^{13}Cにおける単極遷移強度とクラスター構造

山田泰一、船木靖郎

Contents

- **Monopole strength and cluster structure**

  Typical case : ^{16}O (^{12}C)

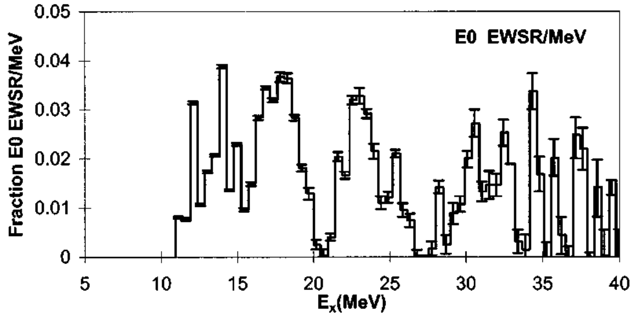
- **Alpha clustering and condensation in ^{13}C**

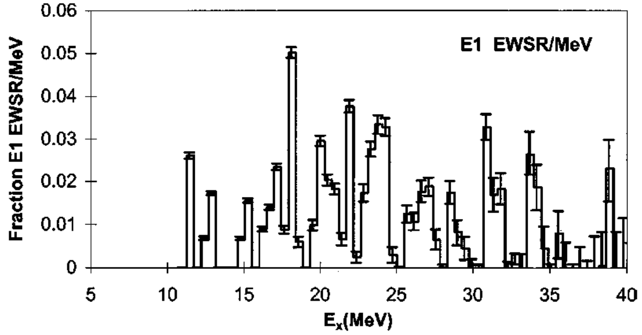
  Monopole strengths in 1/2- states

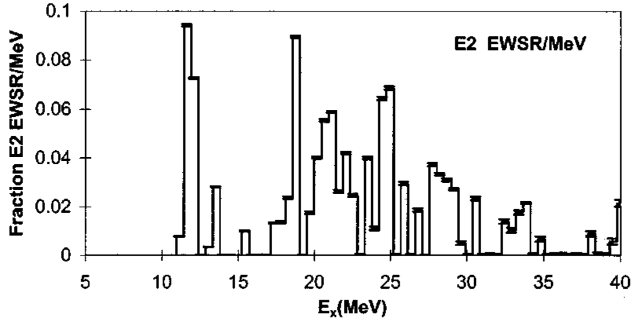
  Structures of ^{13}C: 1/2-,（1/2+）

E0-strength of ^{16}O: Exp. vs Cal.

^{16}O(α,α') TEWSR=48±10%

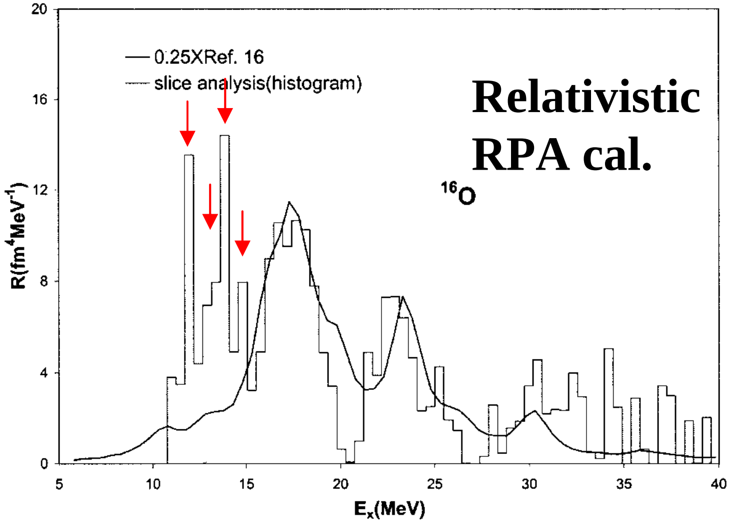






Lui et al., PRC64(2001)

^{16}O(α,α')

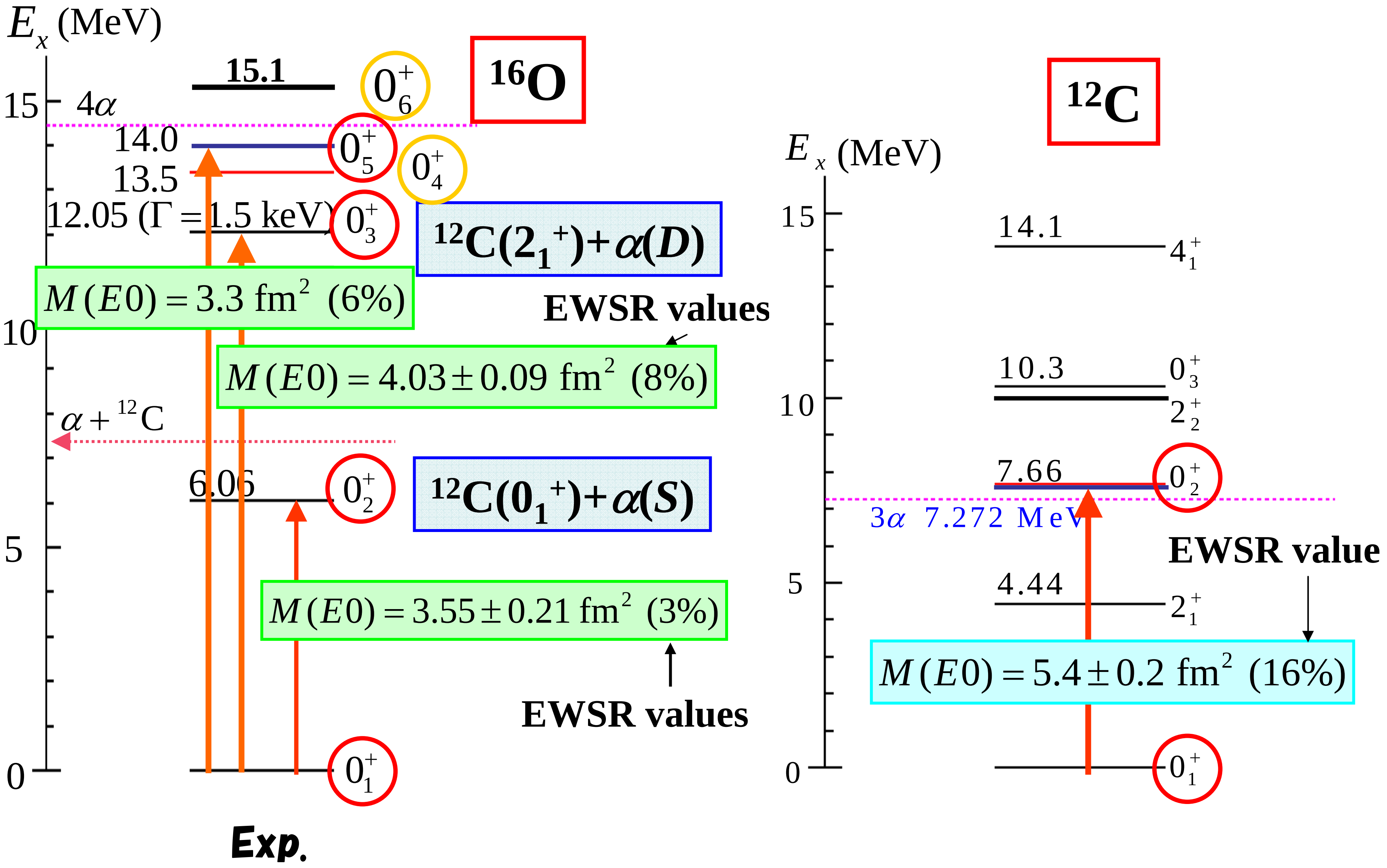


Relativistic RPA cal.

FIG. 7. The histogram is the experimental $E0$ strength converted to monopole response function. The black line shows the monopole response function from Ref. [16] multiplied by 0.25 and shifted by 4.2 MeV.

Exp: Lui et al., PRC 64 (2001)
Cal: Ma et al., PRC 55 (1997)
Multiplied by 0.25
Shifted by 4.2 MeV

Monopole matrix elements *M(E0)* in ^{16}O and ^{12}C



^{12}C$(2_1^+)+\alpha(D)$

$M(E0) = 3.3$ fm^2 (6%)

EWSR values

$M(E0) = 4.03 \pm 0.09$ fm^2 (8%)

^{12}C$(0_1^+)+\alpha(S)$

$M(E0) = 3.55 \pm 0.21$ fm^2 (3%)

EWSR values

Exp.

EWSR value

$M(E0) = 5.4 \pm 0.2$ fm^2 (16%)

Monopole Strengths

		Exp. [fm^2]	**Cal.** [fm^2]
^{16}O	$0^+_1 - 0^+_2$	3.55±0.21	4.1 (3.98*)
	$0^+_1 - 0^+_3$	4.03±0.09	2.6 (3.50*)
	$0^+_1 - 0^+_5$	3.3±0.7	3.0 (—*)
^{12}C	$0^+_1 - 0^+_2$	5.4±0.2	6.7

No effective charge!

^{16}O: ^{12}C+α OCM* **Suzuki, PTP56, 111 (1976)**

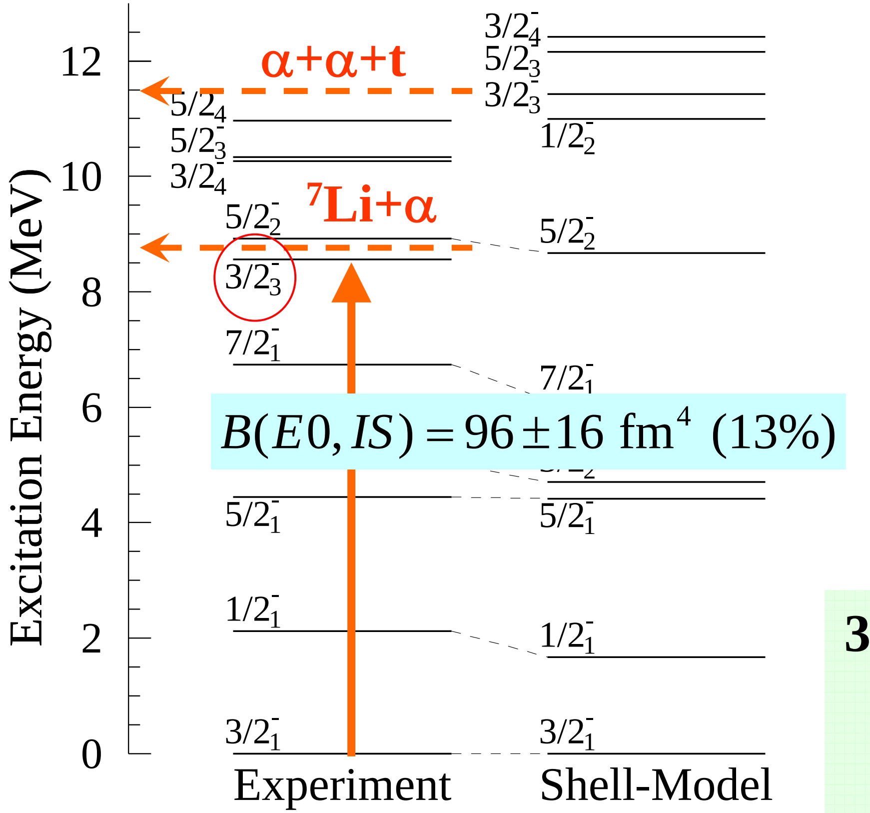
4α OCM **Funaki, Yamada et al., PRL 101 (2008)**

^{12}C: 3α RGM **Kamimura, Nucl. Phys. A 351, 456 (1981)**

^{12}Be: $0^+_1 - 0^+_2$ (shell-model structure), <r^2>=0.83 fm^2, S. Shimoura et al., PLB560 (2002)

Exotic characters of $3/2^-_3$ of ^{11}B

T. Kawabata et al., Phys. Lett. B 646, 6 (2007).



$$B(E0, IS) = 96 \pm 16\ \mathrm{fm}^4\ \ (13\%)$$

E_x (MeV)	B(GT)	
	Experiment	Shell Model
0.000 ($3/2^-$)	0.345±0.008	0.588
2.125 ($1/2^-$)	0.401±0.032	0.782
4.445 ($5/2^-$)	0.453±0.029	0.616
5.020 ($3/2^-$)	0.487±0.029	0.745
8.104 ($3/2^-$)	< 0.003	–
8.420 ($5/2^-$)	0.398±0.031	0.483

$3/2^-_3$ state has exotic characters.

- Suppressed GT strength
- Large monopole strength
- Not predicted by the shell-model calculation
- 100-keV below the α-decay threshold.

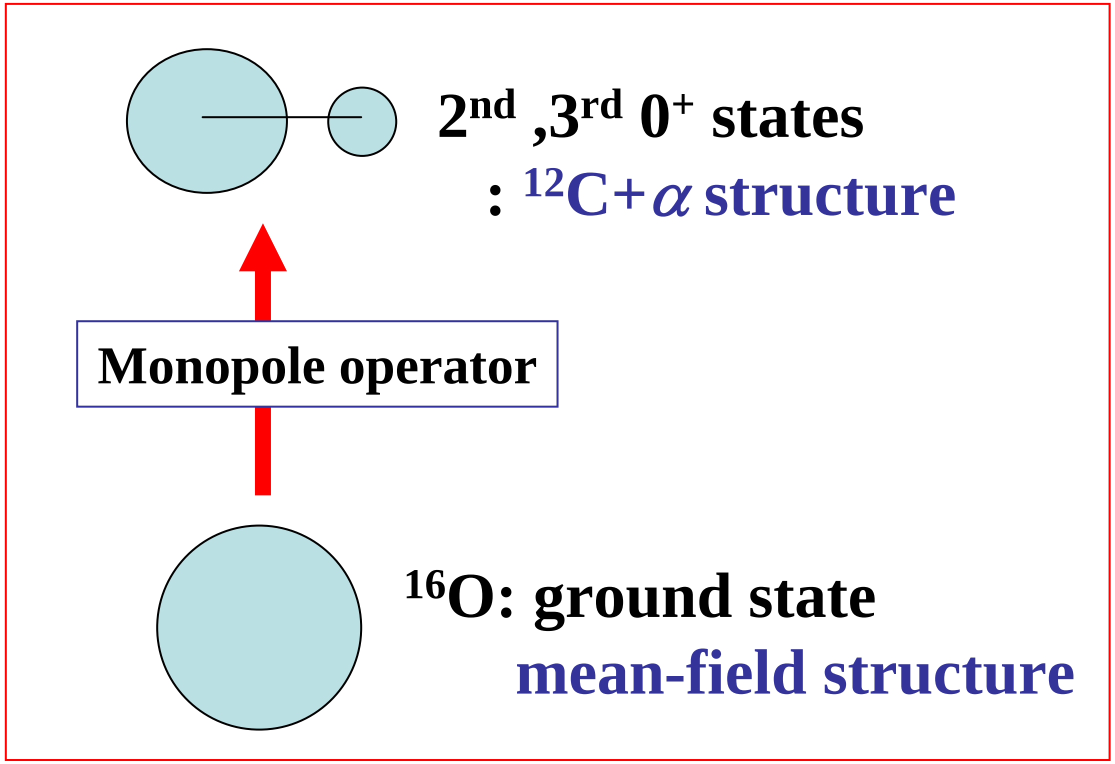
AMD, 2α+t OCM: $3/2^-_3$ state has α+α+t cluster structure

Enyo-Kanada Yamada, Funaki

Monopole strengths to cluster states: ∼20% of EWSR

Here, we have an interesting question.

Why cluster states are populated from the ground states with mean-field structures by the monopole transitions ?



2^{nd} ,3^{rd} 0^+ states

: ^{12}C+α structure

Monopole operator

^{16}O: ground state

mean-field structure

Non trivial problem !

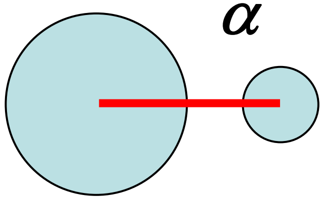
T. Yamada, Y. Funaki, H. Horiuchi,
K. Ikeda, A. Tohsaki,
to be published in Prog. Theor. Phys.

Monopole transition between 0^+_1 and 0^+_2 states

0^+_2 $^{12}\mathrm{C}(0^+)+\alpha$ **cluster structure**

Relative motion is excited.

$^{12}\mathrm{C}(0^+)$



Monopole operator

$$\frac{1}{2}\sum_{i=1}^{16}(\mathbf{r}_i-\mathbf{r}_G)^2=\frac{1}{2}\sum_{i\in{}^{12}\mathrm{C}}(\mathbf{r}_i-\mathbf{r}_C)^2+\frac{1}{2}\sum_{i\in\alpha}(\mathbf{r}_i-\mathbf{r}_\alpha)^2+\overset{\textbf{rel.}}{\frac{1}{2}\frac{12\times4}{16}\mathbf{r}^2}$$

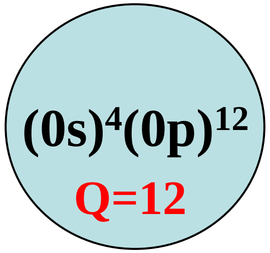
Monopole

Bayman-Bohr theorem

G.S. correlation Q>12
: α clustering

0^+_1

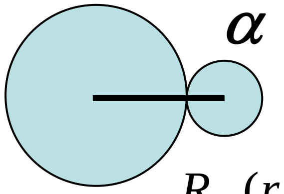
^{16}**O (g.s.)**



SU(3)(00)
90% comp.

=

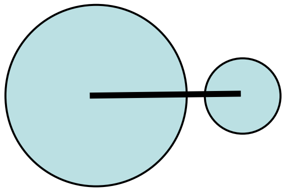
$^{12}\mathrm{C}(0^+)$



$R_{40}(r,3\nu_N)$

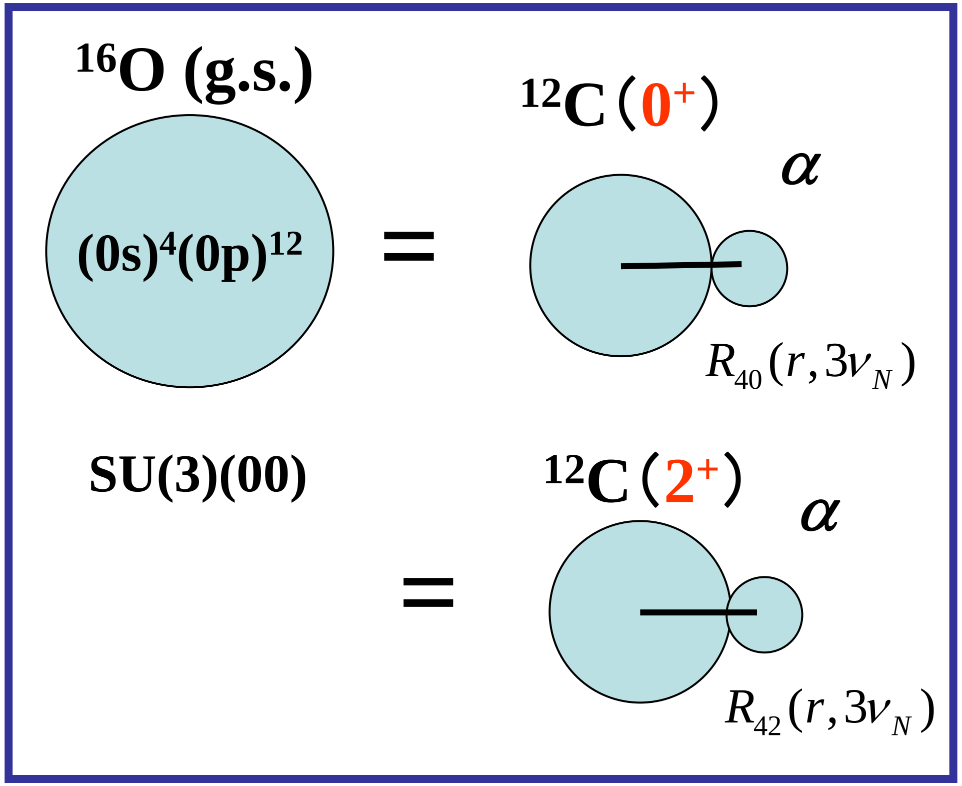
having an seed of α in G.S.

10% comp.



activating the seed of α

Doubly closed shell-model w.f. ^{16}O



^{16}O (g.s.)

$(0s)^4(0p)^{12}$

=

^{12}C (0^+) α

$R_{40}(r, 3\nu_N)$

SU(3)(00)

=

^{12}C (2^+) α

$R_{42}(r, 3\nu_N)$

^{16}O g.s. can be excited through cluster degree of freedom, namely, ^{12}C+α relative motion, from R_{4L} to higher nodal states.

Bayman-Bohr theorem

$$|g.s.\rangle = \frac{1}{\sqrt{16!}} \det\left|(0s)^4(0p)^{12}\right| \times [\phi_G(\mathbf{r}_G)]^{-1}$$

howf

$$= N_{g0}\sqrt{\frac{12!4!}{16!}} A\left\{\left[\phi_{L=0}(^{12}\mathrm{C})R_{40}(r)\right]_{J=0}\phi(\alpha)\right\}$$

$$= N_{g2}\sqrt{\frac{12!4!}{16!}} A\left\{\left[\phi_{L=2}(^{12}\mathrm{C})R_{42}(r)\right]_{J=0}\phi(\alpha)\right\}$$

^{12}C(0^+,2^+,4^+) wf.: SU(3)(04) wf

α cluster: intrinsic $(0s)^4$

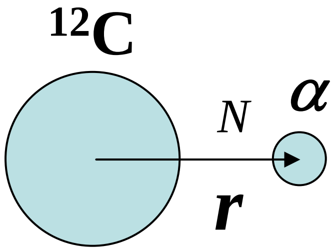
$R_{4L}(r,3\nu_N)$: h.o.w.f. with Q=4

So far, our discussion was qualitatively.

Next, we study the monopole strengths in ^{16}O quantitatively with use of the ^{12}C+α OCM.

^{16}O=^{12}C+α OCM

Y. Suzuki, PTP55(1976)



$$\Phi^J = \sum_{i,N} c_i^{NJ} \Phi_i^{NJ}$$

$$\Phi_i^{NJ} = \sqrt{\frac{12!4!}{16!}} A' \left\{ h_i^J \underline{R_{NL}(r, 3\nu)} \right\}$$

Relative motion expanded by h.o. basis

$$h_i^J = \left[\phi_I(^{12}\mathrm{C}) \phi(\alpha) Y_L(\hat{r}) \right]_J$$

$\phi_L(^{12}C)$: SU(3)(0,4) w.f. $(L = 0, 2, 4)$

$R_{NL}(r, \beta)$: h.o. basis $(N = 2n + L \geq 4)$

N=4 : (0s)^4(0p)^{12} closed shell
Larger N : ^{12}C-α clustering

SU(3) basis : (0,4) × (N,0) = (N-4,0) + (N-2,2) + (N,4)

^{12}C ^{12}C$-\alpha$

^{16}O=^{12}C+α cluster model

Y. Suzuki, PTP55 (1976), 1751

Even-parity

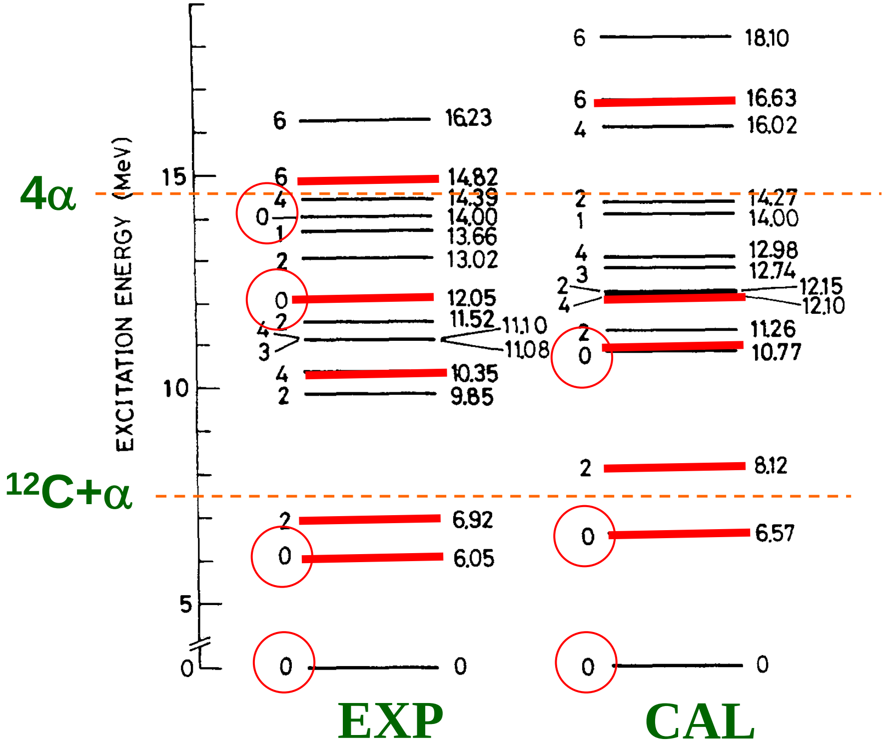


Fig. 2 (a). Energy levels of ^{16}O for the even-parity states [Ref. 30)].

Odd-parity

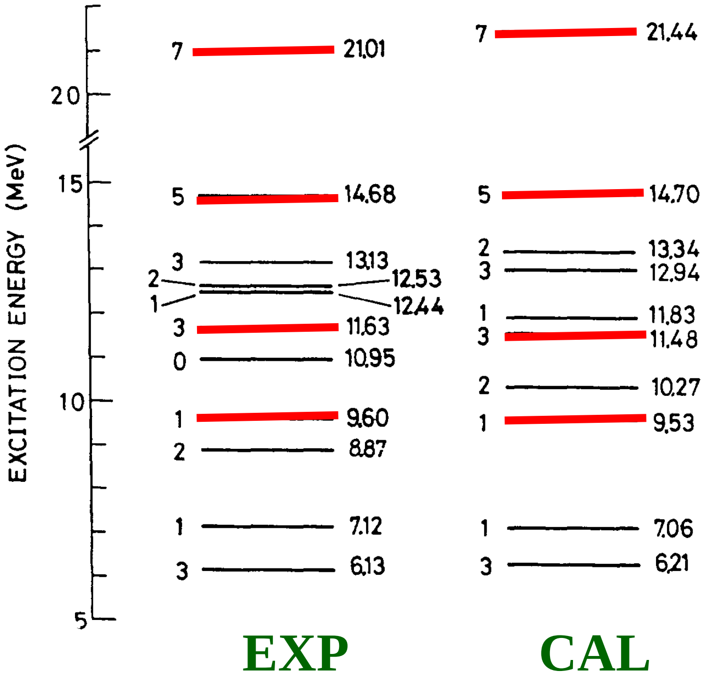
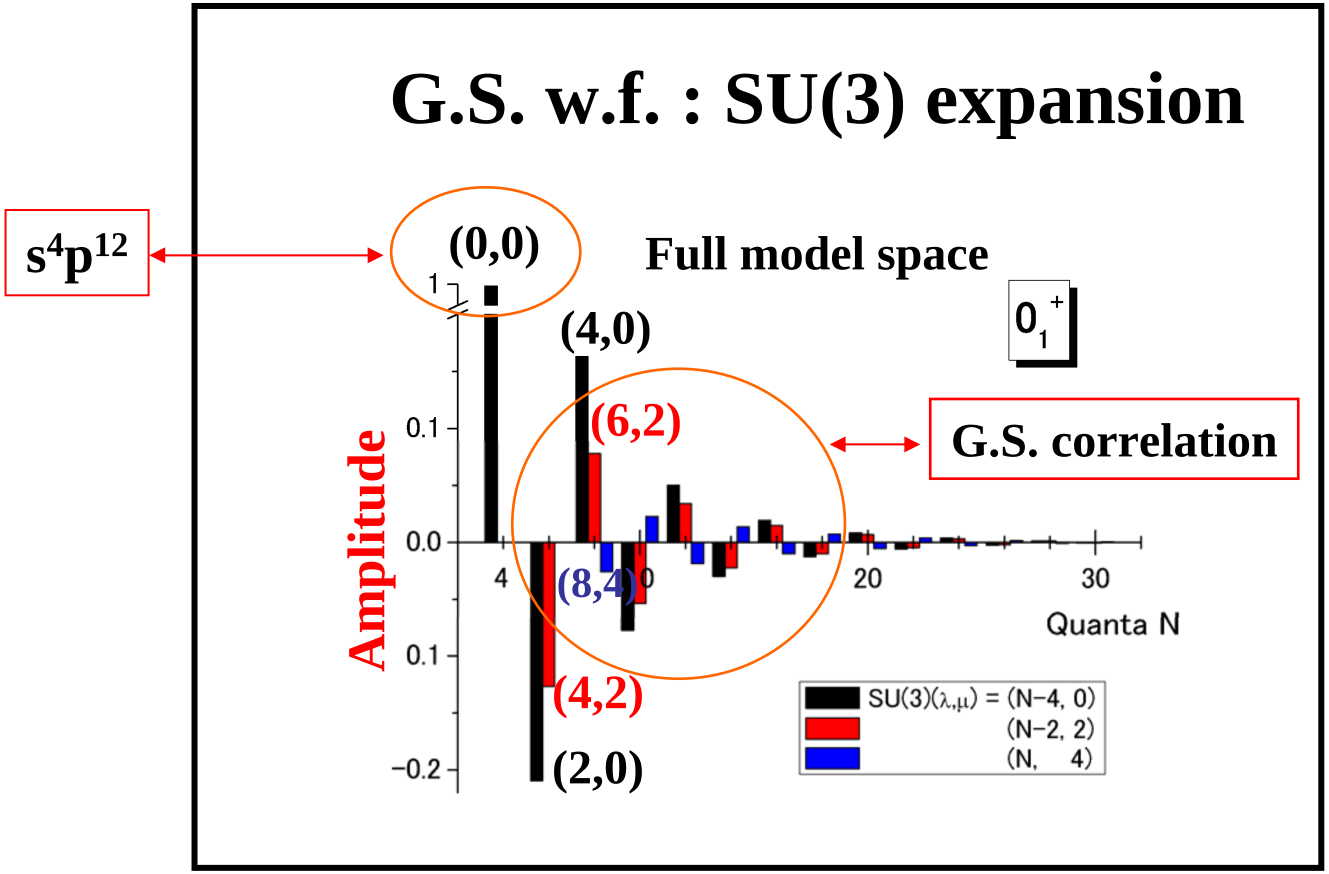


Fig. 2 (b). Energy levels of ^{16}O for the odd-parity states [Ref. 30)].

▬ ^{12}C+α : molecular states

Ground state correlation in ^{16}O (^{12}C+α OCM)



^{12}C+α OCM

^{16}O g.s wf

	comp.
N=4	89 %
N>4	11 %

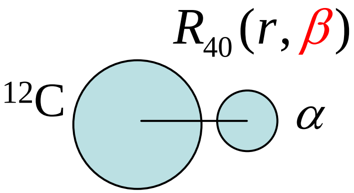
$$M(0_1^+ - 0_k^+) = \left\langle 0_1^+ \middle| \frac{1}{2}\sum_{i=1}^{12}(r_i - r_G)^2 \middle| 0_k^+ \right\rangle \propto \Phi^*(0_1^+)\Phi(0_k^+)$$

Product of amplitudes

Deviation from doubly closed shell w.f.

Modified doubly closed shell w.f. ν_N **: nucleon size parameter**

$$\Phi_{0^+}(\beta) = N(\beta)\sqrt{\frac{12!4!}{16!}}A\left\{\left[\phi_{L=0}(^{12}C)R_{40}(r,\beta)\right]_0 \phi(\alpha)\right\}$$



$\beta/(3\nu_N)=1$ の時、doubly closed shell w.f: $(0s)^4(0p)^{12}$

$\beta/(3\nu_N)<1$ α clustering is activated.

Squared overlap on $\beta/3\nu_N$

$$P(\beta/3\nu_N) = \left|\left\langle \Phi_{0^+}(\beta) \middle| 0_1^+ : \mathrm{OCM} \right\rangle\right|^2$$

α clustering is activated !

$\beta/3\nu_N$	**1**	**···**	**0.847**	**···**
P	**0.890**	↗	**0.958**	↘

最大値になる

Monopole Strengths & G.S. correlation

$|0_1^+;N\rangle$: G.S. wf within N quanta model space (N=4,6,$\cdots$,30)

^{12}C+α OCM

$|0_2^+\rangle,|0_3^+\rangle$: obtained with full model space (N=30)

^{12}C+α structures

$$|\widetilde{0_k^+}\rangle = N_k(1-\widehat{P}_N)|0_k^+\rangle, \quad \mathbf{k=2,3} \qquad \longleftrightarrow \qquad \langle 0_1^+;N|\widetilde{0_k^+}\rangle = 0$$

$$= N_k\left[|0_k^+\rangle - |0_1^+;N\rangle\langle 0_1^+;N|0_k^+\rangle\right]$$

$$M_N(0_1^+ - 0_k^+) = \langle 0_1^+;N|\frac{1}{2}\sum_{i=1}^{12}(r_i - r_G)^2|\widetilde{0_k^+}\rangle \propto \Phi^*(0_1^+;N)\Phi(\widetilde{0_k^+})$$

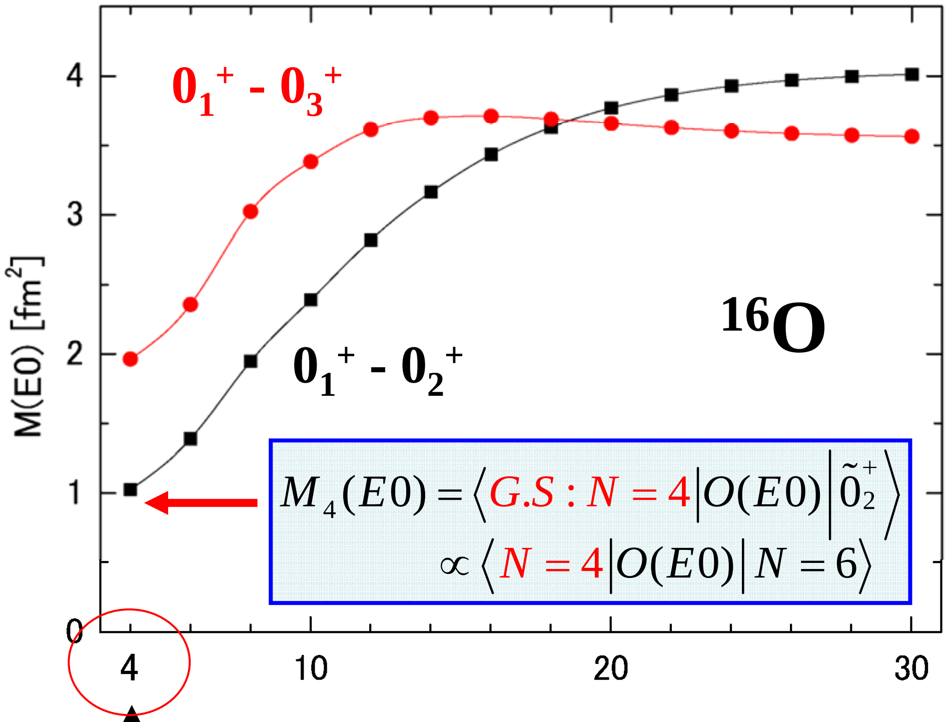
Study effect of the ground-state correlation

(^{12}C-α clustering in g.s.)

Dep. of $M_N(0^+_1-0^+_{2,3})$ on model space of G.S. in ^{16}O

Monopole strengths

^{12}C+α OCM



$$M_4(E0) = \left\langle \color{red}{G.S : N = 4} \middle| O(E0) \middle| \tilde{0}_2^+ \right\rangle$$

$$\propto \left\langle \color{red}{N = 4} \middle| O(E0) \middle| N = 6 \right\rangle$$

CAL **EXP**

3.98 fm^2 vs. 3.55±0.22 fm^2

3.50 fm^2 vs. 4.03±0.09 fm^2

Importance of G.S. correlation
(^{12}C-α clustering in g.s.)

Quanta N (model space of ground state)

Shell-model limit: G.S.=$(0s)^4(0p)^{12}$

T. Yamada et al.,
to be published in Prog. Thor. Phys.

Summary (I)

Mechanism of M(E0) in light nuclei

(1) Structure of ground state

dual aspects in g.s. : mean-field + cluster

originally having a seed of α clustering (Bayman-Bohr theorem）

g.s. correlation

➔ enhanced α clustering or activating the seed

(2) Monopole operator：

exciting relative motions between clusters by 2hw

(3) Cluster states are populated by E0 (about 20% of EWSR).

Monopole strengths are a good tool to explore cluster states.

Cluster structure and α condensation in ^{13}C

T. Yamada and Y. Funaki

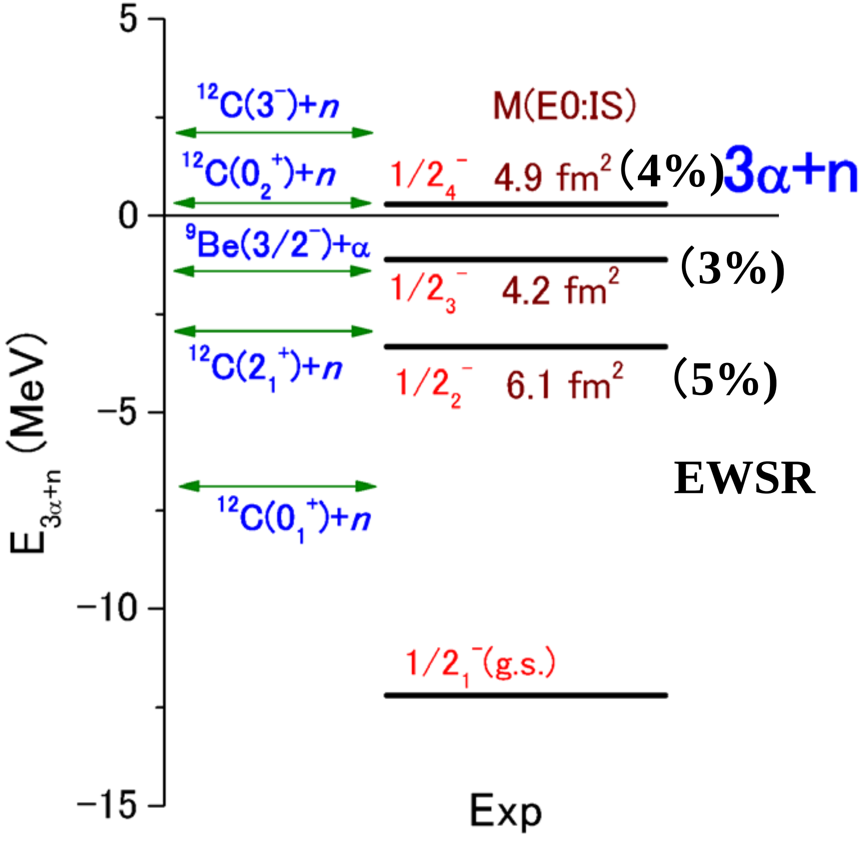
1. Structures of 1/2- states and monopole strengths
2. Structures of 1/2+ states

Motivations

- ^{12}C, 2nd 0^+ (Hoyle); 3α condensate
  ^{16}O, 6th 0^+; 4α condensate
- Addition of an extra neutron to Hoyle state (3α cond.)
  What happens ?
  Which state has the 3α+n gas-like (condensate) structure ?
  : gateway to explore gas-like states composed of bosons and fermions
- $1/2^-$ states excited by monopole transitions in ^{13}C(α,α'):
  What kinds of structures they have ?

Monopole excitations are a good tool to explore cluster structures.

^{13}C: monopole strengths



Shell model

M(E0) < 0.2 fm^2

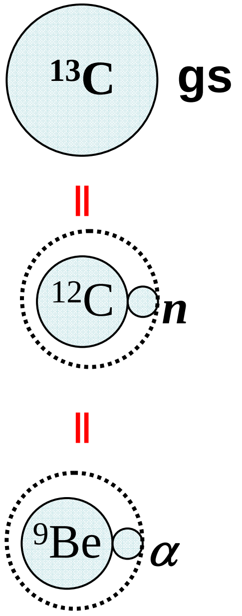
Sasamoto and Kawabata et al.
Mod. Phys. Lett. 21 2393 (2006)
Reanalayses: Mar. 2008

Monopole excitations in ^{13}C

Bayman-Bohr theorem: GS of ^{13}C SU(3)(31) × 1/2

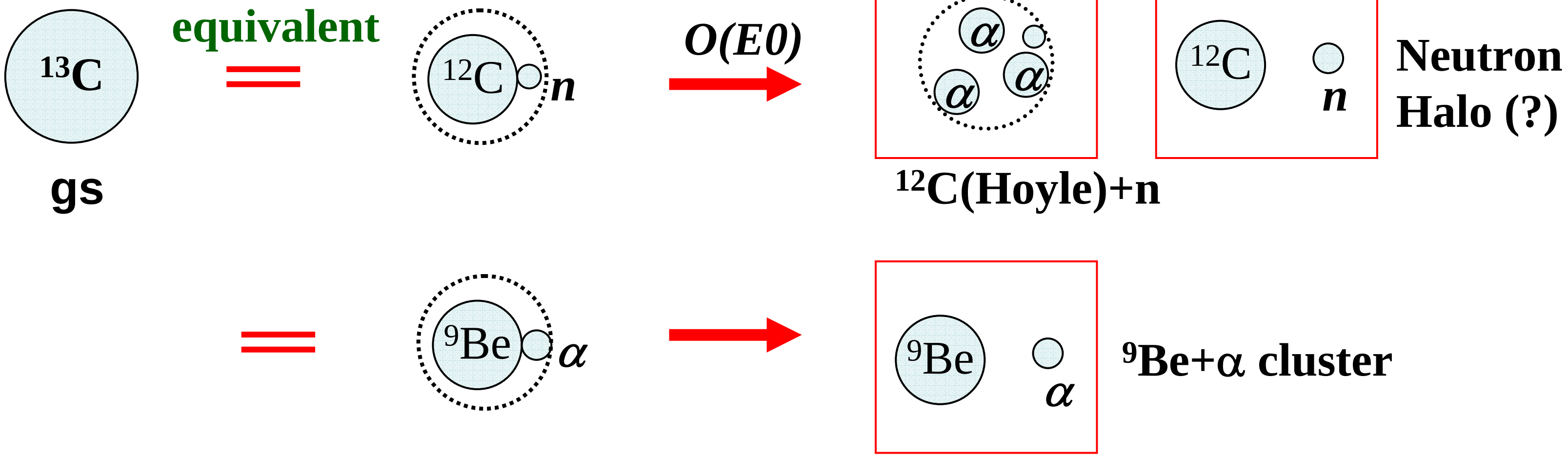
$$\left.\begin{aligned}\Phi_{GS}(^{13}\mathrm{C}) &= \sqrt{\frac{12!}{13!}}\,A\left\{\phi_{0^+}(^{12}\mathrm{C})u_{0p_{1/2}}(\mathbf{R})\right\}_{J=1/2^-}\\ &= \sqrt{\frac{12!}{13!}}\,A\left\{\phi_{2^+}(^{12}\mathrm{C})u_{0p_{3/2}}(\mathbf{R})\right\}_{J=1/2^-}\end{aligned}\right\}\ {}^{12}\mathrm{C}(3\alpha)+\mathrm{n}$$

$$\left.\begin{aligned} &= \sqrt{\frac{9!4!}{13!}}\,A\left\{\phi_{3/2^-}(^{9}\mathrm{Be})\phi(\alpha)u_{N=4,L=2}(\mathbf{R})\right\}_{J=1/2^-}\\ &= \sqrt{\frac{9!4!}{13!}}\,A\left\{\phi_{1/2^-}(^{9}\mathrm{Be})\phi(\alpha)u_{N=4,L=0}(\mathbf{R})\right\}_{J=1/2^-}\end{aligned}\right\}\ {}^{9}\mathrm{Be}+\alpha$$



Monopole operator: O(E0:IS)

Monopole-excited cluster states



Neutron Halo (?)

^{12}C(Hoyle)+n

^{9}Be+α cluster

^{13}C=3α+n OCM with Gaussian basis

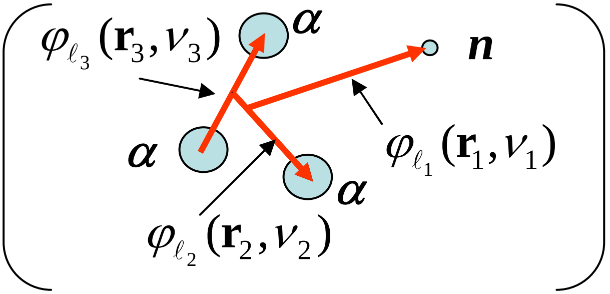
^{12}C=3α OCM, ^{9}Be=2α+n OCM

: successful reproduction of ^{12}C and ^{9}Be

(OCM=Orthogonality Condition Model)

Approximately taken into account: ***Fully Solving 4-body problem***

$$\Psi_J(^{13}\mathrm{C}) = \mathcal{A}$$



Gaussian bais:

$$\varphi_{\ell m}(\mathbf{r},\nu) = N_\ell(\nu) r^\ell \exp\left(-\nu r^2\right) Y_{\ell m}(\mathbf{r})$$

Angular momentum channels:

$$\left[\left[\left[l_{44} l_{84}\right]_I l_n\right]_L \tfrac{1}{2}\right]_J$$ 30 channels

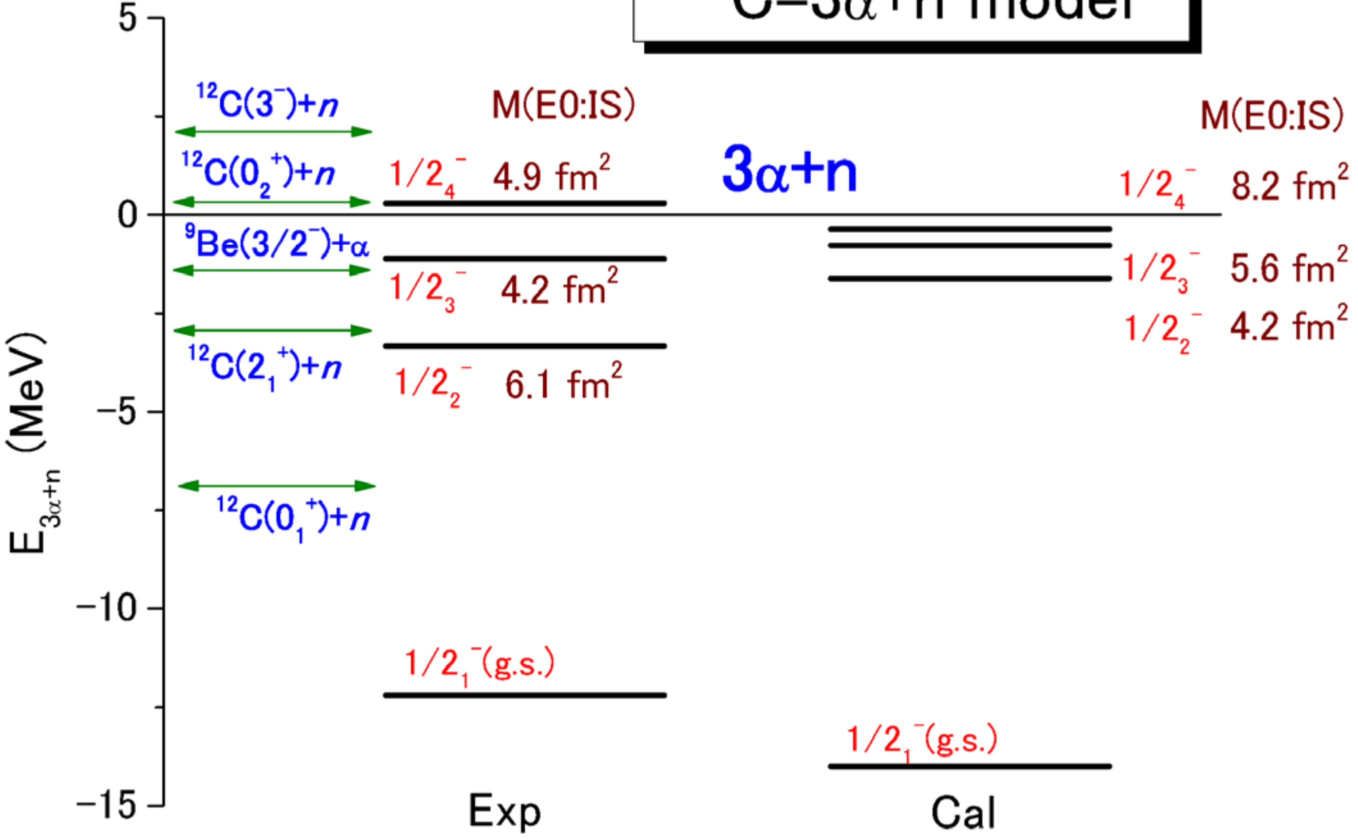
K-H coordinates

$$\boxed{H = T + \sum_{i<j=1}^{3}\left[V_{2\alpha}(r_{ij}) + V_{2\alpha}^{Coul}(r_{ij})\right] + \sum_{i=1}^{3} V_{\alpha n}(r_{in}) + V_{3\alpha} + V_{2\alpha+n} + V_{Pauli}}$$

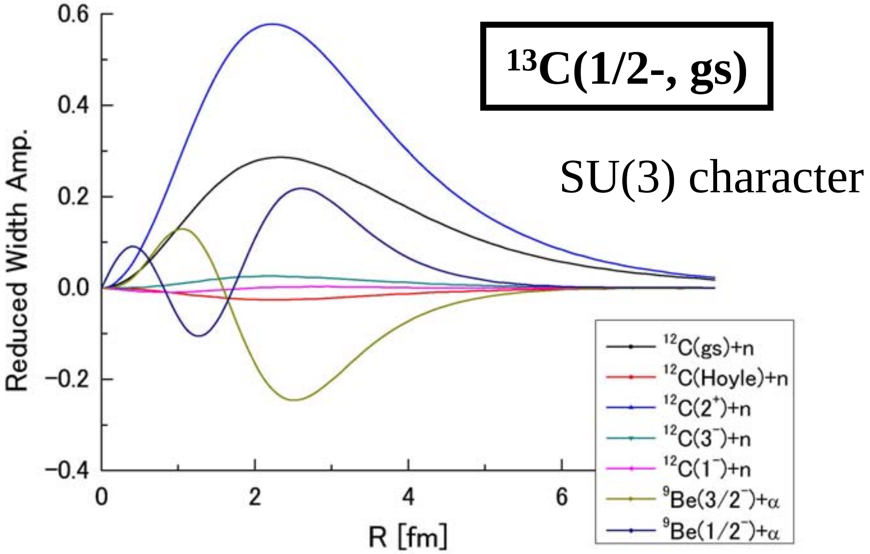
$V_{\alpha\alpha}$: reproduction of α-α phase shifts

$V_{\alpha n}$: reproduction of α-n phase shifts (Kanada-Kaneko pot.)

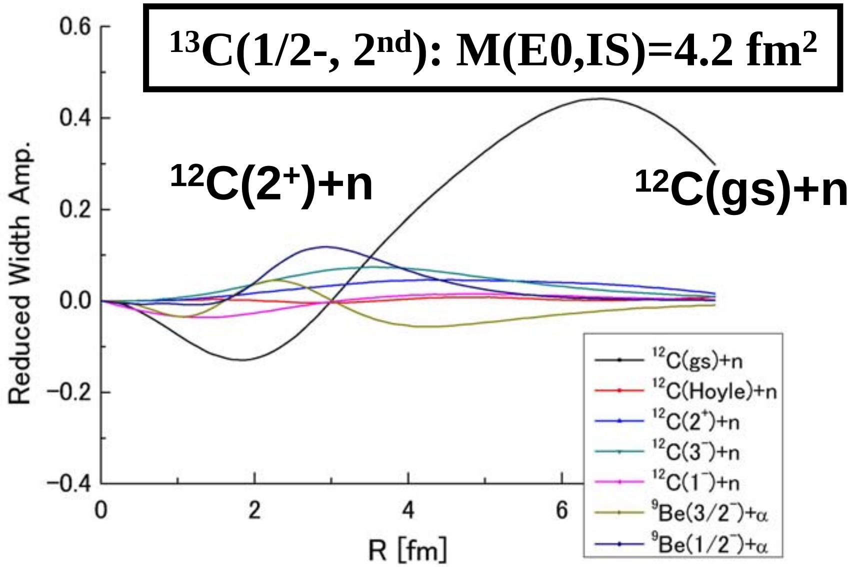
$^{13}C=3\alpha+n$ model



Overlap amplitudes (reduced width amp.)

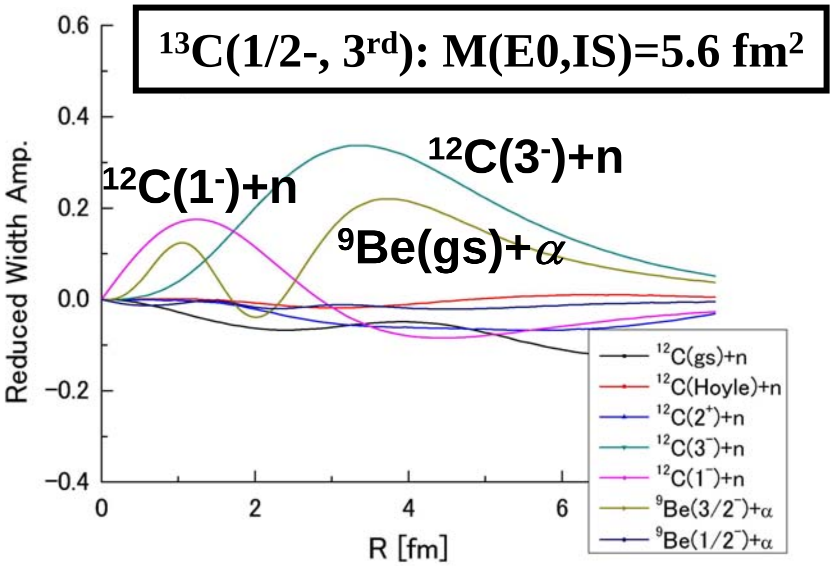

^{13}C(1/2-, gs)

SU(3) character


^{13}C(1/2-, 2nd): M(E0,IS)=4.2 fm^{2}

^{12}C(2^{+})+n

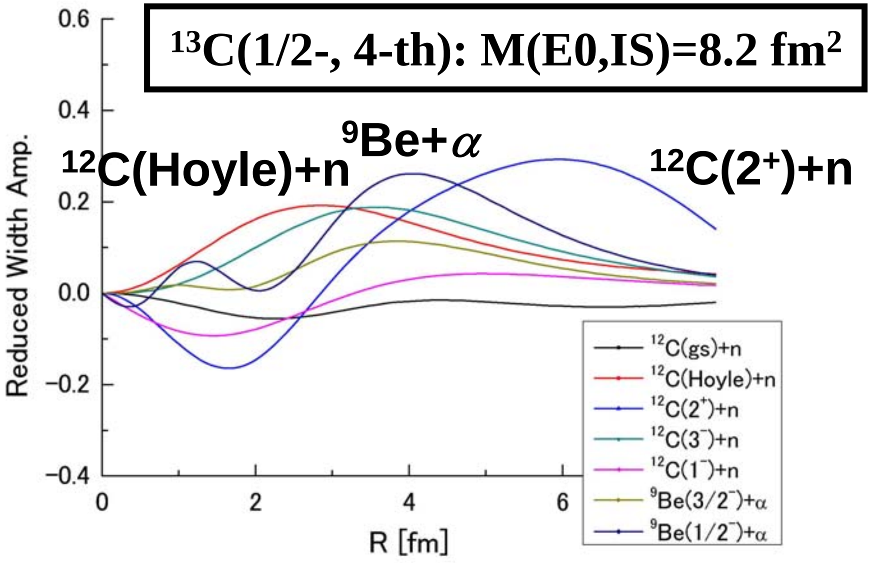
^{12}C(gs)+n


^{13}C(1/2-, 3rd): M(E0,IS)=5.6 fm^{2}

^{12}C(3^{-})+n

^{12}C(1^{-})+n

^{9}Be(gs)+α

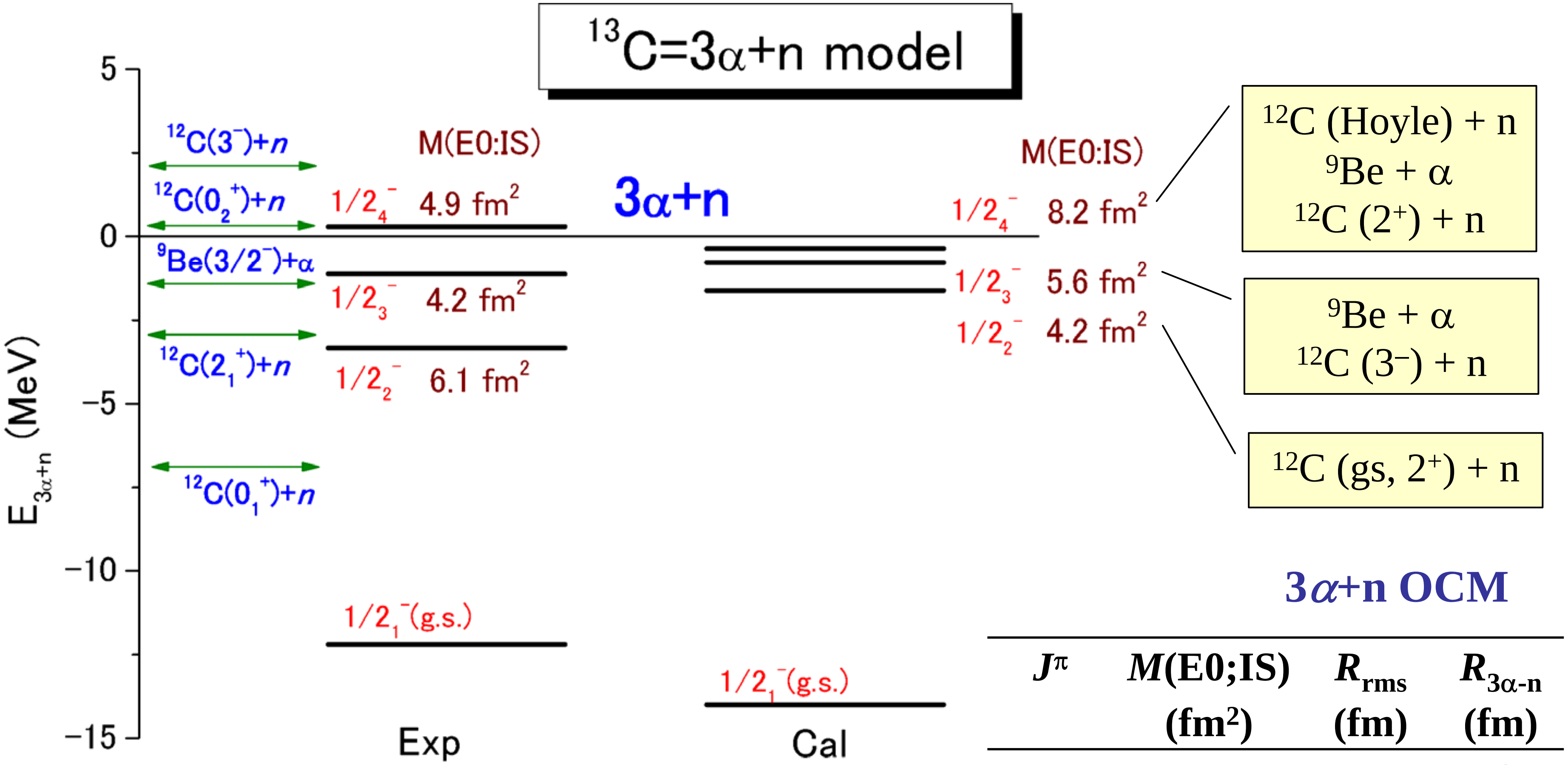

^{13}C(1/2-, 4-th): M(E0,IS)=8.2 fm^{2}

^{9}Be+α

^{12}C(Hoyle)+n

^{12}C(2^{+})+n

M(E0:IS)



^{12}C (Hoyle) + n
^{9}Be + α
^{12}C (2$^+$) + n

^{9}Be + α
^{12}C (3$^-$) + n

^{12}C (gs, 2$^+$) + n

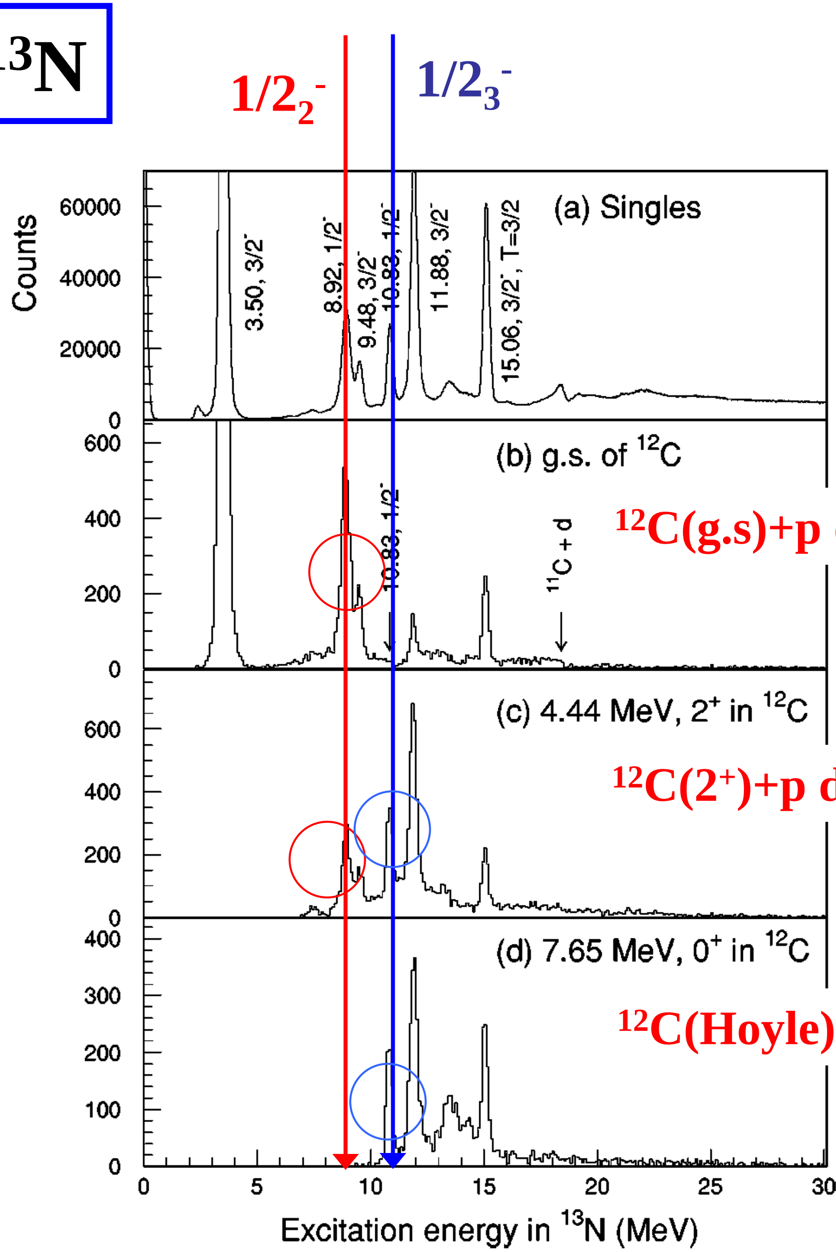
Cluster structure

3α+n OCM

J^{π}	M(E0;IS) (fm^2)	R_{rms} (fm)	$R_{3\alpha\text{-}n}$ (fm)
$1/2^-_1$		2.39	2.82
$1/2^-_2$	4.2	3.36	8.79
$1/2^-_3$	5.6	2.97	4.33
$1/2^-_4$	8.2	3.19	6.37

R_{ms}=4.3 fm for Hoyle state

^{13}N



^{12}C(g.s)+p decay

^{12}C(2^+)+p decay

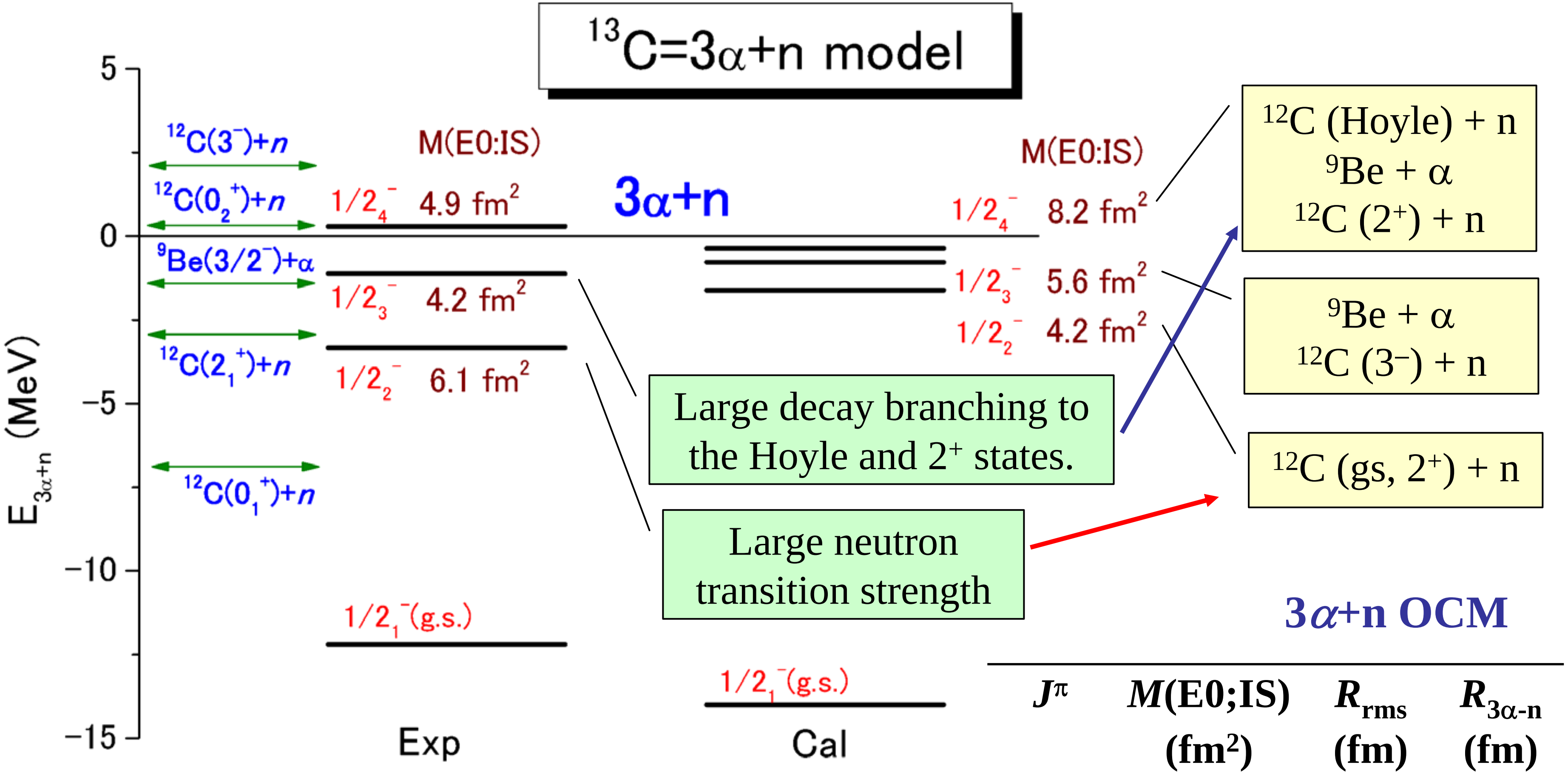
^{12}C(Hoyle)+p decay

^{13}C(^{3}He,t)^{13}N

$\rightarrow$ ^{12}C+p decay

Fujimura and Fujiwara et al., PRC69, 064327 (2004)

M(E0:IS)



^{12}C (Hoyle) + n
^{9}Be + α
^{12}C (2$^+$) + n

^{9}Be + α
^{12}C (3$^-$) + n

^{12}C (gs, 2$^+$) + n

Large decay branching to the Hoyle and 2$^+$ states.

Large neutron transition strength

3α+n OCM

J^π	M(E0;IS) (fm^2)	$R_{\rm rms}$ (fm)	$R_{3\alpha\text{-}n}$ (fm)
$1/2^-_1$		2.39	2.82
$1/2^-_2$	4.2	3.36	8.79
$1/2^-_3$	5.6	2.97	4.33
$1/2^-_4$	8.2	3.19	6.37

$R_{\rm ms}$=4.3 fm for Hoyle state

Summary (2)

- **Mechanism of *M*(E0) transition in light nuclei**

  good tool to explore cluster structures

- **^{13}C（1/2$^-$）, structure and *M*(E0)：3α+n OCM**

  2nd 1/2- ; ^{12}C(0+,2+)+n
  3rd 1/2- ; ^{9}Be+α, ^{12}C(3-,1-)+n
  4-th 1/2- ; ^{12}C(Hoyle)+n, ^{9}Be+α, ^{12}C(2+)+n
  Nuclear radii =3.0~3.4 fm : cluster structure

- **^{13}C(1/2$^+$)**

  3nd 1/2$^+$; strong candidate of dilute α condensation
  ***R*~5 fm, Occu. Prob. ~60%**

- **Future : 3/2-, other states, *LS*-splitting.**