Variational calculation of K-pp system

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1. Introduction

Nucl. Phys. A804, 197 (2008) arXiv:0806.4917v1 To be published in PRC.

- 2. Model wave function
- 3. Effective *K^{bar}N* potential and self-consistency on kaon's energy
- 4. Result of systematic study with effective *s*-wave *K^{bar}N* potential
- 5. Other effects
 - Dispersive effect, *P*-wave interaction, Two nucleons absorption
- 6. Summary and Comments

「少数粒子系物理の現状と今後の展望」研究会 08.12.25 @ RCNP

1. Introduction

People have focused on

K-pp, "a prototype of K^{bar} nuclei"

But, different theoretical studies give different results from each other...



1. Introduction

Experiments concerned to this subject...



- К⁻ absorption on various targets - Лр invariant mass

H. Fujioka's talk (FINUDA collaboration) at PANIC'08

• Re-analysis of KEK E549



- K⁻ stopped on ⁴He target
- Λp invariant mass

T. Suzuki et al (KEK-PS E549 collaboration), arXiv:0711.4943v1[nucl-ex]

DISTO collaboration



- *p* + *p* -> *K*⁺ + Λ + *p* @ 2.85GeV - Λ*p* invariant mass

T. Yamazaki's talk at EXA'08

1. Introduction

✓ Variational method

... Investigate various properties with the obtained wave function.

✓ Av18 NN potential

... a realistic NN potential with strong repulsive core.

✓ K^{bar}N potential based on Chiral SU(3) theory

... Well describe S=-1 meson-baryon scattering and dynamical generation of $\Lambda(1405)$

Thanks, 2. Model wave function Akaishi-san! Model wave function — Simple Correlated Model (Revised) $\left|\Psi_{SCM2}\right\rangle = N^{-1/2} \left[\left|\Phi_{1}\right\rangle + C_{mix} \left|\Phi_{0}\right\rangle \right]$ $J^{\pi} = 0^{-}, T = \frac{1}{2}$ $\left|\left|\Phi_{1}\right\rangle = \Phi_{1}\left(\overrightarrow{r_{1}}, \overrightarrow{r_{2}}, \overrightarrow{r_{K}}\right)\right|S_{N} = 0\right\rangle \left|\left[NN\right]_{T_{N}=1}\overline{K}\right|_{1/2-1/2}\right\rangle$ $\left|\left|\Phi_{0}\right\rangle = \Phi_{0}\left(\vec{r_{1}}, \vec{r_{2}}, \vec{r_{K}}\right)\right|S_{N} = 0\left|\left[NN\right]_{T_{N}=0}\overline{K}\right|_{1/2, 1/2}\right\rangle$ $\Phi_1\left(\vec{r_1}, \vec{r_2}, \vec{r_K}\right) = G\left(\vec{r_1}\right)G\left(\vec{r_2}\right)G'\left(\vec{r_K}\right) \cdot F\left(\vec{r_1}, \vec{r_2}\right)\left\{F_1'\left(\vec{r_1}, \vec{r_K}\right)F_2'\left(\vec{r_2}, \vec{r_K}\right) + F_1'\left(\vec{r_2}, \vec{r_K}\right)F_2'\left(\vec{r_1}, \vec{r_K}\right)\right\}$ $\Phi_0\left(\vec{r_1}, \vec{r_2}, \vec{r_K}\right) = G\left(\vec{r_1}\right)G\left(\vec{r_2}\right)G'\left(\vec{r_K}\right) \cdot F\left(\vec{r_1}, \vec{r_2}\right)\left\{F_1'\left(\vec{r_1}, \vec{r_K}\right)F_2'\left(\vec{r_2}, \vec{r_K}\right) - F_1'\left(\vec{r_2}, \vec{r_K}\right)F_2'\left(\vec{r_1}, \vec{r_K}\right)\right\}$ NN correlation function Single-particle motion $F(\vec{r_1}, \vec{r_2}) = 1 - \sum f_n^{NN} \exp \left| -\lambda_n^{NN} (\vec{r_1} - \vec{r_2})^2 \right|$ $G\left(\vec{r_i}\right) = \exp\left[-\mu \vec{r_i}^2\right]$ Nucleon $G'(\overrightarrow{r_K}) = \exp\left[-\gamma \overrightarrow{r_K}^2\right]$ K^{bar}N correlation function Kaon $F_{a}'\left(\overrightarrow{r_{i}}, \overrightarrow{r_{K}}\right) = 1 + \sum f_{n}^{KN, a} \exp\left|-\lambda_{n}^{KN}\left(\overrightarrow{r_{K}} - \overrightarrow{r_{i}}\right)^{2}\right]$



2. NN potential

A realistic NN potential ... Av18 potential

 Use one-pion-exchange potential (Central and Spin-Spin) and L² potentials in addition to phenomenological central term (= repulsive core).

Central potential



2. Model

Influence of the improvement

	$T_N = 1 + T_N = 0$	T _N =1 only	
C _{mix}	SCM ver2	SCM ver1	ATMS
	Finite	Zero	
Β. Ε.	51.4	39.0	48
Γ(K ^{bar} N → π Y)	61.0	60.0	61
B(K)	80.0	65.8	68
Nucl. E	28.7	26.8	20
Kinetic	162.4	147.0	167
NN pot	-19.2	-19.8	-19
K ^{bar} N pot	-194.6	-166.2	-196
Rel (NN)	1.83	1.75	1.90
Rel (KN)	1.55	1.54	1.57
Mixing ratio	5.9%	0.0%	

NN potential ... Tamagaki potential Effective *K^{bar}N* potential ... Akaishi-Yamazaki potential

Influence of the improvement	$h_{t} = \langle \Phi_{1} \tilde{\nu} \rangle$	$\widetilde{K}_{KN} \Phi_1 \rangle$	$= \frac{3}{4} V_{I=0}^{\overline{K}N} +$	$\frac{1}{4}V_{I=1}^{\overline{K}N}$			
	$\frac{T_{N}}{S} \left\langle \Phi_{0} \right $	$\widehat{\mathcal{V}}_{\overline{K}N} \left \mathbf{\Phi}_{0} \right\rangle$	$= \frac{1}{4} V_{I=0}^{\overline{K}N} +$	$-\frac{3}{4}V_{I=1}^{\overline{K}N}$			
C _{mix}			6				
B. E.	$\Phi_0 i$	$\widehat{\mathcal{V}}_{\overline{K}N} \left \mathbf{\Phi}_{1} \right\rangle$	$= \frac{\sqrt{3}}{4} \left[V_{I=0}^{\overline{K}} \right]$	$V_{D} - V_{I=1}^{\overline{K}N}$			
Additional K ^{bar}	N attra	ction .8	68				
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3. Local *K^{bar}N* potential based on Chiral SU(3) *T. Hyodo and W. Weise, PRC77, 035204(2008)*

Effective K^{bar}N potential ...

reproduce the scattering K^{bar}N amplitude obtained with Chiral unitary model

$$U_{I}^{Eff}\left(r,\sqrt{s}\right) = V_{I}^{Eff}\left(\sqrt{s}\right) \cdot g_{a}\left(r\right)$$

$$g_a(r) = \left(\frac{1}{\pi a^2}\right)^{3/2} \exp\left[-\left(r / a\right)^2\right]$$

- Single channel ... only $K^{bar}N$ channel, $\pi\Sigma$ is eliminated.
- Energy dependent and Complex potential
- Local, Gaussian form

 $T_{ij}^{Ch.U.} = V_{ij} + V_{ik} G_k T_{kj}^{Ch.U.} : \underbrace{\text{Chiral unitary}}_{\text{Relativistic / Coupled Channel}}$ $T_{11}^{Ch.U.} \cdots \text{T-matrix for } K^{bar} N \text{ channel}$ $T_{11}^{Ch.U.} = U^{Eff} + U^{Eff} G_1 T_{11} : \underbrace{\text{Effective local potential}}_{\text{Non-relativistic / Single Channel}}$

3. Local *K^{bar}N* potential based on Chiral SU(3)

I=0 K^{bar}N scatteing amplitude



Chiral unitary; T. Hyodo, S. I. Nam, D. Jido, and A. Hosaka, Phys. Rev. C68, 018201 (2003)

3. Local $K^{bar}N$ potential based on Chiral SU(3)

Four Chiral unitary models:

- "ORB" E. Oset, A. Ramos, and C. Bennhold, Phys. Lett. B527, 99 (2002)
- "HNJH" T.Hyodo, S. I. Nam, D. Jido, and A. Hosaka, Phys. Rev. C68, 018201 (2003)
- "BNW" B. Borasoy, R. Nissler, and W. Weise, Eur. Phys. J. A25, 79 (2005)
- "BMN" B. Borasoy, U. G. Meissner, and R. Nissler, Phys. Rev. C74, 055201 (2006)



3. Self-consistency of kaon's energy

> Hamiltonian $\widehat{H} = \widehat{T} + \widehat{V}_{NN} + \operatorname{Re}\left[\widehat{V}_{KN-S}\left(\sqrt{S}\right)\right] - \widehat{T}_{CM}$

 $\widehat{V}_{NN} = V_{C}(r) + V_{ss}(r)\widehat{\sigma}_{1}\cdot\widehat{\sigma}_{2} + V_{L2}(r)\widehat{\mathbf{L}}^{2}$

Im $\widehat{V}_{KN-S}\left(\sqrt{s}\right)$ treated perturbatively.

✓ Self-consistency for anti-kaon's energy $\longleftrightarrow \widehat{V}_{KN-S}(\sqrt{s})$

Controlled by "Anti-kaon's binding energy"

$$B(K) \equiv -\left\{ \left\langle \widehat{H} \right\rangle - \left\langle \widehat{H}_{Nucl} \right\rangle \right\}$$

 $\widehat{H}_{\mathit{Nucl}}$: Hamiltonian of nuclear part

> Try two ansatz for \sqrt{s}

$$\sqrt{s} = M_N + \omega = \begin{cases} M_N + m_K - B(K) \\ M_N + m_K - B(K)/2 \end{cases}$$



K^{bar} field surrounds two nucleons which are almost static.

of kaon's energy $\begin{bmatrix} \overline{s} \\ \end{array} \end{bmatrix} - \widehat{T}_{CM}$ $\hat{V}_{NN} = V_C(r) + V_{ss}(r) \widehat{\sigma}_1 \cdot \widehat{\sigma}_2 + V_{L2}(r) \widehat{L}^2$

perturbatively.

nergy
$$\leftarrow$$
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ı's el

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 $\widehat{H}_{\it Nucl}$: Hamiltonian of nuclear part

4. Result with effective *s*-wave *K^{bar}N* potential



 $I=0 K^{bar}N$ distance = 1.82 fm $I=1 K^{bar}N$ distance = 2.33 fm

... *I=0 K^{bar}N* potential is more attractive than *I=1* one.



Density distribution K^{bar}N pair in K⁻pp vs Λ(1405)

 $r^{2}
ho_{\overline{K}N}^{Normalized}\left(r
ight)$

K^{bar}N potential based on "HNJH" "Corrected", $\sqrt{s} = M_N + m_K - B(K)$

For comparison, All densities are normalized to 1.





Density distribution K^{bar}N pair in K⁻pp vs Λ(1405)



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5. Other effects

Estimate other possible effects, perturbatively...

I. Dispersive correction (Effect of imaginary part)

II. p-wave K^{bar}N potential

III. Two nucleon absorption

I. Dispersive correction

(Effect of imaginary potential)

Two-body system (*I=0 K^{bar}N*) case

Lippmann-Schwinger eq.

with

the complex potential



Schroedinger eq. with only the real part

B. E. ∼ 10 *M*eV





I. Dispersive correction

(Effect of imaginary potential)

Two-body system (I=0 K^{bar}N) case





Schroedinger eq. with only the real part



Dispersive correction for $K^{bar}N$ system ~ +5 MeV to B. E.

Considering **Four** variants of models...

X2

Dispersive correction for K^{bar}NN



II. P-wave K^{bar}N potential

- Estimate its contribution *perturbatively*.
- Derived from "Full" scattering volume.

$$v_{KN,P-wave}\left(\mathbf{r}_{K}-\mathbf{r}_{N},\omega\right) = -\frac{1}{2\omega} \cdot \frac{4\pi\sqrt{s}}{M_{N}} C_{KN}\left(\omega\right) \cdot \vec{\nabla} \frac{1}{\pi^{3/2} a_{P}^{3}} \exp\left[-\left(\mathbf{r}_{K}-\mathbf{r}_{N}\right)^{2} / a_{P}^{2}\right] \vec{\nabla}$$



R. Brockmann, W. Weise and L. Tauscher, Nucl. Phys. A 308, 365 (1978) For $a_p = 0.4 \sim 0.9$ fm,

$$\checkmark Re V_{KN,P} \sim +3 MeV$$

...small and repulsive

$$\checkmark \Gamma_{p\text{-wave}} = -2 \text{ Im } V_{KN,P} \\ = 10 \ \thicksim 35 \text{ MeV}$$

$$\Delta\Gamma_{abs} = \frac{2\pi\tilde{B}_0}{\omega}\beta_{pp}(\omega)\int d^3\boldsymbol{r}\,\rho_{\bar{K}}(\boldsymbol{r})\,\rho_N^2(\boldsymbol{r}),$$

J. Mares, E. Friedman and A. Gal, Nucl. Phys. A 770, 84 (2006)

- For mean-field model ... no correlation between two nucleons
- Contact interaction



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• Finite range interaction (between NN)



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- $\Delta\Gamma_{abs}(K^{-}pp \rightarrow YN) = \frac{2\pi B_{0}}{\omega}\beta_{pp}(\omega)$ $\times \int d^{3}r \int d^{3}x \rho^{(3)}(r, r, x)G(x r; a)$ Three-body
 Gaussian-type
 Interaction for NN
- Finite range interaction (between NN)

$$\Delta\Gamma_{abs} = 4 \sim 12 \text{ MeV}$$
for $a = 0.6 \text{ fm}$,
 $B_0 = 0.85 \sim 1.5 \text{ fm}^4$

6. Summary

- We studied K⁻pp with a variational method, using a realistic NN potential (Av18) and a Chiral SU(3)-based K^{bar}N potential.
- We tried four variants of Chiral unitary models and two ansatz of K^{bar}N energy in the three-body system. However, the result doesn't depend so much on them. Total binding energy and mesonic decay width are in the small window:

Binding energy and mesonic decay width of K-pp

> Total binding energy and mesonic decay width are in the small window:

Total Binding energy = $20 \pm 3 \text{ MeV}$ $\Gamma(K^{bar}N \rightarrow \pi Y) = 40 \sim 70 \text{ MeV}$... Very shallow binding

Structure of K-pp

- NN distance in K⁻pp is smaller than that of deuteron, rather comparable to that in normal nuclei.
 NN distance = ~2.2 fm K^{bar}N distance = ~2.0 fm
- > The I=0 $K^{bar}N$ component in K-pp is found to be very similar to $\Lambda(1405)$.

Contribution of other effects

Dispersive correction: +6 ~ +18MeV to B.E.
p-wave K^{bar}N potential: ~ -3MeV to B.E., 10 ~ 35MeV to width
Two nucleon absorption: 4 ~ 12MeV to width

6. Summary

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Structure of K-pp

NN distance in rather compare

➤ The I=0 K^{bar}N (

 Λ (1405) almost survives in *K*-*pp*. But, such a state is very short-lived. distance = \sim 2.2 fm distance = \sim 2.0 fm

Contribution of other effects

• Disț	By rough estimation,		
• <i>p-w</i>	Total Binding energy =	20 ~ 40 MeV	widi
• Twc	$\Gamma(total) =$	55 ~ 120 MeV	wid

6. Comment

• Difference from Faddeev calculation with a separable K^{bar}N potential constrained with Chiral SU(3) theory

> Total B. E. = 79 MeV, Decay width = 74 MeV

by Dr. Ikeda and Prof. Sato

Separable potential?

Non-relativistic (semi-relativistic) vs relativistic?

Energy dependence of two-body system (K^{bar}N) in the three-body system (K^{bar}NN)?

Important role of $\pi\Sigma N$ thee-body dynamics?

Y. Ikeda and T. Sato, arXiv:0809.1285

...???

6. Comment

• What is the object measured experimentally?



Just a bound state of K-pp, should $\pi\Sigma N$ be included???

Even if the experimental result is not an artifact, only what we can say at the moment from this is "There is some object with B=2, S=-1, charge=+1"...

- How to distinguish which is the dominant component, K-pp or $\pi\Sigma N$ experimentally?
- Since the signal position is very close to π + Σ +N threshold, it is more natural that the observed state is mainly the bound state of $\pi\Sigma N$?



Coupled channel calculation of $K^{bar}NN-\pi\Sigma N$, using a realistic NN potential and a $K^{bar}N$ and πY potential derived from Chiral SU(3) theory. (NOT separable type)

