## Recent Results from T2K



DBD16 - Osaka $9^{\text {th }}$ Nov

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On behalf of the T2K collaboration

## Three neutrino mixing

## - PMNS framework

Interact as weak process eigenstates Propagate as mass eigenstates
$\xrightarrow[\text { v. }]{\text { v. }}\left(\begin{array}{l}\nu_{e} \\ v_{u} \\ v_{\tau}\end{array}\right)=U_{\text {PMNS }}\left(\begin{array}{l}\nu_{1} \\ v_{2} \\ v_{3}\end{array}\right)$


$$
U_{P M N S}=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & +c_{23} & +s_{23} \\
0 & -s_{23} & +c_{23}
\end{array}\right)\left(\begin{array}{ccc}
+c_{13} & 0 & +s_{13} e^{-i \delta} \\
0 & 1 & 0 \\
-s_{13} e^{i \delta} & 0 & +c_{13}
\end{array}\right)\left(\begin{array}{ccc}
+c_{12} & +s_{12} & 0 \\
-s_{12} & +c_{12} & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
e^{i \alpha_{1}} & 0 & 0 \\
0 & e^{i \alpha_{2}} & 0 \\
0 & 0 & 1
\end{array}\right)
$$



$$
\begin{array}{lll}
\text { Current knowledge: } & \theta_{12} \approx 33^{\circ} & \Delta \mathrm{m}_{21}^{2} \approx 7.5 \times 10^{-5} \mathrm{eV}^{2} \\
& \theta_{23} \approx 45^{\circ} & \left|\Delta \mathrm{m}_{32}^{2}\right| \approx 2.5 \times 10^{-3} \mathrm{eV}^{2} \\
& \theta_{13} \approx 9^{\circ} &
\end{array}
$$

$$
\begin{aligned}
c_{i j} & =\cos \theta_{i j} \\
s_{i j} & =\sin \theta_{i j} \\
\Delta m_{i j}^{2} & =m_{i}^{2}-m_{j}^{2}
\end{aligned}
$$

## Oscillations

## Appearance

$$
\begin{aligned}
P\left(v_{\mu} \rightarrow v_{e}\right)= & 4 c_{13}^{2} s_{13}^{2} s_{23}^{2} \sin ^{2} \Delta_{31} \times\left(1 \pm \frac{2 a}{\Delta m_{31}^{2}}\left(1-s_{13}^{2}\right)\right) \\
& +8 c_{13}^{2} s_{12} s_{13} s_{23}\left(c_{12} c_{23} \cos \delta-s_{12} s_{13} s_{23}\right) \cos \Delta_{32} \sin \Delta_{31} \sin \Delta_{21}
\end{aligned}
$$

Leading term
CP Conserving
vvs. $\bar{v}$
sign change

$$
\begin{aligned}
& \mp 8 c_{13}^{2} s_{13}^{2} s_{23}^{2} \cos \Delta_{32} \sin \Delta_{31} \frac{a L}{4 E}\left(1-2 s_{13}^{2}\right) \\
& \mp 8 c_{13}^{2} c_{12} c_{23} s_{12} s_{13} s_{23} \underline{\sin \delta} \sin \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} \\
& +4 s_{12}^{2} c_{13}^{2}\left(c_{12} c_{23}+s_{12}^{2} s_{13}^{2} s_{23}^{2}-2 c_{12} c_{23} s_{12} s_{13} s_{23} \cos \delta\right) \sin ^{2} \Delta_{21}
\end{aligned}
$$

Matter effect
CP Violating
Solar term
$c_{i j}=\cos \theta_{i j}, \quad s_{i j}=\sin \theta_{i j} \quad \Delta_{i j}=\Delta m_{i j}^{2} \frac{L}{4 E_{v}} \quad a=2 \sqrt{2} G_{F} n_{e} E$
$\theta_{13}$ dependence Octant Sensitivity CP odd phase

## Disappearance

$P\left(v_{u} \rightarrow v_{\mu}\right) \approx 1-\left(\cos ^{4} \theta_{13} \cdot \sin ^{2} 2 \theta_{23}+\sin ^{2} 2 \theta_{13} \cdot \sin ^{2} \theta_{23}\right) \cdot \sin ^{2} \frac{\Delta m_{32}^{2} \cdot L}{4 E_{v}}$
$\theta_{23}$ dependence Octant Sensitivity $\quad P_{P M N S}\left(\bar{v}_{\mu} \rightarrow \bar{v}_{\mu}\right)=P_{P M N S}\left(v_{\mu} \rightarrow v_{\mu}\right)$ Test of CPT

## Questions

- $\theta_{23}$ Octant? $\left(\theta_{23}<,>\right.$ or $\left.=45^{\circ}\right)$
- Sensitivity to $\sin ^{2} \theta_{23}$ (same for $v$ and $\bar{v}$ )
- CP violation?
- $\delta_{\mathrm{CP}}=0, \pi \rightarrow$ no CP violation
$-\delta_{\mathrm{CP}}=-\pi / 2 \rightarrow$ enhance $\left(\nu_{\mu} \rightarrow v_{\mathrm{e}}\right)$, suppress $\left(\bar{v}_{\mu} \rightarrow \bar{v}_{\mathrm{e}}\right)$
$-\delta_{\mathrm{CP}}=\pi / 2 \rightarrow\left(\bar{v}_{\mu} \rightarrow \bar{v}_{\mathrm{e}}\right)$ enhance, suppress $\left(\nu_{\mu} \rightarrow v_{\mathrm{e}}\right)$
- Mass hierarchy?
- enhance $\left(\nu_{\mu} \rightarrow v_{e}\right)$
- suppress $\left(\bar{v}_{\mu} \rightarrow \bar{v}_{e}\right)$
inverted hierarchy (IH)
$\pm 10 \%$ effect at T2K
- enhance $\left(\bar{v}_{\mu} \rightarrow \bar{v}_{\mathrm{e}}\right)$
- suppress $\left(\nu_{\mu} \rightarrow v_{e}\right)$



## T2K Experiment

- Measure $N$ events
- Compare events observed at near and far detector
- Extract oscillation


Near Detector J-PARC probability

$$
\begin{gathered}
N_{N D} \sim \Phi_{N D} \cdot \sigma_{N D} \cdot \epsilon_{N D} \\
\text { Observable } \\
N_{F D} \sim \Phi_{F D} \cdot \underset{\substack{\text { Cross } \\
\text { section } \\
\text { Detector } \\
\text { response }}}{\sigma_{F D} \cdot \epsilon_{F D} \cdot P_{\text {Osc } .} .}
\end{gathered}
$$

## Off-axis Beam

- High purity $\mathrm{v}_{\mathrm{\mu}}$ beam
- Enhanced oscillation - Lower energy beam tuned to maximise oscillations at baseline
- Enhanced CCQE fraction - Energy reconstruction at Super-Kamiokande
- Reduced intrinsic $v_{\mathrm{e}}$ contamination
- Reduced Neutral Current event feed down

Neutrino Cross section


Antineutrino Cross section




## Near Detectors

- 7+7 (+2) identical modules
- Iron and scintillator tracking calorimeter
- Beam direction and stability monitoring



ND280

Downstream ECAL

- Off-axis detector
- Magnet 0.2 T
- Trackers, Calorimeters and muon range detectors
- Active (scintillator) and passive (water) targets


## Super-Kamiokande Detector

- 50kton Water Cherenkov detector
- Optically separate inner and outer (veto) volumes
- Excellent e/ $\mu$ separation, $\pi^{0}$ rejection
- Select single ring, CCQE enriched samples
- $\mathrm{E}_{\mathrm{v}}(C C Q E)$ determined from lepton kinematics


## - Signal

$$
v_{\alpha}+n \rightarrow l_{\alpha}^{-}+p \quad \bar{v}_{\alpha}+p \rightarrow l_{\alpha}^{+}+n
$$

- Background

$$
\begin{aligned}
& v_{\alpha}+n(p) \rightarrow l_{\alpha}^{-}+n(p)+\pi^{+} \\
& v_{\alpha}+n(p) \rightarrow v_{\alpha}+n(p)+\pi^{0}
\end{aligned}
$$

## T2K Oscillation Analysis Overview



## Flux Prediction

- Flux simulation (FLUKA/GEANT3/GCALOR)
- Tuned using external data $\rightarrow$ NA61/SHINE experiment measures hadron production from thin carbon target and T2K replica target
- Large neutrino component in antineutrino flux
- Intrinsic $v_{\mathrm{e}}$ component $\sim 0.5 \%$ at flux peak



## Flux Uncertainties

- Beamline uncertainties
- Proton beam parameters
- Focusing Horn
- Component alignment
- Hadron production uncertainties
- NA61/SHINE uncertainties
- Re-interactions
- Secondary production


High statistics monitoring of beam


Frac. error on $\mathrm{N}_{\mathrm{sk}}$ $\approx 7.7-8.8 \%$ (pre-fit)

SK: Neutrino Mode, $v_{\mu}$



## Cross-section Model

- NEUT* generator tuned to external data from MiniBooNE, MINERvA and Bubble Chambers
- CCQE: Relativistic Fermi Gas (RFG) + rel. Random Phase Approximation (RPA)
- Multinucleon interactions implemented
- $-10 \%$ relative to CCQE
- 2p2h model by Nieves et al. ${ }^{\dagger}$
- Additional freedom for antineutrinos
- 2p-2h normalisation (see right)
- $\bar{v}_{\mathrm{e}} / \bar{\nu}_{\mu}$ cross-section ratio ( $v_{\mathrm{e}} / \bar{v}_{\mathrm{e}}$ cross sections not yet explicitly constrained by the near detector fit)


Large difference in ratio of $2 p-2 h$ crosssection models between neutrinos and antineutrinos

- Fractional error on $\mathrm{N}_{\mathrm{SK}} \approx 7.1-7.7 \%$ (pre-fit)
*Y. Hayato, A neutrino interaction simulation program library NEUT, Acta Phys. Pol. B 40, 2477 (2009)
$\dagger$ J. Nieves, I. R. Simo, and M. J. V. Vacas, The nucleon axial mass and the miniboone quasielastic neutrino-nucleus scattering problem, Phys. Lett. B 707, 72 (2012).

antineutrino



## Near Detector Fit

Flux Parameter Fit


- Each model parameter in the analysis has associated systematic uncertainty
- Near detector fit constrains flux and cross-section uncertainty propagated to far detector as covariance
- Separate "on-water" constraint from ND280 for the first time
- ND280 "wrong sign" constraint in $\bar{v}$-mode

| Total $N_{\text {SK }}$ Fractional Uncertainty |  |  |  |
| :---: | :---: | :---: | :---: |
| Beam mode | Sample type | w/o ND280 | w/ ND280 |
| neutrino | $\mu$-like | $12.0 \%$ | $5.03 \%$ |
| neutrino | e-like | $11.9 \%$ | $5.41 \%$ |
| anti-neutrino | $\mu$-like | $12.5 \%$ | $5.22 \%$ |
| anti-neutrino | e-like | $13.7 \%$ | $6.19 \%$ |

## Accumulation of Data

- Antineutrino exposure doubled in Run7


27 May 2016
POT total: $1.510 \times 10^{21}$
$v$-mode POT: $7.57 \times 10^{20}$ (50.14\%) $\bar{v}$-mode POT: $7.53 \times 10^{20}$ (49.86\%)

- Beam power reached 420 kW
POT = Protons On Target



## Results presented today use:

12,831,370 beam spills
v-mode: $7.482 \times 10^{20}$ POT $\bar{v}$-mode: $7.471 \times 10^{20}$ POT

## Far Detector $\nu_{\mu}$ and $\bar{v}_{\mu}$ Samples

- v mode: 135 events
- $\bar{v}$ mode: 66 events

Single ring $\mu$-like selection

1. Fully contained, Fiducial volume
2. Single ring
3. Muon-like
4. Momentum $>200 \mathrm{MeV}$
5. Zero or one decay electrons


## Far Detector $v_{\mathrm{e}}$ and $\bar{v}_{\mathrm{e}}$ Samples

- v mode: 32 events
- $\bar{v}$ mode: 4 events

Single ring e-like selection

1. Fully contained, Fiducial volume
2. Single ring
3. Electron-like
4. Visible energy $>100 \mathrm{MeV}$
5. Zero decay electrons
6. Reconstructed energy < 1.25 GeV
7. Not piO-like


## T2K Osc. Analysis History

## $v_{\mathrm{e}}$ appearance

Phys. Rev. Lett. 112 (2014) 061802

- Previous analyses
---- Analyses updated in 2016
$v$ joint fit
Phys. Rev. D 91 (2015) 072010


## $\nu_{\mu}$ disappearance <br> Phys. Rev. Lett 112 (2014) 181801

$\bar{v}_{\mu}$ disappearance
Phys. Rev. Lett. 116 (2016) 181801
~Time

## Joint Analysis Results $-\theta_{23} \& \Delta \mathrm{~m}^{2}$

$$
\begin{gathered}
\text { T2K + reactor } \\
\left(\sin ^{2} 2 \theta_{13}=0.085 \pm 0.005\right)
\end{gathered}
$$



- Data consistent with maximal mixing
- Compatible with other experimental results


1D Parameter Constraints

|  | NH. | IH. |
| :---: | :---: | :---: |
| $\boldsymbol{\operatorname { s i n }}^{\mathbf{2}} \boldsymbol{\theta}_{23}$ | $0.532^{+0.046}{ }_{-0.068}$ | $0.534^{+0.043}{ }_{-0.07}$ |
| $\begin{gathered} \left\|\Delta \mathrm{m}_{32}^{2}\right\| \\ \left(\times 1 \mathbf{1 0}^{-3} \mathbf{e V}^{2}\right) \end{gathered}$ | $2.545^{+0.081}{ }_{-0.084}$ | $2.510^{+0.081}{ }_{-0.083}$ |

## Joint Analysis Results - $\theta$

T2K-only


T2K + reactor


- T2K-only measurement consistent with reactor results
- Favours "small" $\sin ^{2} \theta_{13}$ and large CPV


## Joint Analysis Results - $\delta$

## CP

|  | Expected Number of Events (NH) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Observed |  |  |  |  |  |
|  | $\boldsymbol{\delta}_{\mathrm{CP}}=-\pi / 2$ | $\boldsymbol{\delta}_{\mathrm{CP}}=\mathbf{0}$ | $\boldsymbol{\delta}_{\mathrm{CP}}=+\pi / 2$ | $\boldsymbol{\delta}_{\mathrm{CP}}=\pi$ |  |
| $\mathbf{v}_{\mathrm{e}}$-like | 28.7 | 24.2 | 19.6 | 24.2 | 32 |
| $\bar{v}_{\mathrm{e}}$-like | 6.0 | 6.9 | 7.7 | 6.8 | 4 |

- More $v_{\mathrm{e}}$ appearance events than expected + fewer $\bar{v}_{\mathrm{e}}$ appearance events than expected
- Data prefers largest CP asymmetry $\delta_{C P} \approx-\pi / 2$, normal hierarchy

CP conservation ( $\delta_{\mathrm{CP}}=0, \pi$ ) disfavoured at $90 \%$ C.L.



Normal hierarchy: $\mathrm{CP}=[-3.13,-0.39]\left[-179^{\circ},-22^{\circ}\right]$ at $90 \% \mathrm{CL}$
Inverted hierarchy: $\mathrm{CP}=[-2.09,-0.74]\left[-120^{\circ},-42^{\circ}\right]$ at $90 \% \mathrm{CL}$

## Joint Analysis Results - MH

- T2K Bayesian Analysis
- Integrate over posterior likelihood to compare hypotheses

| Bayesian Posterior Probabilities |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $\sin ^{2} \boldsymbol{\theta}_{23}<0.5$ | $\sin ^{2} \theta_{23}<0.5$ | Line Total |
| Inverted <br> Hierarchy | 0.10 | 0.14 | 0.25 |
| Normal | 0.29 | 0.46 | 0.75 |
| Hierarchy <br> Column | 0.39 | 0.61 | 1 |
| Total |  |  |  |

Weak preference for Normal Hierarchy

## Antineutrino Fits

- Methodology
- Allow antineutrinos to oscillate differently to current PMNS description for neutrinos
- Use neutrino samples to constrain wrong sign background parameters
$\bar{v}_{\mu}$ disappearance test of CPT

$$
\bar{\theta}_{23} \neq \theta_{23}, \Delta \bar{m}_{32}^{2} \neq \Delta m_{32}^{2}
$$

Test of $\bar{v}_{\mathrm{e}}$ appearance

$$
P\left(\bar{v}_{\mu} \rightarrow \bar{v}_{e}\right)=\beta \times P_{P M N S}\left(\bar{v}_{u} \rightarrow \bar{v}_{e}\right)
$$

## $v_{\mathrm{e}}$ appearance

- $\bar{v}_{\mathrm{e}}$ appearance not yet observed
- Test the hypothesis

$$
P\left(\bar{v}_{u} \rightarrow \bar{v}_{e}\right)=\beta \times P_{P M N S}\left(\bar{v}_{u} \rightarrow \bar{v}_{e}\right)
$$

- Consider cases $\beta=0, \beta=1$
- Rate only and Rate+Shape
- Data preference inconclusive

| Results Summary |  |  |
| :---: | :---: | :---: |
|  | P-value $(\beta=0)$ | P-value $(\beta=1)$ |
| Rate Only | 0.41 | 0.21 |
| Rate+shape | 0.46 | 0.07 |

- Two events more background-like



Sensitivity:
$65 \%$ of $\beta=1$ toy experiments return $P$-value $(\beta=0) \leq 0.05$ in rate+shape analysis

## $\bar{v}_{\mu}$ disappearance

- Test CPT invariance $\bar{\theta}_{23} \neq \theta_{23}, \Delta \bar{m}_{32}^{2} \neq \Delta m_{32}^{2}$
- Good agreement between $P_{P M N S}\left(\bar{v}_{u} \rightarrow \bar{v}_{u}\right)$ and $P_{P M N S}\left(v_{u} \rightarrow v_{u}\right)$


Inverted Hierarchy


## Future of T2K

- Staged beam upgrade to increase intensity
- $420 \mathrm{~kW} \rightarrow 750 \mathrm{~kW} \rightarrow 1.3 \mathrm{MW}$
- Sample selection development to increase statistics
- Fiducial volume expansion, CC nonQE event samples, multi-ring ( $\sim+40 \%$ )
- Analysis improvements to reduce sys. Uncertainties $6 \% \rightarrow 4 \%$
- Proposal for extension of T2K with phase II
- Increase POT collected $7.8 \times 10^{21} \rightarrow 20.0 \times 10^{21}$
- Beam Upgrade
- To 750 kW - Decrease bunch interval ( $2.48 \mathrm{sec} \rightarrow 1.3 \mathrm{sec}$ )
- Replace Main Ring Power Supply
- Upgrade MR RF
- To 1.3 MW Further decrease in bunch interval ( $1.3 \mathrm{sec} \rightarrow 1.16 \mathrm{sec}$ ) and Increase protons per bunch $2.7 \times 10^{14} \rightarrow 3.2 \times 10^{14}$
- Increase horn current $250 \mathrm{kA} \rightarrow 320 \mathrm{kA}$ ( $\sim+10 \%$ stats)



## Oscillation Analysis Prospects

## - With $+50 \%$ effective statistics/POT and reduced uncertainties



## Summary

- Accumulated $15 \times 10^{20}$ POT ( $\sim 20 \%$ of total)
- Data taking has resumed following the Summer break
- First fully-joint oscillation analysis completed
- $v_{\mathrm{e}} / \bar{v}_{\mathrm{e}}$ appearance and $v_{\mu} / \bar{\nu}_{\mu}$ disappearance joint fit
- Water target and "wrong sign" constraints from near detector
- Data consistent with $\theta_{23}$ at maximal mixing, $\delta_{\mathrm{CP}} \sim-\pi / 2$, normal hierarchy
- CP conservation $\delta_{\mathrm{CP}}=(0, \pi)$ disfavoured at $90 \%$ C.L.
- Beam power continues to increase - anticipate 750 kW in near future, first stage approved
- T2K-phase II
- Accelerator and beamline upgrades $\rightarrow 1.3 \mathrm{MW}$
- Run to 2026, accumulate $20 \times 10^{21}$ POT


## The T2K Collaboration

## ~ 500 members, 61 Institutes, 11 countries

| Canada | Italy | Poland | Switzerland | USA |
| :--- | :--- | :--- | :--- | :--- |
| TRIUMF | INFN, U. Bari | IFJ PAN, Cracow | ETH Zurich | Boston U. |
| U. B. Columbia | INFN, U. Napoli | NCBJ, Warsaw | U. Bern | Colorado S. U. |
| U. Regina | INFN, U. Padova | U. Silesia, Katowice | U. Geneva | Duke U. |
| U. Toronto | INFN, U. Roma | U. Warsaw |  | Louisiana State U. |
| U. Victoria |  | Warsaw U. T. | United Kingdom | Michigan S.U. |
| U. Winnipeg | Japan | Wroclaw U. | Imperial C. London | Stony Brook U. |
| York U. | ICRR Kamioka |  | Lancaster U. | U. C. Irvine |
|  | ICRR RCCN |  | Oxford U. | U. Colorado |
| France | Kavli IPMU | Russia | Queen Mary U. L. | U. Pittsburgh |
| CEA Saclay | KEK | INR | Royal Holloway U.L. U. Rochester |  |
| IPN Lyon | Kobe U. |  | STFC/Daresbury | U. Washington |
| LLR E. Poly. | Kyoto U. | Spain | STFC/RAL |  |
| LPNHE Paris | Miyagi U. Edu. | IFAE, Barcelona | U. Liverpool |  |
| Okayama U. | IFIC, Valencia | U. Sheffield |  |  |
| Gachen | Osaka City U. | U. Autonoma Madrid | U. Warwick |  |
|  | Tokyo Metropolitan U. |  |  |  |

## Supplemental

## Neutrino Beamline



## T2K Event Timing

## - Extraction of T2K events

All data extracted


Fully-contained \&\& $\Delta \mathrm{T}_{0}-2-10 \mu \mathrm{~s}$
Tue Jul 5 01:1s
w.r.t. bunch centre


## ND280 Fit

## - FGD1 - post-fit

## $\checkmark$ beam mode




CC 1pi


CC other

## $\bar{v}$ beam mode



Nu CC 1trck


Nu CC 2trck


Nubar CC 1trck


Nubar CC 2trck

## Event Selection at Super-K

- e-like sample selection


## $v$ beam mode








## Event Selection at Super-K

## - $\mu$-like sample selection

## $v$ beam mode






## $\bar{v}$ beam mode






## P-theta distributions

## - Nue and Nuebar samples






## Joint Analysis Results $-v_{e}$ and $\bar{v}_{e}$

## - Comparison to NOvA Best-Fit Spectra




## Joint Analysis Results $-\nu_{\mu}$ and $\bar{v}_{\mu}$

## - Comparison to NOvA Best-Fit Spectra




## Vacuum Oscillations

$$
\begin{aligned}
P\left(v_{\mu} \rightarrow v_{e}\right)= & 4 c_{13}^{2} s_{13}^{2} s_{23}^{2} \sin ^{2} \Delta_{31} \\
& +8 c_{13}^{2} s_{12} s_{13} s_{23}\left(c_{12} c_{23} \cos \delta-s_{12} s_{13} s_{23}\right) \cos \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} \\
& -8 c_{13}^{2} c_{12} c_{23} s_{12} s_{13} s_{23} \sin \delta \sin \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} \\
& +4 s_{12}^{2} c_{13}^{2}\left(c_{12} c_{23}+s_{12}^{2} s_{13}^{2} s_{23}^{2}-2 c_{12} c_{23} s_{12} s_{13} s_{23} \cos \delta\right) \sin ^{2} \Delta_{21} \\
c_{i j}=\cos \theta_{i j} \quad, \quad s_{i j}= & \sin \theta_{i j} \quad \Delta_{i j}=\Delta m_{i j}^{2} \frac{L}{4 E_{v}}
\end{aligned}
$$

$P\left(\nu_{\mu} \rightarrow v_{\mu}\right) \approx 1-\left(\cos ^{4} \theta_{13} \cdot \sin ^{2} 2 \theta_{23}+\sin ^{2} 2 \theta_{13} \cdot \sin ^{2} \theta_{23}\right) \cdot \sin ^{2} \frac{\Delta m_{32}^{2} \cdot L}{4 F}$


