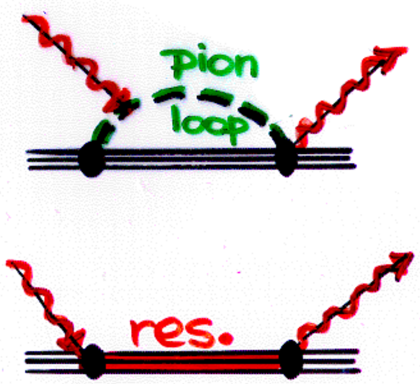
$$I_1(Q^2) = \frac{2M^2}{Q^2}\int g_1(x,Q^2)\,dx$$

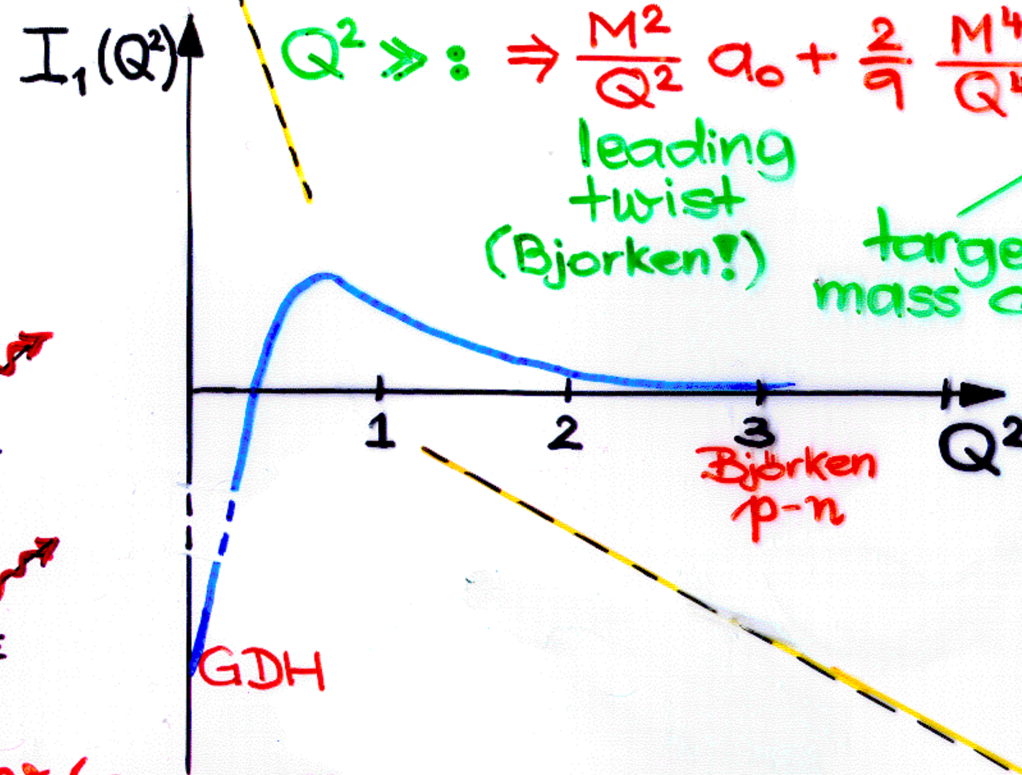
$$Q^2 \gg: \Rightarrow \frac{M^2}{Q^2} a_0 + \frac{2}{9}\frac{M^4}{Q^4}\left(a_2 + 4d_2 + 4f_2\right) + \mathcal{O}\left(\frac{M^6}{Q^6}\right)$$

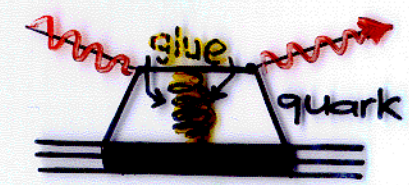
leading twist (Bjorken!)

target mass corr.

higher twists







response of gluon field to applied e.m. field, e.g.

$$d_2 = \int (2g_1 + 3g_2)\,x^2\,dx$$

color e.m. polarizability

$$I_1(Q^2) = \int \frac{d\nu}{\nu}\left(\frac{\sigma_{1/2}(\nu,Q^2) - \sigma_{3/2}(\nu,Q^2)}{2} - \frac{Q}{\nu}\sigma_{LT}(\nu,Q^2)\right)$$

$$Q^2 \to 0: \Rightarrow -\frac{1}{4}\kappa^2 \text{ (GDH!)}$$

$$\left.\begin{aligned}\gamma_0(Q^2) &= \ldots \int \frac{d\nu}{\nu^3}\left(\sigma_{1/2} - \sigma_{3/2}\right)\\ \delta_{LT}(Q^2) &= \ldots \int \frac{d\nu}{\nu^2 Q}\,\sigma_{LT}(\nu,Q^2)\end{aligned}\right\} \rightarrow$$

spatial distribution of polarizabilities ($Q^2 < 4M^2$!)

response of π-loops & resonances

incoherent scattering, strong correlations