Theory Prospective on $(\beta\beta)^{0\nu}_{0\nu}$-Decay
(The Quest for the Nature of Massive Neutrinos)

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Double Beta Decay and Underground Science (DBD16)
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Determining the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos is one of the most challenging and pressing problems in present day elementary particle physics.
\( \nu_j \) – Dirac or Majorana particles, \textit{fundamental problem}

\( \nu_j \) – Dirac: \textit{conserved lepton charge exists},
\[ L = L_e + L_\mu + L_\tau, \nu_j \neq \bar{\nu}_j \]

\( \nu_j \) – Majorana: \textit{no lepton charge is exactly conserved},
\[ \nu_j \equiv \bar{\nu}_j \]

The observed patterns of \( \nu \)–mixing and of \( \Delta m^2_{\text{atm}} \) and \( \Delta m^2_\odot \) can be related to Majorana \( \nu_j \) and a \textit{new fundamental (approximate) symmetry}.

\[ L' = L_e - L_\mu - L_\tau \]

\textit{See-saw mechanism:} \( \nu_j \) – Majorana

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S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
Establishing that the total lepton charge $L = L_e + L_\mu + L_\tau$ is not conserved in particle interactions by observing the $(\beta\beta)_{0\nu}$ decay would be a fundamental discovery (similar to establishing baryon number nonconservation (e.g., by observing proton decay)).

Establishing that $\nu_j$ are Majorana particles would be of fundamental importance, as important as the discovery of $\nu-$ oscillations, and would have far reaching implications.
Current Challenging Problems:

- determination of the neutrino mass ordering (T2K + NOνA; JUNO; PINGU, ORCA; T2HKK, DUNE);
- determination of the absolute neutrino mass scale, or \(\min(m_j)\) (KATRIN, new ideas; cosmology);
- determination of the status of the CP symmetry in the lepton sector (T2K, NOνA; DUNE, T2HK).
There have been remarkable discoveries in neutrino physics in the last $\sim 18$ years.
Compellings Evidence for $\nu$–Oscillations

$-\nu_{\text{atm}}$: SK \text{ UP-DOWN ASYMMETRY}

$\theta_z$, $L/E$– dependences of $\mu$–like events

Dominant $\nu_\mu \rightarrow \nu_\tau$  K2K, MINOS, T2K; CNGS (OPERA)

$-\nu_{\odot}$: Homestake, Kamiokande, SAGE, GALLEX/GNO

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$  BOREXINO

$-\bar{\nu}_e$ (from reactors): Daya Bay, RENO, Double Chooz

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS ($\nu_\mu$ from accelerators): $\nu_\mu \rightarrow \nu_e$
Compelling Evidences for $\nu$–Oscillations: $\nu$ mixing

$$|\nu_l >= \sum_{j=1}^{n} U_{lj}^* |\nu_j >, \quad \nu_j : m_j \neq 0; \quad l = e, \mu, \tau; \quad n \geq 3;$$

$$\nu_{lL}(x) = \sum_{j=1}^{n} U_{lj} \nu_{jL}(x), \quad \nu_{jL}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$  

B. Pontecorvo, 1957; 1958; 1967;  
Z. Maki, M. Nakagawa, S. Sakata, 1962;

$U$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

$\nu_j, m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 $\nu$s are light: $\nu_{1,2,3}, m_{1,2,3} \lesssim 1$ eV.
The Charged Current Weak Interaction Lagrangian:

\[ \mathcal{L}_{CC}^{CC}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x) \gamma^\alpha (1 - \gamma^5) \nu_{lL}(x) W^\alpha(x) + \text{h.c.}, \]

\[ \nu_{lL}(x) = \sum_{j=1}^{n} U_{lj} \nu_{jL}(x), \quad \nu_{jL}(x) : m_j \neq 0; \quad l = e, \mu, \tau. \]
Dr. T. Kajita, Prof. A. McDonald, Nobel Prize for Physics winners, 2015

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
LAUREATES

Breakthrough Prize  Special Breakthrough Prize  New Horizons Prize  Physics Frontiers Prize


Kam-Biu Luk and the Daya Bay Collaboration  Yifang Wang and the Daya Bay Collaboration  Koichiro Nishikawa and the K2K and T2K Collaboration  Atsuto Suzuki and the KamLAND Collaboration

Arthur B. McDonald and the SNO Collaboration  Takaaki Kajita and the Super K Collaboration  Yoichiro Suzuki and the Super K Collaboration
These data imply that

\[ m_{\nu_j} \ll m_{e,\mu,\tau, q}, \quad q = u, c, t, d, s, b \]

For \( m_{\nu_j} \lesssim 1 \text{ eV} \):

\[ m_{\nu_j}/m_{l,q} \lesssim 10^{-6} \]

For a given family:

\[ 10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^{2} \]
These discoveries suggest the existence of New Physics beyond that of the ST.
The New Physics can manifest itself (can have a variety of different “flavours”):

- In the existence of more than 3 massive neutrinos: $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...).
- In the Majorana nature of massive neutrinos.
- In the observed pattern of neutrino mixing and in the values of the CPV phases in the PMNS matrix.
- In the existence of new particles, e.g., at the TeV scale: heavy Majorana Neutrinos $N_j$, doubly charged scalars,...

- In the existence of LFV processes: $\mu \rightarrow e + $, $\mu \rightarrow 3e$, $\mu - e$ conversion, etc., which proceed with rates close to the existing upper limits.
- In the existence of new (FChNC, FCFNSNC) neutrino interactions.
- In the existence of “unknown unknowns”...
We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \ldots$) if, e.g., sterile $\nu_R$, $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos $\nu_l$ ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,\ldots} \sim 1$ eV; in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analyses of short baseline (SBL) reactor neutrino oscillation data (“reactor neutrino anomaly”), data of radioactive source callibration of the solar neutrino SAGE and GALLEX experiments (“Gallium anomaly”); tests (STEREO, SOX, CeLAND, DANS, ICARUS (at Fermilab), … under way).

ii) $M_{4,5,\ldots} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models; $M_{4,5,\ldots} \sim (10^9 - 10^{13})$ GeV, “classical” seesaw models.
All compelling data compatible with 3-ν mixing:

\[ \nu_{lL} = \sum_{j=1}^{3} U_{lj} \nu_{jL} \quad l = e, \mu, \tau. \]

The PMNS matrix $U$ - 3 × 3 unitary to a good approximation (at least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

$\nu_j$, $m_j \neq 0$: Dirac or Majorana particles.

3-ν mixing: 3-flavour neutrino oscillations possible.

$\nu_\mu$, $E$; at distance $L$: $P(\nu_\mu \to \nu_\tau (e)) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$
Three Neutrino Mixing

\[ \nu_{iL} = \sum_{j=1}^{3} U_{ij} \nu_{jL} . \]

\( U \) is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

\[
U = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\]

- \( U \) - \( n \times n \) unitary:
  \[
  \begin{array}{cccc}
  n & 2 & 3 & 4 \\
  \end{array}
  \]

mixing angles:
  \[ \frac{1}{2}n(n-1) \quad 1 \quad 3 \quad 6 \]

CP-violating phases:

- \( \nu_{j} \) - Dirac:
  \[ \frac{1}{2}(n-1)(n-2) \quad 0 \quad 1 \quad 3 \]

- \( \nu_{j} \) - Majorana:
  \[ \frac{1}{2}n(n-1) \quad 1 \quad 3 \quad 6 \]

\( n = 3 \): 1 Dirac and 2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980
PMNS Matrix: Standard Parametrization

\[ U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}, \]

\[ V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & s_{12}c_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} & c_{23}c_{13} \end{pmatrix} \]

- \( s_{ij} \equiv \sin \theta_{ij}, \ c_{ij} \equiv \cos \theta_{ij}, \ \theta_{ij} = [0, \pi/2], \)

- \( \delta \) - Dirac CPV phase, \( \delta = [0, 2\pi]; \) CP inv.: \( \delta = 0, \pi, 2\pi; \)

- \( \alpha_{21}, \alpha_{31} \) - Majorana CPV phases; CP inv.: \( \alpha_{21(31)} = k(k')\pi, \ k(k') = 0, 1, 2... \)

- \( \Delta m_{21}^2 \equiv \Delta m_{31}^2 \approx 7.37 \times 10^{-5} \text{ eV}^2 > 0, \ \sin^2 \theta_{12} \approx 0.297, \ \cos 2\theta_{12} \gtrsim 0.29 \ (3\sigma), \)

- \( |\Delta m_{31(32)}^2| \approx 2.53 \ (2.43) \times 10^{-3} \text{ eV}^2, \ \sin^2 \theta_{23} \approx 0.437 \ (0.569), \ NO \ (IO), \)

- \( \theta_{13} \) - the CHOOZ angle: \( \sin^2 \theta_{13} = 0.0214 \ (0.0218), \ Capozzi \ et \ al. \ NO \ (IO). \)

F. Capozzi et al. (Bari Group), arXiv:1601.07777v1.
\[
\Delta m^2_\odot \equiv \Delta m^2_{21} \approx 7.54 \times 10^{-5} \text{ eV}^2 > 0, \ \sin^2 \theta_{12} \approx 0.308, \ \cos 2\theta_{12} \gtrsim 0.28 \ (3\sigma),
\]

\[
|\Delta m^2_{31(32)}| \approx 2.47 \ (2.42) \times 10^{-3} \text{ eV}^2, \ \sin^2 \theta_{23} \approx 0.437 \ (0.455), \ NO \ (IO),
\]

\[
\theta_{13} - \text{the CHOOZ angle: } \sin^2 \theta_{13} = 0.0234 \ (0.0240), \ NH \ (IH).
\]

\[
1\sigma(\Delta m^2_{21}) = 2.6\%, \ 1\sigma(\sin^2 \theta_{12}) = 5.4\% ;
3\sigma(\Delta m^2_{21}) : (6.93 - 7.97) \times 10^{-5} \text{ eV}^2 ; \ 3\sigma(\sin^2 \theta_{12}) : (0.250 - 0.354) ;
\]

\[
3\sigma(|\Delta m^2_{31(23)}|) : 2.27(2.23) - 2.65(2.60) \times 10^{-3} \text{ eV}^2 ;
(2.40(2.30) - 2.66(2.57) \times 10^{-3} \text{ eV}^2 ;
3\sigma(\sin^2 \theta_{23}) : 0.374(0.380) - 0.628(0.641) ;
(3\sigma(\sin^2 \theta_{23}) : 0.379(0.383) - 0.616(0.637))
\]

\[
3\sigma(\sin^2 \theta_{13}) : 0.0176(0.0178) - 0.0296(0.0298)
(3\sigma(\sin^2 \theta_{13}) : 0.0185(0.0186) - 0.0246(0.0248)).
\]

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)
(F. Capozzi et al. (Bari Group), arXiv:1601.07777v1.)
• $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31(32)})$ not determined

\[ \Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \text{ normal mass ordering (NO)} \]

\[ \Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \text{ inverted mass ordering (IO)} \]

Convention: \( m_1 < m_2 < m_3 \) - NO, \( m_3 < m_1 < m_2 \) - IO

\[ \Delta m^2_{31}(\text{NO}) = -\Delta m^2_{32}(\text{IO}), \quad \Delta m^2_{32}(\text{NO}) = -\Delta m^2_{31}(\text{IO}) \]

\[ m_1 \ll m_2 < m_3, \quad \text{NH,} \]

\[ m_3 \ll m_1 < m_2, \quad \text{IH,} \]

\[ m_1 \approx m_2 \approx m_3, \quad m^2_{1,2,3} >> |\Delta m^2_{31(32)}|, \quad \text{QD;} \quad m_j \geq 0.10 \text{ eV.} \]

• \( m_2 = \sqrt{m^2_1 + \Delta m^2_{21}}, \quad m_3 = \sqrt{m^2_1 + \Delta m^2_{31}} \) - NO;

• \( m_1 = \sqrt{m^2_3 + \Delta m^2_{23} - \Delta m^2_{21}}, \quad m_2 = \sqrt{m^2_3 + \Delta m^2_{23}} \) - IO;
The (Mass)$^2$ Spectrum

$\Delta m^2_{\text{sol}} \approx 7.6 \times 10^{-5} \text{ eV}^2$, \hspace{1cm} $\Delta m^2_{\text{atm}} \approx 2.4 \times 10^{-3} \text{ eV}^2$

Are there more mass eigenstates, as LSND suggests, and MiniBooNE recently hints?
\[ m^2 \]

\[ m_1^2 \quad m_2^2 \quad m_3^2 \]

\[ \text{atmospheric} \sim 2 \times 10^{-3} \text{eV}^2 \]

\[ \text{solar} \sim 7 \times 10^{-5} \text{eV}^2 \]

\[ \nu_e \quad \nu_\mu \quad \nu_\tau \]

S. King, Ch. Luhn, 2013
• **Dirac phase** $\delta$: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$

  $J_{CP} = \text{Im} \{U_{e1} U_{\mu2} U_{e2}^* U_{\mu1}^*\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$

  Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \approx -0.035$.

• **Majorana phases** $\alpha_{21}, \alpha_{31}$:
  
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

  - $|<m>|$ in $(\beta\beta)_{0\nu}$–decay depends on $\alpha_{21}, \alpha_{31}$;

  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;

  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$ !
Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{\text{atm}}} < \text{Mass}[\text{Heaviest } \nu_i]$
Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on $^{3}\text{H} \rightarrow ^{3}\text{He} + e^{-} + \bar{\nu}_{e}$:

\[ m_{\nu_{e}} < 2.2 \text{ eV} \quad \text{(95\% C.L.)} \]

We have \( m_{\nu_{e}} \cong m_{1,2,3} \) in the case of QD spectrum. The upcoming KATRIN experiment is planned to reach sensitivity

\[ \text{KATRIN: } m_{\nu_{e}} \sim 0.2 \text{ eV} \]

i.e., it will probe the region of the QD spectrum.
Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum j m_j$: the Planck + WMAP (low $l \leq 25$) + ACT (large $l \geq 2500$) CMB data + $\Lambda$CDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

$$\sum_j m_j : \quad \delta \cong (0.01 - 0.04) \text{ eV.}$$

**NH:** $\sum_j m_j \leq 0.05 \text{ eV} \quad (3\sigma)$;

**IH:** $\sum_j m_j \geq 0.10 \text{ eV} \quad (3\sigma)$.  

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
Future Progress

- Determination of the nature - Dirac or Majorana, of $\nu_j$.
- Determination of $\text{sgn}(\Delta m^2_{\text{atm}})$, type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \quad \text{NH},$$

$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 > \Delta m^2_{\text{atm}}, \quad \text{QD}; \quad m_j \geq 0.10 \text{ eV}.$$  

- Determining, or obtaining significant constraints on, the absolute scale of $\nu_j$-masses, or $\text{min}(m_j)$.
- Status of the CP-symmetry in the lepton sector: violated due to $\delta$ (Dirac), and/or due to $\alpha_{21}, \alpha_{31}$ (Majorana)?
- High precision determination of $\Delta m^2_\odot, \theta_{12}, \Delta m^2_{\text{atm}}, \theta_{23}, \theta_{13}$
- Searching for possible manifestations, other than $\nu_l-$oscillations, of the non-conservation of $L_l, l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.
• Understanding at fundamental level the mechanism giving rise to the $\nu$– masses and mixing and to the $L_l$–non-conservation. Includes understanding
  – the origin of the observed patterns of $\nu$-mixing and $\nu$-masses ;
  – the physical origin of $CPV$ phases in $U_{PMNS}$ ;
  – Are the observed patterns of $\nu$-mixing and of $\Delta m_{21,31}^2$ related to the existence of a new symmetry?
    – Is there any relations between $q$–mixing and $\nu$– mixing? Is $\theta_{12} + \theta_c = \pi/4$ ?
    – Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?
    – Is there any correlation between the values of $CPV$ phases and of mixing angles in $U_{PMNS}$?

• Progress in the theory of $\nu$-mixing might lead to a better understanding of the origin of the BAU.
  – Can the Majorana and/or Dirac CPVP in $U_{PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?
The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos \( ((\beta\beta)_0\nu^-) \)-decay exps: GERDA, CUORE, EXO, KamLAND-Zen, SNO+, SuperNEMO, MAJORANA, AMORE,...).

- determination of the status of the CP symmetry in the lepton sector (T2K, NO\(\nu\)A; DUNE, T2HK)

- determination of the neutrino mass ordering (JUNO, RENO50; ORCA, PINGU (IceCube), HK, INO; T2K + NO\(\nu\)A; DUNE (future); + T2HKK (future)) ;

- determination of the absolute neutrino mass scale, or \( \min(m_j) \) (KATRIN, new ideas; cosmology);

The program of research extends beyond 2030.

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
$\delta \cong 3\pi/2?$
\[ J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} \]
\[ = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \]
Status and prospects of global analyses of neutrino mass-mixing parameters

A. Marrone
Univ. of Bari & INFN

4-9 July 2016 — London — United Kingdom
Our “pre-London” reference analysis in the standard 3ν mixing scenario:

Bari group, arXiv:1601.07777 (NPB Special Issue on ν Oscillations)

Updated in a preliminary way with some data presented here at Neutrino 2016 (thanks in particular to F. Capozzi):
- New NOvA neutrino data in appearance and disappearance channels
- New T2K anti-neutrino data in appearance channel

Other updates not (yet) included

Please focus only on “trends” of the global analysis: numbers may change when a more refined and proper analysis of the new data will be performed in due time.
Bounds on single oscillation parameters
(preliminary update)

CP phase trend:
- $\delta \sim 1.4\pi$ at best fit
- CP-conserving cases ($\delta = 0, \pi$) disfavored at $\sim 2\sigma$ level or more
- Significant fraction of the $[0, \pi]$ range disfavored at $>3\sigma$

$\theta_{23}$ trend:
- Maximal mixing disfavored at about $\sim 2\sigma$ level
- Best-fit octant flips with mass ordering

$$\Delta\chi^2_{10-NO} = 3.1$$
Inverted ordering slightly disfavored
LBL Acc + Solar + KL + SBL Reactors + SK Atm

\[ N \sigma \]

\[ \delta m^2 / 10^{-5} \text{ eV}^2 \]

\[ \Delta m^2 / 10^{-3} \text{ eV}^2 \]

\[ \delta / \pi \]

\[ \sin^2 \theta_{12} \]

\[ \sin^2 \theta_{23} \]

\[ \sin^2 \theta_{13} \]

F. Capozzi, E. Lisi et al., arXiv:1312.2878

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
\[ P^{3\nu}(\bar{\nu}_e \to \bar{\nu}_e) = P^{3\nu}(\theta_{13}, \Delta m_{31}^2; \theta_{12}, \Delta m_{21}^2) \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2}{4E} L \right), \text{ no dependence on } \theta_{23}, \delta. \]

- **Daya Bay, July 2016 (Nu2016):**
  \[ \sin^2 2\theta_{13} = 0.0841 \pm 0.0033. \]

- **RENO, July 2016 (Nu2016):**
  \[ \sin^2 2\theta_{13} = 0.087 \pm 0.011. \]

- **Double Chooz, March 2016:**
  \[ \sin^2 2\theta_{13} = 0.111 \pm 0.018. \]
T2K: Search for $\nu_\mu \rightarrow \nu_e$ oscillations
T2K: Search for $\nu_{\mu} \rightarrow \nu_e$ oscillations

T2K: first results March 2011 (2 events); June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5$\sigma$; July, 2013 (28 events).

For $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV$^2$, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO (IO) spectrum:

$\sin^2 2\theta_{13} = 0.14$ (1.7), best fit.

This value is by a factor of $\sim 1.6$ (1.9) bigger than the value obtained in the Daya Bay and RENO experiments.

\[ P^3_{\nu_m}(\nu_\mu \rightarrow \nu_e) = P^3_{\nu_m}(\theta_{13}, \Delta m^2_{31(32)}, \theta_{12}, \Delta m^2_{21}, \theta_{23}, \delta). \]

After 2013 data also on $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations; July 2016 (Nu2016), 32 $\nu_\mu \rightarrow \nu_e + 4 \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ events.
The results on $\nu_\mu \rightarrow \nu_e$ oscillations from NO$\nu$A (August 6, 2015: 6 (12) events; July 2016 (Nu2016): 33 (34) events) are compatible with, and strengthened, the hint that $\delta \cong 3\pi/2$. 
Large $\sin\theta_{13} \cong 0.15 + \delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in $\nu$-oscillations; for the b.f.v. one has $J_{CP} \cong -0.035$;

- Important implications also for the “flavoured” leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to $\delta$, a necessary condition for reproducing the observed BAU is

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.09$$

Determining the Nature of Massive Neutrinos
Dirac CP-Nonconservation: $\delta$ in $U_{\text{PMNS}}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21}, \alpha_{31}$

S.M. Bilenky, J. Hosek, S.T.P., 1980; P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U_{jl}^\dagger$$

$$U = VP : \quad P_j e^{-i(E_j t - p_j x)} P_j^* = e^{-i(E_j t - p_j x)}$$

$P$ - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: $\nu_l$ oscillations are not sensitive to the nature of $\nu_j$. 

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
If $\nu_j$—Majorana particles, $U_{PMNS}$ contains ($3-\nu$ mixing)

$\delta$-Dirac, $\alpha_{21}$, $\alpha_{31}$ - Majorana physical CPV phases

$\nu$-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,
- are not sensitive to the nature of $\nu_j$,

• provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of $\nu_j$ masses.

The Majorana nature of $\nu_j$ can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + (A, Z) \rightarrow \mu^+ + (A, Z - 2)$$

The process most sensitive to the possible Majorana nature of $\nu_j$ - $(\beta \beta)_{0\nu}$-decay

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$$

of even-even nuclei, $^{48}\text{Ca}, ^{76}\text{Ge}, ^{82}\text{Se}, ^{100}\text{Mo}, ^{116}\text{Cd}, ^{130}\text{Te}, ^{136}\text{Xe}, ^{150}\text{Nd}$.  

$2n$ from $(A,Z)$ exchange a virtual Majorana $\nu_j$ (via the CC weak interaction) and transform into $2p$ of $(A,Z+2)$ and two free $e^-$. 

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
Nuclear $0\nu\beta\beta$-decay

strong in-medium modification of the basic process

$$dd \rightarrow uue^-e^- (\bar{\nu}_e \bar{\nu}_e)$$

virtual excitation of states of all multipolarities in $(A,Z+1)$ nucleus

V. Rodin, talk at Gran Sasso, 2006
$(\beta\beta)_{0\nu}$–Decay Experiments:
- $L$–nonconservation, Majorana nature of $\nu_j$.
- Type of $\nu$–mass spectrum (NH, IH, QD).
- Absolute neutrino mass scale.

$^3$H $\beta$-decay , cosmology: $m_\nu$ (QD, IH),
- Majorana CPV phases.
\[ A(\beta\beta)_{0\nu} \sim \langle m \rangle \quad M(A,Z), \quad M(A,Z) - \text{NME}, \]

\[ |\langle m \rangle| = |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 \quad e^{i\alpha_{21}} + m_3 |U_{e3}|^2 \quad e^{i\alpha_{31}}| \]

\[ = |m_1 \ c_{12}^2 \ c_{13}^2 + m_2 \ s_{12}^2 \ c_{13}^2 \ e^{i\alpha_{21}} + m_3 \ s_{13}^2 \ e^{i\alpha_{31}}|, \quad \theta_{12} \equiv \theta_\odot, \ \theta_{13} - \text{CHOOZ} \]

\[ \alpha_{21}, \ \alpha_{31} \quad ((\alpha_{31} - 2\delta) \rightarrow \alpha_{31}) - \text{the two Majorana CPVP of the PMNS matrix.} \]

**CP-invariance:** \( \alpha_{21} = 0, \pm \pi, \ \alpha_{31} = 0, \pm \pi; \)

\[ \eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1 \]

relative CP-parities of \( \nu_1 \) and \( \nu_2 \), and of \( \nu_1 \) and \( \nu_3 \) .

L. Wolfenstein, 1981;
S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;
\[ A(\beta \beta)_{0\nu} \sim <m> \quad \text{M}(A,Z), \quad \text{M}(A,Z) - \text{NME}, \]

\[ |<m>| \cong \sqrt{\Delta m^2_{\odot}} \sin^2 \theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}} \sin^2 \theta_{13} e^{i\beta M}|, \quad m_1 \ll m_2 \ll m_3 \quad \text{(NH)}, \]

\[ |<m>| \cong \sqrt{m^2_3 + \Delta m^2_{13}} |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_3 < (\ll)m_1 < m_2 \quad \text{(IH)}, \]

\[ |<m>| \cong m |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV} \quad \text{(QD)}, \]

\[ \theta_{12} \equiv \theta_{\odot}, \quad \theta_{13}-\text{CHOOZ}; \quad \alpha \equiv \alpha_{21}, \quad \beta_M \equiv \alpha_{31}. \]

**CP-invariance:** \[ \alpha = 0, \pm \pi, \quad \beta_M = 0, \pm \pi; \]

\[ |<m>| \cong 5 \times 10^{-3} \text{ eV}, \quad \text{NH}; \]

\[ \sqrt{\Delta m^2_{13}} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |<m>| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055 \text{ eV}, \quad \text{IH}; \]

\[ m \cos 2\theta_{12} \lesssim |<m>| \lesssim m, \quad m \gtrsim 0.10 \text{ eV}, \quad \text{QD}. \]
$\sin^2 \theta_{13} = 0.0214 \pm 0.0010; \ \delta = 0.$

$1\sigma(\Delta m^2_{21}) = 2.3\%, \ 1\sigma(\sin^2 \theta_{12}) = 5.6\%, \ 1\sigma(|\Delta m^2_{31(23)}|) = 1.7\%.$

F. Capozzi et al. (Bari Group), arXiv:1601.07777

$2\sigma(|<m>| \ )$ used.
Results from IGEX ($^{76}$Ge), NEMO3 ($^{100}$Mo), CUORICINO+CUORE-0 ($^{130}$Te):

**IGEX $^{76}$Ge:** $|<m>| < (0.33 - 1.35)$ eV (90% C.L.).

**Data from NEMO3 ($^{100}$Mo), CUORICINO+CUORE-0 ($^{130}$Te):**

$T(^{100}Mo) > 1.1 \times 10^{24}$ yr, $|<m>| < (0.3-0.6)$ eV;

$T(^{130}Te) > 4.0 \times 10^{24}$ yr.
Best Sensitivity Results from 2012-2016:

\[ T(^{136}\text{Xe}) > 1.6 \times 10^{25}\text{yr at 90\% C.L.}, \ \text{EXO} \]

\[ T(^{136}\text{Xe}) > 1.07 \times 10^{26}\text{yr at 90\% C.L.}, \ \text{KamLAND – Zen} \]

\[ |<m>| < (0.061 - 0.165) \text{ eV} . \]

\[ T(^{76}\text{Ge}) > 5.2 \times 10^{25}\text{yr at 90\% C.L.}, \ \text{GERDA II} \]

\[ |<m>| < (0.16 - 0.26) \text{ eV} . \]

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

\[ T(^{76}\text{Ge}) = 2.23^{+0.44}_{-0.31} \times 10^{25} \text{ yr at 90\% C.L.} \]

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
Large number of experiments: $|<m>| \sim (0.01-0.05)\text{ eV}$

CUORE - $^{130}\text{Te}$;  
GERDA-II - $^{76}\text{Ge}$; 
MAJORANA - $^{76}\text{Ge}$; 
KamLAND-ZEN - $^{136}\text{Xe}$; 
(n)EXO - $^{136}\text{Xe}$; 
SNO+ - $^{130}\text{Te}$; 
AMoRE - $^{100}\text{Mo}$ (S. Korea); 
CANDLES - $^{48}\text{Ca}$; 
SuperNEMO - $^{82}\text{Se}$, $^{150}\text{Nd}$; 
MAJORANA - $^{76}\text{Ge}$; 
NEXT - $^{136}\text{Xe}$; 
DCBA - $^{82}\text{Se}$, $^{150}\text{Nd}$; 
XMASS - $^{136}\text{Xe}$; 
PANDAX-III - $^{136}\text{Xe}$; 
ZICOS - $^{96}\text{Zr}$; 
MOON - $^{100}\text{Mo}$; 
...

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
GERDA: Experimental Setup

- Cleanroom and lock
- Control room
- Ge-strings
- LAr cryostat
- Ge-detectors
- Water purification & Radon monitor
- Muon Cherenkov veto

Kai Freund
Gerifil with GERDA
DPG 2012
Majorana CPV Phases and $|<m>|$

CPV can be established provided

- $|<m>|$ measured with $\Delta \lesssim 15\%$ ;

- $\Delta m_{\text{atm}}^2$ (IH) or $m_0$ (QD) measured with $\delta \lesssim 10\%$ ;

- $\xi \lesssim 1.5$ ;

- $\alpha_{21}$ (QD): in the interval $\sim [\pi/4 - 3\pi/4]$, or $\sim [5\pi/4 - 3\pi/2]$ ;

- $\tan^2 \theta \circ \gtrsim 0.40$ .

S. Pascoli, S.T.P., W. Rodejohann, 2002
S. Pascoli, S.T.P., L. Wolfenstein, 2002

No “No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay”

V. Barger et al., 2002
NMEs for Light $\nu$ Exchange

![Graph showing $0\nu\beta\beta$ NMEs - status 2016](image)

<table>
<thead>
<tr>
<th>Large model space</th>
<th>mean field meth.</th>
<th>ISM</th>
<th>IBM</th>
<th>QRPA</th>
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<td>Constr. Interm. States</td>
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<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Nucl. Correlations</td>
<td>limited</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

F. Simkovic, September, 2016
The $g_A$ Quenching Problem

$g_A$: related to the weak charged axial current which is not conserved and therefore can be and is renormalised, i.e., quenched, by the nuclear medium. Effectively, this implies that $g_A$ is reduced from its current standard value $g_A = 1.269$.

The reduction of $g_A$ can have important implications for the $(\beta\beta)_{0\nu}$-decay searches since $T_{1/2}^{0\nu} \propto (g_{A}^{eff})^{-4}$.

The reduction of $g_A$ necessary in various model NME calculations of $T_{1/2}^{2\nu}$ to reproduce the data; does not imply the same reduction of $g_A$ takes place in the $(\beta\beta)_{0\nu}$-decay NME, there are indications that the reduction is much smaller.

The mechanism of quenching is not understood at present. Thus, the degree of quenching cannot be firmly determined quantitatively and is subject to debates.
Quenching of $g_A$ (from theory: $T_{1/2}^{0\nu}$ up 50 x larger)

$(g_{\text{eff}}^A)^4 \approx 0.66 ({^{48}}\text{Ca})$, 0.66 ($^{76}\text{Ge}$), 0.30 ($^{76}\text{Se}$), 0.20 ($^{130}\text{Te}$) and 0.11 ($^{136}\text{Xe}$).

The Interacting Shell Model (ISM), which describes qualitatively well energy spectra, does reproduce experimental values of $M^{2\nu}$ only by consideration of significant quenching of the Gamow-Teller operator, typically by 0.45 to 70%.

$(g_{\text{eff}}^A)^4 \approx (1.269 \text{ A}^{-0.18})^4 = 0.063$ (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.

It has been determined by theoretical prediction for the $2\nu\beta\beta$-decay half-lives, which were based on within closure approximation calculated corresponding NMEs, with the measured half-lives.

(g_{eff}^A)^4 = 0.30 and 0.50 for ^{100}\text{Mo} and ^{116}\text{Cd}, respectively (The QRPA prediction).

g_{eff}^A was treated as a completely free parameter alongside g_{pp} (used to renormalize particle-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of g_{eff}^A and g_{pp}, where possible, to the $\beta$-decay rate and $\beta^+/EC$ rate of the $J = 1^+$ ground state in the intermediate nuclei involved in double-beta decay in addition to the $2\nu\beta\beta$ rates of the initial nuclei, leads to an effective $g_{eff}^A$ of about 0.7 or 0.8.

Extended calculation also for neighbour isotopes performed by

Simkovic
Dependence of $g_{eff}^A$ on $A$ was not established.
Quenching of $g_A$, two-body currents and QRPA
(Suppression of the 0νββ-decay NME of about 20%)
Engel, Vogel, Faessler, F.Š., PRC 89 (2014) 064308

But, a strong suppression of 2νββ-decay half-life, ($g_A^{\text{eff}} = g_A \delta(p=0) = 0.7$-1.0)
New Physics and $(\beta\beta)_{0\nu}$-Decay
Light Sterile Neutrinos and $(\beta\beta)_{0\nu}$-Decay
One Sterile Neutrino: the 3 + 1 Model

\[|<m>| = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2e^{i\alpha} + m_3|U_{e3}|^2e^{i\beta} + m_4|U_{e4}|^2e^{i\gamma}|.\]

\[U_{e1} = c_{12}c_{13}c_{14}, \quad U_{e2} = e^{i\alpha/2}c_{13}c_{14}s_{12},\]

\[U_{e3} = e^{i\beta/2}c_{14}s_{13}, \quad U_{e4} = e^{i\gamma/2}s_{14},\]

\[\sin^2 \theta_{14} = 0.0225, \quad \Delta m_{41(43)}^2 = 0.93 \text{ eV}^2 \quad (A),\]

\[J. \, Kopp \, et \, al., \, 2013\]

\[\sin^2 \theta_{14} = 0.023 \, (0.028), \quad \Delta m_{41(43)}^2 = 1.78 \, (1.60) \text{ eV}^2 \quad (B).\]

\[J. \, Kopp \, et \, al., \, 2013 \, (\nu_e, \bar{\nu}_e \, \text{disappearnce data});\]

\[C. \, Giunti \, et \, al., \, 2013 \, (\text{global, except for MiniBooNE results at } E_\nu \leq 0.475 \text{ GeV})\]
NO spectrum; green, red and orange lines: \((\alpha, \beta, \gamma) = (0, 0, 0), (0, 0, \pi), (\pi, \pi, \pi)\);
five gray lines: the other five sets of CP conserving values.
Left panel: \(\Delta m_{41}^2 = 0.93 \text{ eV}^2\), \(\sin \theta_{14} = 0.15\).
Right panel: \(\Delta m_{41}^2 = 1.78 \text{ eV}^2\), \(\sin \theta_{14} = 0.15\).

IO spectrum; green and orange lines: $(\alpha, \beta, \gamma) = (0, 0, 0), (\pi, \pi, \pi)$; six gray lines: the other six sets of CP conserving values.
Left panel: $\Delta m_{43}^2 = 0.93 \text{ eV}^2$, $\sin \theta_{14} = 0.15$.
Right panel: $\Delta m_{43}^2 = 1.78 \text{ eV}^2$, $\sin \theta_{14} = 0.15$.
Heavy Majorana Neutrino Exchange Mechanisms

\[ V - A \]
\[ \begin{array}{c}
W_L \\
\chi_{jL}, N_{kL} \\
W_L \\
V - A
\end{array} \]
\[ V - A \]
\[ e^- \]
\[ \begin{array}{c}
W_L \\
\chi_{jL}, N_{kL} \\
W_L \\
V - A
\end{array} \]
\[ V - A \]
\[ e^- \]
\[ \begin{array}{c}
W_R \\
N_{kR} \\
W_R \\
V + A
\end{array} \]
\[ V + A \]
\[ e^- \]
\[ \begin{array}{c}
W_R \\
N_{kR} \\
W_R \\
V + A
\end{array} \]
\[ V + A \]

Light Majorana Neutrino Exchange

\[ \eta_\nu = \frac{\langle m \rangle}{m_e} . \]

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH \( N_k, M_k > 10 \text{ GeV} \):

\[ \eta_{N}^L = \sum_k^{\text{heavy}} U_{ek}^2 \frac{m_p}{M_k}, m_p - \text{proton mass}, U_{ek} - \text{CPV} . \]
NMEs for Heavy Majorana Neutrino Exchange

F. Simkovic, September, 2016

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
(ββ)_{0ν}-Decay and TeV Scale Type I See-Saw Mechanism
The Seesaw Mechanisms of Neutrino Mass Generation \( M_\nu \) from the See-Saw Mechanism

- M. Gell-Mann, P. Ramond, R. Slansky, 1979;
- T. Yanagida, 1979;

- Explain the smallness of \( \nu \)-masses.

- Through leptogenesis theory link the \( \nu \)-mass generation to the generation of baryon asymmetry of the Universe.

Type I Seesaw Mechanism

- Requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$.
- Dirac+Majorana Mass Term: $M^{LL} = 0$, $|M_D = vY^\nu/\sqrt{2}| << |M^{RR}|$.
- Diagonalising $M^{RR}$: $N_j$ - heavy Majorana neutrinos, $M_j \sim$ TeV; or $(10^9 - 10^{13})$ GeV in GUTs.

For sufficiently large $M_j$, Majorana mass term for $\nu_{lL}(x)$:

$$M_\nu \simeq v_u^2 (Y^\nu)^T M_j^{-1} Y^\nu = U^*_{\text{PMNS}} m_{\nu}^{\text{diag}} U^\dagger_{\text{PMNS}}.$$ 

$\nu_u Y^\nu = M_D$, $M_D \sim 1$ GeV, $M_j = 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.
\( \phi \) \hspace{1cm} \nu^c_l R \hspace{1cm} N_j \hspace{1cm} \nu^l_L \\nonumber

- \( \nu_{lR}(x) \): Majorana mass term at “high scale” (\(~\text{TeV}; \text{ or} \ (10^9 - 10^{13}) \text{ GeV}) \) in SO(10) GUT

\[ \mathcal{L}^\nu_M(x) = + \frac{1}{2} \nu^T_{lR}(x) C^{-1} (M^{RR})^\dagger_{vl} \nu_{lR}(x) + h.c. = - \frac{1}{2} \sum_j \bar{N}_j M_j N_j , \]

- Yukawa type coupling of \( \nu_{lL}(x) \) and \( \nu_{lR}(x) \) involving \( \Phi(x) \):

\[ \mathcal{L}_Y(x) = \bar{Y}_{vl} \nu^\nu_{lR}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c., \]

\[ = Y^\nu_{jl} \bar{N}_{jR}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c., \]

\[ M_D = \frac{v}{\sqrt{2}} Y^\nu, \quad v = 246 \text{ GeV}. \]
TeV Scale Type I See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos $N_j$ at the TeV scale:
$m_\nu \simeq - M_D \bar{M}^{-1}_N M_D^T$, $\bar{M} = \text{diag}(M_1, M_2, M_3)$, $M_j \sim (100 - 1000) \text{ GeV}$.

\[ L_{CC}^N = - \frac{g}{2\sqrt{2}} \bar{\ell} \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k W^\alpha + \text{h.c.}, \quad (RV)_{\ell k} \equiv U_{\ell 3+k}, \]

\[ L_{NC}^N = - \frac{g}{2c_w} \nu_{\ell L} \gamma_\alpha (RV)_{\ell k} N_{kL} Z^\alpha + \text{h.c.} \]

- $|m_{\ell\ell}| \cong |\sum_k (RV)_{\ell k}^* M_k (RV)_{k\ell}^\dagger| \lesssim 1 \text{ eV}, \quad \ell', \ell = e, \mu, \tau.$

One $N_1$, $M_1 \sim 100$ GeV: $|(RV)_1|^2 \lesssim 10^{-11}$

- All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos $N_{1,2}$ with $|(RV)_{1,2}|^2 \lesssim 10^{-3}$, if $N_{1,2}$ form a pseudo-Dirac pair:
  $M_2 = M_1 (1 + z), \quad 0 < z \ll 1.$

- Only NH and IH $\nu$ mass spectra possible: $\min(m_j) = 0.$
Requirements: \(|(RV)_{\ell k}|\) “sizable”

+ reproducing correctly the neutrino oscillation data:

\[
| (RV)_{\ell 1} |^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_3}{m_2 + m_3} | U_{\ell 3} + i \sqrt{m_2/m_3} U_{\ell 2} |^2, \quad \text{NH},
\]

\[
| (RV)_{\ell 1} |^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_2}{m_1 + m_2} | U_{\ell 2} + i \sqrt{m_1/m_2} U_{\ell 1} |^2 \approx \frac{1}{4} \frac{y^2 v^2}{M_1^2} | U_{\ell 2} + i U_{\ell 1} |^2, \quad \text{IH},
\]

\[
(RV)_{\ell 2} = \pm i (RV)_{\ell 1} \sqrt{\frac{M_1}{M_2}}, \quad \ell = e, \mu, \tau,
\]

\[y- \text{the maximum eigenvalue of } Y^\nu, \nu_u \approx 174 \text{ GeV}.
\]

4 parameters: \(M, z, y \) and a phase \(\omega\).

Low energy data:

\[
| (RV)_{\ell 1} |^2 \lesssim 2 \times 10^{-3},
\]

\[
| (RV)_{\ell 1} |^2 \lesssim 0.8 \times 10^{-3},
\]

\[
| (RV)_{\ell 1} |^2 \lesssim 2.6 \times 10^{-3}.
\]

Observation of \(N_{1,2}\) at LHC - problematic.


S. Antusch et al., 2008

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
The exchange of virtual $N_j$ contributes to $|<m>|$: 

$$
|<m>| \equiv \left| \sum_i (U_{PMNS})_{ei}^2 m_i - \sum_k f(A, M_k) (RV)^2_{ek} \frac{(0.9 \text{ GeV})^2}{M_k} \right|
$$

$$
f(A, M_k) \approx f(A).
$$

For, e.g., $^{48}\text{Ca}$, $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{130}\text{Te}$ and $^{136}\text{Xe}$, the function $f(A)$ takes the values $f(A) \approx 0.035, 0.028, 0.028, 0.033$ and $0.032$, respectively.

The Predictions for $|<m>|$ can be modified significantly: we can have

$$
|<m>| \left(\frac{NH}{\nu+N}\right) \sim |<m>| \left(\frac{IH}{\nu}\right),
$$

or

$$
|<m>| \left(\frac{IH}{\nu+N}\right) \sim |<m>| \left(\frac{NH}{\nu}\right).
$$

$$
M_{1,2} \sim 1 \text{ TeV}, |<m>|^{\text{heavy}} \sim 0.1 \text{ eV}.
$$

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay

Based on:
If the decay \((A, Z) \rightarrow (A, Z + 2) + e^- + e^- ((\beta\beta)_{0\nu}-\text{decay})\) will be observed, the question will inevitably arise:

Which mechanism is triggering the decay?

How many mechanisms are involved?

“Standard Mechanism”: light Majorana \(\nu\) exchange.

Fundamental parameter - the effective Majorana mass:

\[
\langle m \rangle = \sum_{j}^{\text{light}} (U_{ej})^2 m_j , \text{ all } m_j \geq 0 ,
\]

\(U\) - the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix, \(m_j\) - the light Majorana neutrino masses, \(m_j \lesssim 1\) eV.

\(U\) - CP violating, in general: \((U_{ej})^2 = |U_{ej}|^2 e^{i\alpha_j} , j = 2, 3 , \alpha_{21}, \alpha_{31}\) - Majorana CPV phases.

S.M. Bilenky, J. Hosek, S.T.P.,1980
A number of different mechanisms possible. For a given mechanism $\kappa$ we have in the case of $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-:

\frac{1}{T_{1/2}^{0\nu}} = |\eta_{\kappa}^{LNV}|^2 \ G^{0\nu}(E_0, Z)|M^{0\nu}_\kappa|^2,

$\eta_{\kappa}^{LNV}$ - the fundamental LNV parameter characterising the mechanism $\kappa$,

$G^{0\nu}(E_0, Z)$ - phase-space factor (includes $g_A^4 = (1.25)^4$, as well as $R^{-2}(A), R(A) = r_0 A^{1/3}$ with $r_0 = 1.1 \ fm$),

$M^{0\nu}_\kappa = (g_A/1.25)^2 M^{0\nu}_\kappa$ - NME (includes $R(A)$ as a factor).
Different Mechanisms of $(\beta\beta)_{0\nu}$-Decay

Light Majorana Neutrino Exchange

$$\eta_N = \frac{<m>}{m_e}.$$  

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH $N_k$, $M_k \gtrsim 10$ GeV:

$$\eta_N^L = \sum_k^{heavy} U_{ek}^2 \frac{m_p}{M_k}, \; m_p - \text{proton mass}, \; U_{ek} - \text{CPV}.$$  

S.T. Petcov, DBD16, Osaka Univ., 08/11/2016
\((V+A)\) Weak Interaction, RH \(N_k, M_k \gtrsim 10\) GeV:

\[ \eta^R_N = \left( \frac{M_W}{M_{WR}} \right)^4 \sum_k \text{heavy} \, V_{ek}^2 \frac{m_p}{M_k} \; ; \; V_{ek}: N_k - e^- \text{ in the CC} \; . \]

\(M_W \cong 80\) GeV; \(M_{WR} \gtrsim 2.5\) TeV; \(V_{ek} - \text{CPV}, \) in general.

A comment.

\((V-A)\) CC Weak Interaction:

\[ \bar{e}(1 + \gamma_5)e^c \equiv 2\bar{e}_L (e^c)_R , \; e^c = C(\bar{e})^T , \]

\(C - \) the charge conjugation matrix.

\((V+A)\) CC Weak Interaction:

\[ \bar{e}(1 - \gamma_5)e^c \equiv 2\bar{e}_R (e^c)_L . \]

The interference term: \(\propto m_e, \) suppressed.

\[ S.T. \text{ Petcov, DBD16, Osaka Univ., 08/11/2016} \]
The Gluino Exchange Dominance Mechanism

\[ \eta \chi' = \frac{\pi \alpha_s}{6} \frac{\lambda_{111}'}{G_F m_{\tilde{d} R}^4 m_{\tilde{g}}} \left[ 1 + \left( \frac{m_{\tilde{d} R}}{m_{\tilde{u} L}} \right)^2 \right]^2, \]

\( G_F \) - the Fermi constant, \( \alpha_s = \frac{g_3^2}{4\pi} \), \( g_3 \) - the SU(3)\(_c\) gauge coupling constant, \( m_{\tilde{u} L}, m_{\tilde{d} R} \) and \( m_{\tilde{g}} \) - the masses of the LH u-squark, RH d-squark and gluino.

The Squark-Neutrino Mechanism

\[ \eta \tilde{q} = \sum_k \frac{\lambda_{11k}^' \lambda_{1k1}^1}{2\sqrt{2}G_F} \sin 2\theta^d(k) \left( \frac{1}{m_{\tilde{d} 1}(k)} - \frac{1}{m_{\tilde{d} 2}(k)} \right), \]

\( d(k) = d, s, b; \theta^d: \tilde{d}_k L - \tilde{d}_k R \) - mixing (3 light Majorana neutrinos assumed).

The 2e\(^-\) current in both mechanisms:
\[ \bar{e}(1 + \gamma_5) e^c \equiv 2\bar{e}_L (e^c)_R, \] as in the “standard” mechanism.

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Two “Non-Interfering” Mechanisms

Example: light LH and heavy RH Majorana $\nu$ exchanges

The corresponding LNV parameters, $|\eta_\nu|$ and $|\eta_R|$ -
from “data” on $T_{1/2}^{0\nu}$ of two nuclei:

$$\frac{1}{T_{1G_1}} = |\eta_\nu|^2 |M'_{1,\nu}^0|^2 + |\eta_R|^2 |M'_{1,N}^0|^2,$$

$$\frac{1}{T_{2G_2}} = |\eta_\nu|^2 |M'_{2,\nu}^0|^2 + |\eta_R|^2 |M'_{2,N}^0|^2.$$  

The solutions read:

$$|\eta_\nu|^2 = \frac{|M'_{2,N}^0|^2/T_{1G_1} - |M'_{1,N}^0|^2/T_{2G_2}}{|M'_{1,\nu}^0|^2 |M'_{2,N}^0|^2 - |M'_{1,N}^0|^2 |M'_{2,\nu}^0|^2},$$

$$|\eta_R|^2 = \frac{|M'_{1,\nu}^0|^2/T_{2G_2} - |M'_{2,\nu}^0|^2/T_{1G_1}}{|M'_{1,\nu}^0|^2 |M'_{2,N}^0|^2 - |M'_{1,N}^0|^2 |M'_{2,\nu}^0|^2}.$$
Solutions giving $|\eta_\nu|^2 < 0$ and/or $|\eta_R|^2 < 0$ are unphysical. Given a pair $(A_1, Z_1)$, $(A_2, Z_2)$ of the three $^{76}\text{Ge}$, $^{100}\text{Mo}$ and $^{130}\text{Te}$ we will be considering, and $T_1$, and choosing (for convenience) always $A_1 < A_2$, positive solutions for $|\eta_\nu|^2$ and $|\eta_R|^2$ - possible for the following range of values of $T_2$:
The positivity conditions

\[
\frac{T_1 G_1 |M'_{1,N}|^2}{G_2 |M'_{2,N}|^2} \leq T_2 \leq \frac{T_1 G_1 |M'_{1,\nu}|^2}{G_2 |M'_{2,\nu}|^2}
\]

(|M'_{1,\nu}|^2/|M'_{2,\nu}|^2 > |M'_{1,N}|^2/|M'_{2,N}|^2 (from Table 1) used.)

Using \(G_{1,2}\), and QRPA \(M'_{i,\nu}, M'_{i,N}, i = 1,2\), from Table 1 (“CD-Bonn, large, \(g_A=1.25\ (1.0)\)” in arXiv:1103.2434, we get the positivity conditions for the 3 ratios of pairs of \(T_{1/2}^{0\nu}\):

\[
0.15 \leq \frac{T_{1/2}^{0\nu}(100\text{Mo})}{T_{1/2}^{0\nu}(76\text{Ge})} \leq 0.18 \ (0.17),
\]

\[
0.17 \leq \frac{T_{1/2}^{0\nu}(130\text{Te})}{T_{1/2}^{0\nu}(76\text{Ge})} \leq 0.22 \ (0.23),
\]

\[
1.14 \ (1.16) \leq \frac{T_{1/2}^{0\nu}(130\text{Te})}{T_{1/2}^{0\nu}(100\text{Mo})} \leq 1.24 \ (1.30).
\]
Similar results with Argonne, large, $g_A = 1.25(1.0)$ NMEs:

\[
0.15 \leq \frac{T^{0\nu}_{1/2}(^{100}Mo)}{T^{0\nu}_{1/2}(^{76}Ge)} \leq 0.18,
\]

\[
0.18 \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{76}Ge)} \leq 0.24 \ (0.25),
\]

\[
1.22 \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{100}Mo)} \leq 1.36 \ (1.42).
\]

The physical solutions possible only for remarkably narrow intervals of $T_2/T_1$. If any of the ratios is shown to lie outside the relevant intervals, the case - excluded.

Conditions for only one mechanism being active:

\[
|\eta_R|^2 = 0 : \quad |M'_{1,\nu}^{0\nu}|^2 T_1 G_1 = |M'_{2,\nu}^{0\nu}|^2 T_2 G_2,
\]

\[
|\eta_\nu|^2 = 0 : \quad |M'_{1,N}^{0\nu}|^2 T_1 G_1 = |M'_{2,N}^{0\nu}|^2 T_2 G_2.
\]
Comments.

- The feature discussed above - common to all cases of two “non-interfering” mechanisms considered.
- The indicated specific half-life intervals for the various isotopes, are stable with respect to the change of the NMEs.
- Assuming two “non-interfering” mechanisms are operative in $(\beta\beta)_{0\nu}$-decay, say light LH and heavy RH Majorana $\nu$ exchanges, from the measured half-lives of two nuclei $(A_1, Z_1)$ and $(A_2, Z_2)$, given the corresponding NMEs, one can derive the values of the two relevant LNV constants, $|\eta_\nu|^2$ and $|\eta_R|^2$. Using these as input one can predict the half-life of any third nucleus $(A_3, Z_3)$. If the predicted half-life does not correspond to the measured one, the given pair of mechanisms will be ruled out.
- The intervals of $T_2/T_1$ depend on the type of the two “non-interfering” mechanisms. However, the differences in the cases of the $(\beta\beta)_{0\nu}$-decays of $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{100}\text{Mo}$ and $^{130}\text{Te}$, triggered by the exchange of heavy Majorana neutrinos coupled to $(V+A)$ currents and i) light Majorana neutrino exchange, or ii) the gluino exchange mechanism, or iii) the squark-neutrino exchange mechanism, are extremely small and cannot be used to distinguish experimentally between the indicated three pairs of $(\beta\beta)_{0\nu}$-decay mechanisms.
For each mechanism $\kappa$ discussed, the NMEs for the nuclei considered differ relatively little:

$$|M'_{\kappa i} - M'_{\kappa j}| << M'_{\kappa i}, M'_{\kappa j}, \text{ typically}$$

$$\frac{|M'_{\kappa i} - M'_{\kappa j}|}{0.5(M'_{\kappa i} + M'_{\kappa j})} \sim 0.1, \ i \neq j = ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te.$$ 

- One of the consequences - if it will be possible to rule out one pair of these mechanisms as the cause of $\beta\beta_{0\nu}$-decay, most likely one will be able to rule out all three of them.
The constraints under discussion will not be valid, in general, if the $(\beta\beta)_{0\nu}$-decay is triggered by two "interfering" mechanisms with a non-negligible (destructive) interference term, or by more than two mechanisms none of which plays a subdominant role in $(\beta\beta)_{0\nu}$-decay.
The degeneracy between the intervals of allowed values of $T_2/T_1$ (determined by the positivity conditions) corresponding to different pairs of “non-interfering” mechanisms can be lifted by using in the analysis isotopes with largely different NMEs, e.g., $^{136}Xe$, $^{48}Ca$, $^{96}Zr$, and any of $^{76}Ge$, $^{82}Se$, $^{100}Mo$, $^{130}Te$.

Constraints from $^3H$ $\beta$-decay data.

Important in the cases of light $\nu$ exchange + “nonstandard” mechanisms.

Moscow, Mainz: $m(\bar{\nu}_e) < 2.3$ eV; $|\eta_\nu|^2 \times 10^{10} < 0.21$.

KATRIN: $m(\bar{\nu}_e) < 0.2$ eV; $|\eta_\nu|^2 \times 10^{10} < 1.6 \times 10^{-3}$. 
Theoretical Model Predictions
$T'$ model of lepton flavour: $U_{TBM}, \, \delta \approx 3\pi/2$ or $\pi/2$.

(The prediction

- Light neutrino masses: type I seesaw mechanism.

- $\nu_j$ - Majorana particles.

- Diagonalisation of $M_\nu$: $U_{TBM}\Phi, \, \Phi = \text{diag}(1, 1, 1(i))$

- $U_{TBM}$ “corrected” by

$U_{\text{lep}}^\dagger \, Q = R_{12}(\theta_{12}^\ell) \, R_{23}(\theta_{23}^\ell) \, Q, \quad Q = \text{diag}(1, e^{i\phi}, 1)$
\( T' \) model of lepton flavour: \( U_{\text{TBM}} \), \( \delta \cong 3\pi/2 \) or \( \pi/2 \).

- \( T' \): double covering of \( A_4 \) (tetrahedral symmetry group).

- \( T' \): \( 1, 1', 1''; 2, 2', 2''; 3 \).

- \( T' \) model: \( \psi_{eL}(x), \psi_{\mu L}(x), \psi_{\tau L}(x) \) - triplet of \( T' \);
  \( e_R(x), \mu_R(x) \) - a doublet, \( \tau_R(x) \) - a singlet, of \( T' \);
  \( \nu_{eR}(x), \nu_{\mu R}(x), \nu_{\tau R}(x) \) - a triplet of \( T' \);
  the Higgs doublets \( H_u(x), H_d(x) \) - singlets of \( T' \).

- The discrete symmetries of the model are \( T' \times H_{\text{CP}} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2 \), the \( Z_n \) factors being the shaping symmetries of the superpotential required to forbid unwanted operators.
Predictions of the $T'$ Model

- $m_{1,2,3}$ determined by 2 real parameters $+ \Phi^2$:

  NO spectrum A : $(m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3}$
  NO spectrum B : $(m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3}$
  IO spectrum : $(m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3}$

  \[
  \text{NO A : } \sum_{j=1}^{3} m_j = 6.29 \times 10^{-2} \text{ eV ,} \\
  \text{NO B : } \sum_{j=1}^{3} m_j = 6.52 \times 10^{-2} \text{ eV ,} \\
  \text{IO : } \sum_{j=1}^{3} m_j = 12.11 \times 10^{-2} \text{ eV ,}
  \]
• $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ determined by 3 real parameters.

Given the values of $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ are predicted:

\[
\delta \cong \frac{3\pi}{2} \ (266^\circ) \ (\text{or} \ \frac{\pi}{2} \ (94^\circ));
\]

**NO A**: $\alpha_{21} \cong +47.0^\circ \ (\text{or} \ -47.0^\circ) \ (+2\pi),$

\[
\alpha_{31} \cong -23.8^\circ \ (\text{or} \ +23.8^\circ) \ (+2\pi).
\]
Conclusions

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The \((\beta\beta)_{0\nu}\)-decay experiments:

- Are testing the status of \(L\) conservation, can establish the Majorana nature of \(\nu_j\);
- Can provide unique information on the \(\nu\) mass spectrum;
- Can provide unique information on the absolute scale of \(\nu\) masses;
- Can provide information on the Majorana CPV phases;
- Provide critical tests of neutrino-related BSM theoretical ideas.

\[
T_{1/2}^{0\nu} = 10^{25} \text{ yr probes } |\langle m \rangle| \sim 0.1 \text{ eV};
\]
\[
T_{1/2}^{0\nu} = 10^{25} \text{ yr probes } \Lambda_{LNV} \sim 1 \text{ TeV}.
\]
- Synergy with searches of BSM physics at LHC.

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