# *Journey to the World of Quantum Electrodynamics*

*Learning to count*

**Paul Seignac, French (1826-1904)**

# *Preface*

***Quantum electrodynamics (QED)*** is the relativistic quantum field theory. In essence, it describes how light and matter interact and it is the first theory where full agreement between quantum mechanics and special relativity is achieved. R. Feynman, one of the founding fathers of QED, has called it ***"the jewel of physics"*** for its extremely accurate predictions of many quantities.

QED mathematically describes all phenomena involving electrically charged particles interacting by means of exchange of photons.

In technical terms, QED can be described as a perturbation theory of the electromagnetic quantum processes. Perturbation theory may be called (symbolically) as a linear QED. The small parameter (expansion parameter) is $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ .

The next step is studying essentially non-linear, non-perturbative effects of QED which becomes important when $\alpha$ is compensated by large value of external EM field. This subject is very popular now because it is related to many aspects of laser physics….

# *Contents*

# *References:*


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# Part I. Equations of QED

***(theory of photon – electron-positron interactions)***

**QED** is the theory of interaction of the bi-spinor (four component) electron field,

$$\psi_\alpha(x) = \begin{pmatrix} \phi(x) \\ \chi(x) \end{pmatrix}, \quad \phi(x) \text{ and } \chi(x) \text{ - two-components fields}$$

and vector electromagnetic field

$$A_\mu(x) = \left(A_0(x), \vec{A}(x)\right)$$

***Gauge invariance of Lagrangians and equations of motion is the basis of QED***

***Consider transformations*** $\psi'(x) = \mathrm{e}^{-i\omega(x)}\psi(x)$ ***and*** $A'_\mu(x) + A_\mu(x) - \frac{1}{e}\partial_\mu\omega(x)$

***where*** $\omega(x)$ ***is an arbitrary phase,*** $\partial_\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial x_0}, -\nabla\right)$ ***operator of partial derivative***

Different choice of $\omega(x)$ means the different gauge conditions

It can be seen that ***the tensor of EM field*** $F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\mu A_\mu$

and Lorentz invariant forms

$$\sum_\alpha \bar{\psi}_\alpha(x)\psi_\alpha(x) \equiv \left(\bar{\psi}(x)\psi(x)\right) \quad \text{and} \quad \left(\bar{\psi}(x)\gamma_\mu[i\partial_\mu - eA_\mu(x)]\psi(x)\right)$$

***does not depend on choice of*** $\omega(x)$ ***(or gauge) !!!***

***Gauge invariance of these combinations:*** $F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\mu A_\mu$

$$\sum_\alpha \bar{\psi}_\alpha(x)\psi_\alpha(x) \equiv \left(\bar{\psi}(x)\psi(x)\right) \quad \text{and} \quad \left(\bar{\psi}(x)\gamma_\mu[i\partial_\mu - eA_\mu(x)]\psi(x)\right)$$

***gives an unambiguous prediction for the Lagrangian of the electron field, photon field and their mutial interaction***

$$\boxed{\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma_\mu A^\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

where

$\gamma_\mu$ are the Dirac matrices $\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, $\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$ **Pauli matrices**

with $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\psi$ is a four component spinor field (electron – positron field) $\begin{bmatrix} a^1 \\ a^2 \\ a^3 \\ a^4 \end{bmatrix}$ ***(four - column)***

$\bar{\psi} = \psi^\dagger\gamma_0$ called "psi-bar" or Dirac ad joint field $[a^{*1}, a^{*2}, -a^{*3}, -a^{*4}]$

***(four -component string (row))***

## *QED equations are consequence of Euler-Lagrange equations*

**(a) electron** $$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0$$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - e\bar{\psi}\gamma_\mu A^\mu \psi - m\bar{\psi}\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \right) = \partial_\mu (i\bar{\psi}\gamma^\mu) \qquad \frac{\partial \mathcal{L}}{\partial \psi} = -e\bar{\psi}\gamma_\mu A^\mu - m\bar{\psi}$$

**substituting these two back to the Euler-Lagrange equation results in**

$$i\partial\bar{\psi}\gamma^\mu + e\bar{\psi}\gamma_\mu A^\mu + m\bar{\psi} = 0$$

**with complex conjugate**

$$i\gamma^\mu \partial_\mu \psi - e\gamma_\mu A^\mu \psi - m\psi = 0$$

**or**

$$\boxed{i\gamma^\mu \partial_\mu \psi - m\psi = e\gamma_\mu A^\mu \psi}$$

*equation of motion for electron in the presence of electromagnetic field*

**original Dirac equation** **interaction with electromagnetic field**

## Euler-Lagrange equations: (b) photons

$$\partial_\nu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)} \right) - \frac{\partial \mathcal{L}}{\partial A_\mu} = 0$$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\mu A_\mu$$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - e\bar{\psi}\gamma_\mu A^\mu \psi - m\bar{\psi}\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

$$\partial_\nu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)} \right) = \partial_\nu \left( \partial^\mu A^\nu - \partial^\nu A^\mu \right) = \partial_\nu F^{\mu\nu}(x)$$

$$\frac{\partial \mathcal{L}}{\partial A_\mu} = -e\bar{\psi}\gamma^\mu \psi$$

**substituting these two back to the Euler-Lagrange equation results in**

$$\partial_\nu F^{\mu\nu} = e\bar{\psi}\gamma^\mu \psi$$

**Lorentz-gauge condition:** $\partial_\mu A^\mu = 0$

$$\boxed{\Box A^\mu = e\bar{\psi}\gamma^\mu \psi}$$

***equation of motion for photon in the presence of electric charge***

$$\Box = \partial_\mu \partial^\mu$$

***d'Alembertian operator***

***electron charge current***

## Summary:

**coupling equations**

$$\begin{cases} i\gamma^\mu \partial_\mu \psi - m\psi = e\gamma_\mu A^\mu \psi \\ \Box A^\mu = e\bar{\psi}\gamma^\mu \psi \end{cases}$$

**are the starting point for QED calculations**

## States with "negative energy": lack or achievement of Dirac theory

***let's make sure that the theory predicts states with negative energy***

**electron is in rest** $\psi(t,x)=\psi(t)$

$$i\gamma^{\mu}\partial_{\mu}\psi - m\psi = 0$$

$$i\gamma_0\frac{\partial\psi}{\partial t} + \underbrace{i\vec{\gamma}\cdot\vec{\nabla}\psi}_{0} - I\,m\psi = 0 \quad \textbf{with} \quad \gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\frac{\partial\psi}{\partial t} + im\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\psi = 0$$

***solutions with***

$$\overbrace{\psi^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}\mathrm{e}^{-imt}, \quad \psi^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}\mathrm{e}^{-imt}}^{\textit{positive energy}}, \quad \overbrace{\psi^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\mathrm{e}^{+imt}, \quad \psi^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\mathrm{e}^{+imt}}^{\textit{negative energy}}$$

***Physical interpretation of solutions with negative energy may be done in a presence of EM field!!***

$$(i\gamma^\mu\partial_\mu - e\gamma_\mu A^\mu - m)\,\psi_{e^-} = 0$$ *equation for particle with negative charge*

$$(i\gamma^\mu\partial_\mu + e\gamma_\mu A^\mu - m)\,\psi_c = 0$$ *equation for particle with positive charge*

***Question: what is connection between*** $\psi_c$ *and* $\psi_{e^-}$ ***?***

***1: complex conjugation*** $$\left(-i\gamma_\mu^*\partial^\mu - e\gamma_\mu^* A^\mu - m\right)\psi_{e^-}^* = 0$$

***2: act by "charge conjugated operator"*** $C$ ***with properties***

$$\left.\begin{array}{l} C\gamma_\mu^* C^{-1} = -\gamma_\mu \\ CC^{-1} = 1 \end{array}\right\} \longrightarrow C = i\gamma_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\left.\begin{array}{l} C\left(-i\gamma_\mu^*\partial^\mu - e\gamma_\mu^* A^\mu - m\right)C^{-1}C\psi_{e^-}^* = 0 \\ \longrightarrow (i\gamma_\mu\partial^\mu + e\gamma_\mu A^\mu - m)\,C\psi_{e^-}^* = 0 \end{array}\right\} \boxed{\psi_{e^+} \equiv \psi_c = C\psi_{e^-}^*}$$

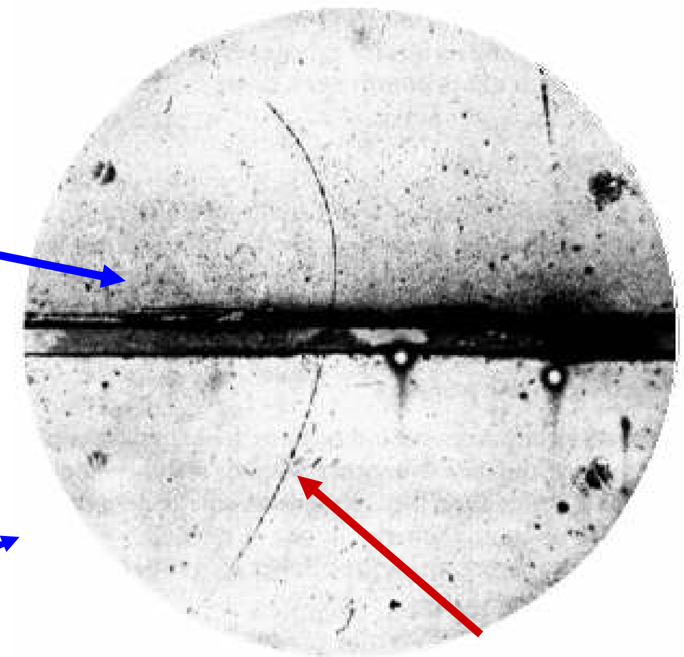
$$\psi_{e^-} = \psi_{e^-}^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{+imt} \longrightarrow \psi_{e^+} = C\psi_{e^-}^{(4)*} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-imt}$$

***electron with "negative energy" and spin down*** ***positron with "positive energy" and spin up***

# **Positron e+ (1932) C. D. Anderson**
## *(Nobel Prize 1936)*

Lead plate

Cloud Chamber: (1911)
C.T.R. Wilson (Nobel Prize, 1927)
[Cloud chamber contains a gas supersaturated with water or alcohol vapor + Photographic device]



positron

One of the first positron tracks was observed by Anderson in 1933. It was taken in a cloud chamber in the presence of a magnetic field (so the particle paths are curved).
In the presence of a charged particle (such as a positron), the water vapor condenses into droplets - these droplets mark out the path of the particle.

## Spin of electron:

**Dirac equation** $$i\gamma^\mu\partial_\mu\psi - m\psi = e\gamma_\mu A^\mu\psi$$

$$\gamma_\mu = (\gamma_0, \vec{\gamma})\,,\ A_\mu = \left(\Phi, \vec{A}\right)\,,\ \partial_\mu = \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right) \equiv \left(\frac{\partial}{\partial t}, -i\vec{p}\right)\,,\ \vec{p} \equiv -i\vec{\nabla}$$

$$i\gamma_0\frac{\partial\psi}{\partial t} = -\vec{\gamma}\cdot\vec{\nabla}\psi + m\psi + e\gamma_0\Phi\psi - e\vec{\gamma}\cdot\vec{A}\psi$$

$$\psi = \begin{pmatrix}\varphi \\ \chi\end{pmatrix} \mathrm{e}^{-imt} \qquad \gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I\end{pmatrix},\ \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0\end{pmatrix}$$

$$\boxed{\begin{aligned} i\frac{\partial\varphi}{\partial t} &= \vec{\sigma}(\vec{p} - e\vec{A})\chi + e\Phi\varphi \\ i\frac{\partial\chi}{\partial t} &= \vec{\sigma}(\vec{p} - e\vec{A})\varphi + e\Phi\chi - 2m\chi \end{aligned}} \qquad \chi \simeq \frac{\vec{\sigma}\cdot(\vec{p} - e\vec{A})}{2m}\varphi$$

$$i\frac{\partial\varphi}{\partial t} = \frac{[\vec{\sigma}(\vec{p} - e\vec{A})][\vec{\sigma}(\vec{p} - e\vec{A})]}{2m}\varphi + e\Phi\varphi$$

$$\left\{\begin{aligned} &[\vec{\sigma}\cdot\vec{X}][\vec{\sigma}\cdot Y] = \vec{X}\cdot\vec{Y} + i\vec{\sigma}\cdot[\vec{X}\times\vec{Y}] \\ &\vec{B} = [\vec{\nabla}\times\vec{A}]\ \ (\vec{B} = \mathrm{curl}\,\vec{A}) \end{aligned}\right.$$

$$\boxed{i\frac{\partial\varphi}{\partial t} = \left[\frac{(\vec{p} - e\vec{A})^2}{2m} - \frac{e}{2m}\vec{\sigma}\cdot\vec{B} + e\Phi\right]\varphi}$$

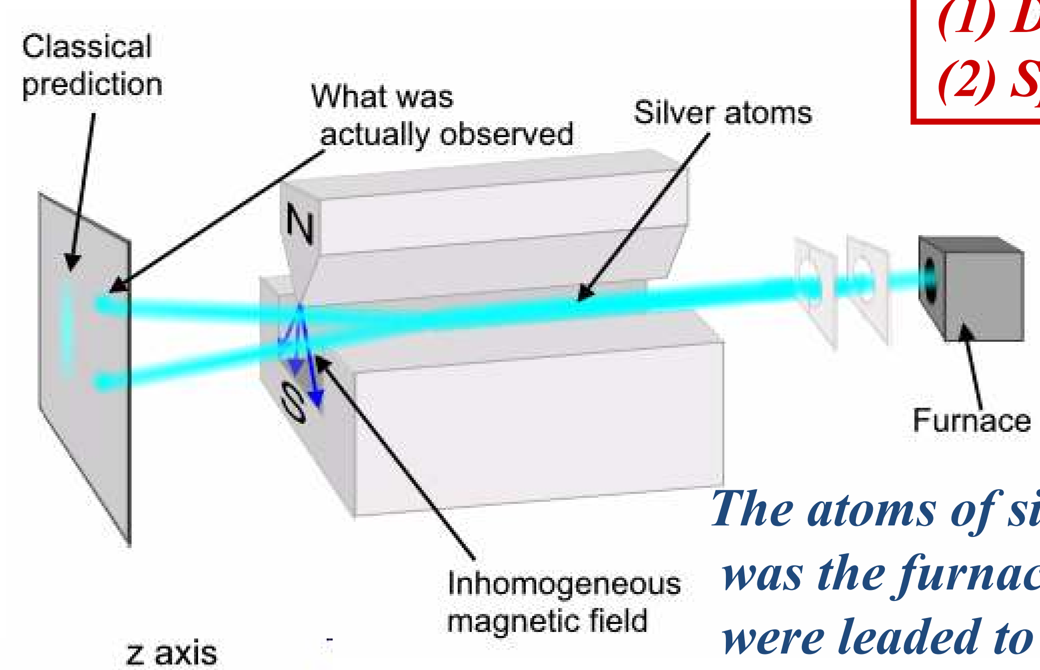
**Pauli equation (1927)**

***electron spin***
***& magnetic moment***

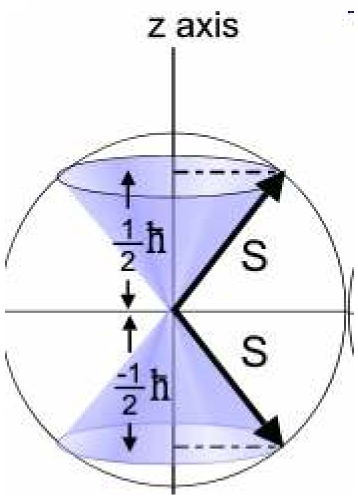
***[Stern-Gerlach -term]***

# Discovery of electron's spin:  Stern–Gerlach experiment (1922)

***Two discoveries:***
***(1) Detection of spin***
***(2) Spin is a quantum number***




***The atoms of silver from the source which was the furnace with boiling silver were leaded to the vacuum space.***




**Spin values for fermions.**

# Part II. Calculating technique

★ **properties of the bound states: energy levels, excitation/decay rates, etc** } **Pauli & Schrödinger equation(s)**

★ **scattering, annihilations, pair productions, etc** } **Perturbation approaches**

**Transition amplitude** $M_{fi} = \langle f \,| U |\, i \rangle$

***Bosonic and fermionic sectors in the initial and the final states are treated as free***

***Transition operator*** $U = T\exp\left[-i \int_{t_0}^{t} dt' V(t')\right]$ $T$ *is the time ordering operator*

**with** $V = e \int d\vec{x} \bar{\psi}(x) \gamma_\mu \psi(x) A^\mu(x)$ $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

$$T\exp[\int Z] = 1 + \int Z + T\int\int \frac{Z \cdot Z}{2!} + T\int\int\int \frac{Z \cdot Z \cdot Z}{3!} + \ldots$$

***arranged in descending order of time***

**Time ordering is *necessity to iterate through all possible options***

**Example: Time ordering for Compton scattering in second order of PT**

$$\gamma + e \to \gamma' + e'$$

***option* 1**

**1. photon absorption in**

$x = x_1$ *with* $t = t_1$

**2. photon emission in**

$x = x_2$ *with* $t = t_2$

***option* 2**

**1. photon emission in**

$x = x_1$ *with* $t = t_1$

**2. photon absorption in**

$x = x_2$ *with* $t = t_2$

$$N^n_{\text{options}} \sim n!$$

***n* is order of expansion**

# *Feynman Rules for evaluation of*

$$M_{fi} = \langle f \, | U | \, i \rangle$$

***(1) fixing of basis states***

***(2) rules for the construction of matrix elements convenient for practical calculations***

# Basis states in pQED

$$\left.\begin{array}{l} i\gamma^\mu\partial_\mu\psi - m\psi = 0 \\ \Box A^\mu = 0 \end{array}\right\}$$

*Equations for the free fermion and photon fields*

## electron

$$\psi(x) = ae^{-ipx}u^s(p)$$

$$a = \frac{1}{\sqrt{2E_p}}$$

$$(\gamma^\mu p_\mu - m)u = 0$$

$$\bar{u}(\gamma^\mu p_\mu - m) = 0$$

$$\bar{v} = v^\dagger\gamma_0$$

$$\bar{u} = u^\dagger\gamma_0$$

$$\bar{u}^i_\alpha u^j_\alpha = 2m\delta_{ij}$$

$$\sum_{s=1,2} u^s_\alpha\bar{u}^s_\beta = (\gamma_\mu p^\mu + m)_{\alpha\beta}$$

*projection operator*

## positron

$$\psi(x) = ae^{ipx}v^s(p)$$

$$(\gamma^\mu p_\mu + m)v = 0$$

$$\bar{v}(\gamma^\mu p_\mu + m) = 0$$

$$\bar{v}v = -2m$$

$$\sum_{s=1,2} v^s\bar{v}^s = \gamma_\mu p^\mu - m$$

$$u = \sqrt{E_p + m}\begin{bmatrix} I \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+m} \end{bmatrix}\chi_s, \qquad \chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

# *Basis states in pQED (continue)*

## *photon*

$$A_\mu(x) = \frac{1}{\sqrt{2E_\gamma}} e^{-ipx} \epsilon_\mu(\lambda)$$

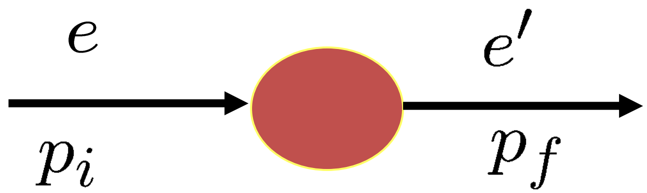
$\epsilon_\mu p^\mu = 0$, *Lorentz condition*

$\vec{\epsilon} \cdot \vec{p} = 0$, $\epsilon^0 = 0$, *Coulomb gauge*

$\vec{\epsilon}(\lambda) = \frac{-\lambda}{\sqrt{2}}(\vec{x} + i\lambda\vec{y})$, ***circular polarization***

$\vec{\epsilon}_x = \hat{\vec{x}}$, $\vec{\epsilon}_y = \hat{\vec{y}}$, ***linear polarization***

# *Structure of matrix elements*




$$M_{fi} = \langle f\,|U|\,i\rangle$$

$$M_{fi} \simeq \frac{(2\pi)^4\delta^4(p_i - p_f)}{\sqrt{2E_p 2E_{p'}}}\left[\bar{u}_f(p_f)\,\widehat{M}\,u_i(p_i)\right] \equiv \frac{(2\pi)^4\delta^4(p_i - p_f)}{\sqrt{2E_p 2E_{p'}}}\,T_{fi}$$

$$T_{fi} = \bar{u}_f(p_f)\,\widehat{M}\,u_i(p_i) =$$

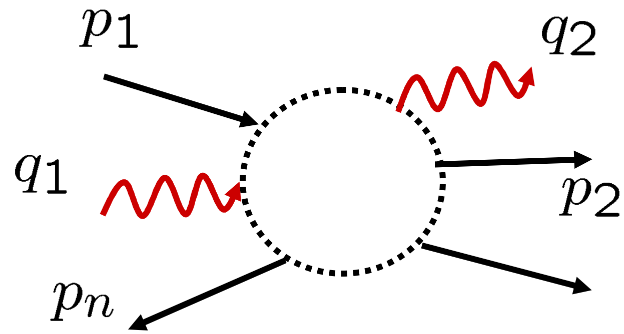
$$= [u_f*^1, u_f^{*2}, -u_f^{*3}, -u_f^{*4}]\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{pmatrix}\begin{bmatrix} u_i^1 \\ u_i^2 \\ u_i^3 \\ u_i^4 \end{bmatrix}$$ **= *scalar function (or) number***

$$\widehat{M} \sim I,\ R_\mu\gamma^\mu,\ R_\mu S_\nu\gamma^\mu\gamma^\nu,\ \ldots.$$

**Q.: how to evaluate $\widehat{M}$ ?**  **A.: use the Feynman Rules**

***Feynman Rules of QED explain how to calculate corresponding transition operators***

**(1) Graphically construct all possible diagrams with given the initial and the finite states (assuming time ordering!)**




**(2) Make proper labeling of electrons and positrons in external lines ("in-" and "out-" states)!**

**(3) For each external line put corresponding spinor (polarization operator)**

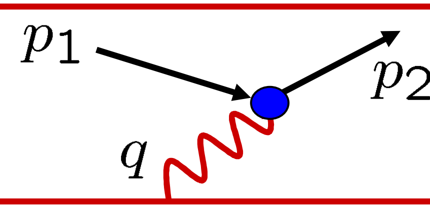
*electron* $\left\{\begin{array}{ll} \textit{incoming} & u \\ \textit{outgoing} & \bar{u} \end{array}\right.$

*positron* $\left\{\begin{array}{ll} \textit{incoming} & \bar{v} \\ \textit{outgoing} & v \end{array}\right.$

*photon* $\varepsilon_\lambda^\mu \ (\varepsilon_\lambda^{\mu *})$

**(4) Vertex factor**

$$-ie\gamma_\mu$$




$$-ie\bar{u}_{p_2}\gamma_\mu u_{p_1}\varepsilon_\lambda^\mu(q)$$

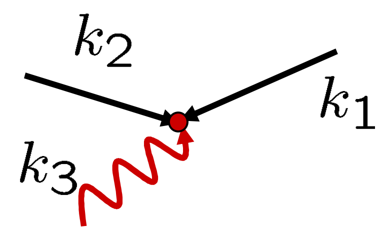
## *Feynman rules (continue)*

**(5) Propagators** **(describe propagation of the virtual particle from $x_1$ to $x_2$)**

***electron (positron)*** $i\frac{\gamma_\mu p^\mu \pm m}{p^2 - m^2}$; ***photon*** $-i\frac{g_{\mu\nu}}{q^2}$

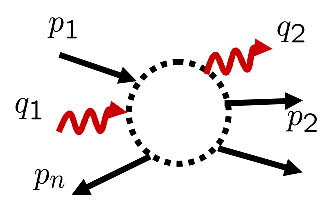
**(6) Conservation** ***of total four momentum in each vertex***

$$(2\pi)^4 \delta(k_1 + k_2 + k_3)$$



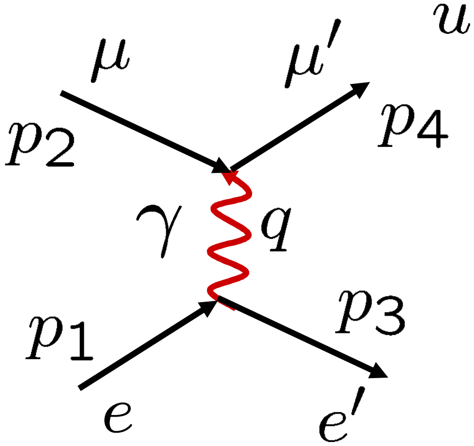

**(7) Cancel the final delta function**

$$(2\pi)^4 \delta(p_1 + p_2 ... - p_n)$$




**(8) Antisymmetrization.** ***Include a minus sign between diagrams that differ only in the interchange of two incoming (or outgoing) electrons***

# Example 1: $e\mu \to e\mu$ scattering




$$\bar{u}_\mu(p_4)(-ie\gamma_\nu)u_\mu(p_2)(2\pi^4)\delta(p_2 - p_4 + q)$$

$$-i\frac{g^{\mu\nu}}{q^2}\frac{d^4q}{(2\pi)^4}$$

$$\bar{u}_e(p_3)(-ie\gamma_\mu)u_e(p_1)(2\pi^4)\delta(p_1 - p_3 - q)$$

$$T_{fi} = -i\frac{e^2}{q^2}[\bar{u}_\mu(p_4)\gamma_\nu u_\mu(p_2)] \cdot [\bar{u}_e(p_3)\gamma^\nu u_e(p_1)],$$

where $q^2 = (p_1 - p_3)^2$

$$T^{ABC} = \frac{g^2}{q^2 - M_C^2}$$

The number of $M$ is $2 \times 2 \times 2 \times 2 = 16$

**Question:** **How to calculate cross section of $a+b \rightarrow c+d$ reaction for unpolarized $a,b$ if we know the corresponding invariant amplitude $M_{fi}$ ?**

***If the amplitude for reaction*** $a+b\to c+d$ ***has a form as***

$$M_{fi}=\frac{(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)}{\sqrt{2E_a2E_b2E_c2E_d}}T_{fi}$$

***then the cross section follows Fermi's golden rule***

$$d\sigma = \frac{1}{2\sqrt{\lambda(s,M_a^2,M_b^2)}}|T_{fi}|^2\frac{d\vec{p}_c}{(2\pi)^3 2E_c}\frac{d\vec{p}_d}{(2\pi)^3 2E_d}$$
$$\times(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)$$

*flux factor*

*invariant amplitude*

*phase space factors*

*total four momentum conservation*

*where*

$$s=(p_a+p_b)^2$$

***"triangle function"***

$$\lambda(s,M_a^2,M_b^2)=(s-(M_a+M_b)^2)(s-(M_a-M_b)^2)\equiv\lambda(i) \longrightarrow |\vec{p}_a|=\frac{\sqrt{\lambda(s,M_a^2,M_b^2)}}{2\sqrt{s}}$$

$$d\sigma \;=\; \underbrace{\frac{(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)}{2\sqrt{\lambda(s,M_a^2,M_b^2)}}\,\frac{d\vec{p}_c}{(2\pi)^3 2E_c}\,\frac{d\vec{p}_d}{(2\pi)^3 2E_d}} \times |T_{fi}|^2$$

***For calculation of the cross section we have to***

***(1) evaluate the "overall phase space factor"***

***(2) and determine*** $|T_{fi}|^2$

***( squared + sum over polarizations )***

## *evaluation of the phase space factor*

- $$\frac{1}{2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{(2\pi)^3 2E_c}\frac{d\vec{p}_d}{(2\pi)^3 2E_d}(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)$$

## *evaluation of the phase space factor*

- $$\frac{1}{2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{(2\pi)^3 2E_c}\frac{d\vec{p}_d}{(2\pi)^3 2E_d}(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}d\vec{p}_d dE_d\delta(E_d^2-\vec{p}_d^2-M_d^2)\delta^4(p_a+p_b-p_c-p_b)$$

## *evaluation of the phase space factor*

- $$\frac{1}{2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{(2\pi)^3 2E_c}\frac{d\vec{p}_d}{(2\pi)^3 2E_d}(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}d\vec{p}_d dE_d\delta(E_d^2-\vec{p}_d^{\,2}-M_d^2)\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}\delta((p_a+p_b-p_c)^2-M_d^2)$$

## *evaluation of the phase space factor*

- $$\frac{1}{2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{(2\pi)^3 2E_c}\frac{d\vec{p}_d}{(2\pi)^3 2E_d}(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}d\vec{p}_d dE_d \delta(E_d^2-\vec{p}_d^2-M_d^2)\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}\delta((p_a+p_b-p_c)^2-M_d^2)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}\delta(s-2\sqrt{s}E_c+M_c^2-M_d^2)\quad \left(s=(p_a+p_b)^2\right)$$

## *evaluation of the phase space factor*

- $$\frac{1}{2\sqrt{\lambda(i)}}\int\frac{d\vec{p}_c}{(2\pi)^3 2E_c}\frac{d\vec{p}_d}{(2\pi)^3 2E_d}(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int\frac{d\vec{p}_c}{2E_c}d\vec{p}_d dE_d\delta(E_d^2-\vec{p}_d^2-M_d^2)\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int\frac{d\vec{p}_c}{2E_c}\delta((p_a+p_b-p_c)^2-M_d^2)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int\frac{d\vec{p}_c}{2E_c}\delta(s-2\sqrt{s}E_c+M_c^2-M_d^2)\qquad\left(s=(p_a+p_b)^2\right)$$
- $$=\frac{1}{64\pi^2 s}\frac{|\vec{p}_c|}{|\vec{p}_a|}d\Omega$$

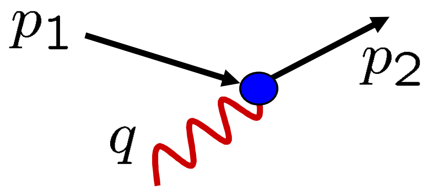
## *evaluation of the phase space factor*

- $$\frac{1}{2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{(2\pi)^3 2E_c}\frac{d\vec{p}_d}{(2\pi)^3 2E_d}(2\pi)^4\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}d\vec{p}_d dE_d \delta(E_d^2-\vec{p}_d^2-M_d^2)\delta^4(p_a+p_b-p_c-p_b)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}\delta((p_a+p_b-p_c)^2-M_d^2)$$
- $$=\frac{1}{8\pi^2\sqrt{\lambda(i)}}\int \frac{d\vec{p}_c}{2E_c}\delta(s-2\sqrt{s}E_c+M_c^2-M_d^2) \qquad \left(s=(p_a+p_b)^2\right)$$
- $$=\frac{1}{64\pi^2 s}\frac{|\vec{p}_c|}{|\vec{p}_a|}d\Omega$$
- $$=\frac{1}{32\pi s}\frac{|\vec{p}_c|}{|\vec{p}_a|}d\cos\theta$$

$$\boxed{\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{1}{64\pi^2 s}\frac{p_c}{p_a}|T_{fi}|^2\\ \frac{d\sigma}{d\cos\theta} &= \frac{1}{32\pi s}\frac{p_c}{p_a}|T_{fi}|^2\end{aligned}}$$

## *evaluation of square of matrix element*

$$\sum_{\text{spins } i,f} |T_{fi}|^2$$




$$\sum |T|^2 = \sum_{s_1 s_2 \lambda} |e\bar{u}^{s_2}(p_2)\gamma^\mu u^{s1}(p_1)\varepsilon^\mu_\lambda|^2$$

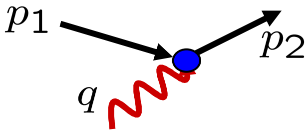
$$\to e^2 \sum [\bar{u}^{s_2}(p_2)\gamma_\mu u^{s1}(p_1) \times \bar{u}^{s_1}(p1)\gamma_\nu u^{s_2}(p_2)]\, \varepsilon^\mu_\lambda \varepsilon^{\nu *}_\lambda$$

$$\to e^2 \sum [u^{s_2}(p_2)\bar{u}^{s_2}(p_2)]\gamma_\mu [u^{s1}(p_1)\, \bar{u}^{s_1}(p1)]\gamma_\nu\, \varepsilon^\mu_\lambda \varepsilon^{\nu *}_\lambda$$

$$\sum_s u^s_\beta \bar{u}^s_\alpha = (\not{p} + m)_{\beta\alpha} \qquad \sum_\lambda \varepsilon^\mu_\lambda \varepsilon^{\nu *}_\lambda = -g^{\mu\nu}$$

$$\not{p} \equiv \gamma \cdot p$$

$$\sum_{s_1 s_2 \lambda} |T_{fi}|^2 = -e^2 \left( (\not{p}_2 + m)_{\beta'\alpha} \gamma^\mu_{\alpha\beta} (\not{p}_1 + m)_{\beta\alpha'} \gamma^\nu{}_{\alpha'\beta'} \right) g_{\mu\nu}$$




## *evaluation of* $\sum_{\text{spins}\, i,f} |T_{fi}|^2$ *(continue)*

$$\sum_{s_1 s_2 \lambda} |T_{fi}|^2 = -e^2 \left( (\not{p}_2 + m)_{\beta'\alpha} \gamma^{\mu}_{\alpha\beta} (\not{p}_1 + m)_{\beta\alpha'} {\gamma^{\nu}}_{\alpha'\beta'} \right) g_{\mu\nu}$$

$$\sum_{s_1 s_2 \lambda} |T_{fi}|^2 = -e^2 \mathrm{Tr}\left( (\not{p}_2 + m) \gamma^{\mu} (\not{p}_1 + m) \gamma_{\mu} \right)$$

$$I = \begin{pmatrix} I^{(2)} & 0 \\ 0 & I^{(2)} \end{pmatrix}, \quad \gamma_0 = \begin{pmatrix} I^{(2)} & 0 \\ 0 & -I^{(2)} \end{pmatrix}, \quad \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

$\mathrm{Tr}(I)=4$

$\mathrm{Tr}(\gamma_\mu)=0$

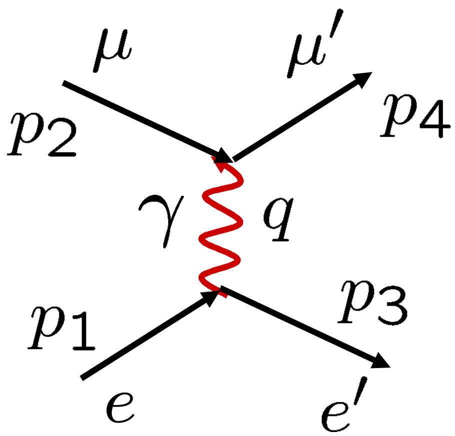
$\mathrm{Tr}(\gamma_\mu \gamma_\nu) = 4 g_{\mu\nu}$ $\quad\quad$ $\mathrm{Tr}(\not{p}_1 \not{p}_2) = \mathrm{Tr}[(p_1 \cdot \gamma)\,(p_2 \cdot \gamma)] = 4(p_1 \cdot p_2)$

$\mathrm{Tr}(\gamma_\alpha \gamma_\mu \gamma_\nu)=0$

$\mathrm{Tr}(\gamma_\alpha \gamma_\beta \gamma_\mu \gamma_\nu) = 4(g_{\alpha\beta} g_{\mu\nu} + g_{\alpha\nu} g_{\beta\mu\mu} - g_{\alpha\mu} g_{\beta\nu})$

**Summary:** ***utilization of the Feynman rules reduces evaluation of*** $|T_{fi}|^2$ ***to calculating traces of sum of the products of the Dirac gamma matrices …***

# $e\mu \to e\mu$ scattering (cross section)




$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|T_{fi}|^2$$

$$\sum_{s_1 s_2 \lambda} |T_{fi}|^2 = \frac{e^4}{q^4}\frac{1}{4}\mathrm{Tr}\left((\not{p}_4 + m_\mu)\gamma^\alpha(\not{p}_2 + m_\mu)\gamma^\beta\right)$$
$$\times \ \mathrm{Tr}\left((\not{p}_3 + m_e)\gamma_\alpha(\not{p}_1 + m_e)\gamma_\beta\right)$$

$$\frac{2e^4}{q^4}\left[(p_1p_2))(p_3p_4) + (p_1p_4))(p_2p_3) - (p_1p_3)m_\mu^2 - (p_2p_4))m_e^2 + 2m_\mu^2 m_e^2\right]$$

$$(p_1+p_2)^2 = (p_3+p_4)^2 = s, \ \ (p_1-p_3)^2 = (p_2-p_4)^2 = t$$

$$|T_{fi}|^2 \to \frac{2e^4}{t^2}\left(2(m_\mu^2 + m_e^2)^2 - 4s(m_\mu^2+m_e^2) + 2s^2 + st + t^2\right)$$

at high energies with $s \gg m_i^2, \ |t|$

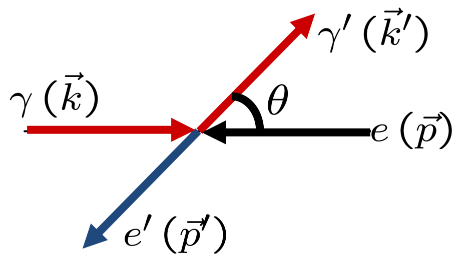
$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{t^2}s \simeq \frac{\alpha^2}{4E_e^2 \sin^4\frac{\theta}{2}}}$$

at low energies with $s \sim m_\mu^2$

$$\boxed{\frac{d\sigma}{d\Omega'} \simeq \frac{\alpha^2}{(2m_e v)^2 \sin^4\frac{\theta}{2}}}$$

# Example 2: $\gamma e \to \gamma' e'$ Compton scattering

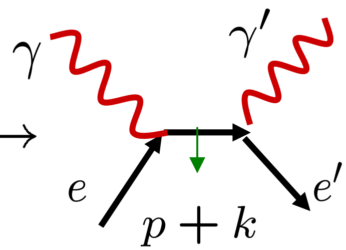
***(Klein-Nishina equation)***



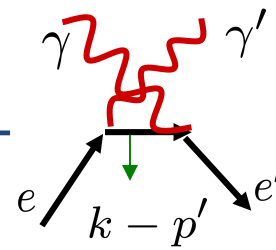

$$\boxed{\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s}|T_{fi}|^2}$$

*with* $s = (p+k)^2 = M_e^2 + 2p \cdot k$

$M_{fi}^{\gamma e \to \gamma e} \to$




$+$




$$= \overbrace{e^2 \epsilon^*_\mu(\gamma')\epsilon_\nu(\gamma) \cdot [\bar{u}(e')\widehat{M}^{\mu\nu}\, u(p)]}^{T_{fi}} \times \frac{(2\pi)^4 \delta(p+k-p'-k')}{\sqrt{2E_p 2E_{p'} 2\omega 2\omega'}}$$

$$\widehat{M}^{\mu\nu} = \gamma^\mu \frac{\gamma \cdot (p+k) + M_e}{(p+k)^2 - M_e^2}\gamma^\nu + \gamma^\nu \frac{\gamma \cdot (k-p') + M_e}{(p'-k)^2 - M_e^2}\gamma^\mu$$

$$= \gamma^\mu \frac{\gamma \cdot p + \gamma \cdot k + M_e}{2p \cdot k}\gamma^\nu - \gamma^\nu \frac{\gamma \cdot k - \gamma \cdot p' + M_e}{2p' \cdot k}\gamma^\mu$$

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s}|T|^2$$

$$|T|^2 = \frac{e^4}{4} \sum_{\lambda_\gamma,\lambda'_\gamma,m_e,m'_e} |\epsilon^*_\mu(\gamma')\,[\bar{u}(e')M^{\mu\nu}\,u(p)]\epsilon_\nu(\gamma)|^2$$

$$|T|^2 = T_d^2 + T_{exch}^2 + 2T_{interf}^2$$ *with* $$T_i^2 \sim \mathrm{Tr}[\underbrace{\gamma \ldots \gamma}_{4\ldots 8}]$$

$$\frac{d\sigma}{d\cos\theta} = \pi r_0^2\, F(p, p', k),$$

$$r_0 = \frac{e^2}{4\pi M_e} \equiv \frac{\alpha}{M_e} \simeq 2.82 \text{ fm}, \;\; (1\text{fm} = 10^{-13}\text{cm})$$

*"classic electron radius"*

and function

$$F(p,p',k) = \frac{4M_e^2}{s}\left\{\left(\frac{M_e^2}{2k\cdot p} - \frac{M_e^2}{2k\cdot p'}\right)^2 + \left(\frac{M_e^2}{2k\cdot p} - \frac{M_e^2}{2k\cdot p'}\right) + \frac{1}{4}\left(\frac{k\cdot p}{k\cdot p'} + \frac{k\cdot p'}{k\cdot p}\right)\right\}$$

## For further applications we introduce an invariant variable

$$u = \frac{k \cdot k'}{k \cdot p'} = v \frac{1 - \cos\theta}{1 + v\cos\theta}$$

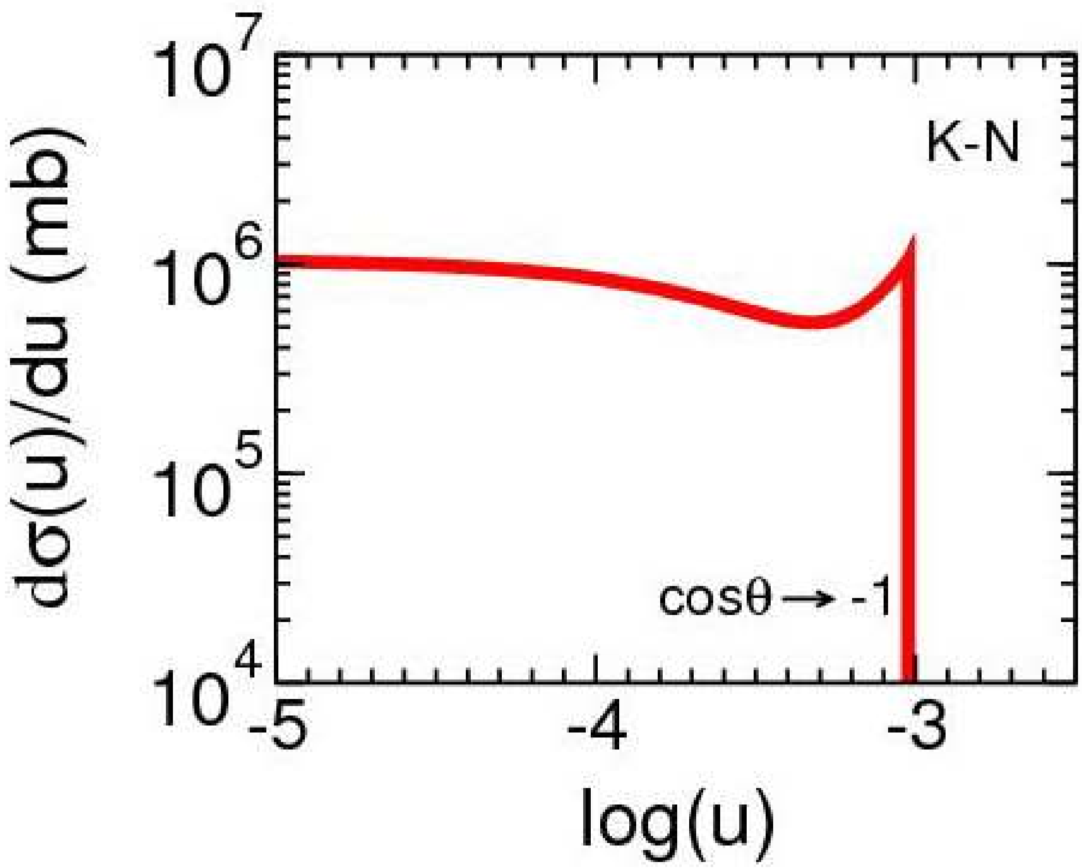
with $\quad v = \frac{p_e}{E_e} \quad$ *is the electron velocity in c.m.s.*

and $$d\cos\theta = \frac{2s}{s - M_e^2} \frac{du}{(1+u)^2}$$

$$\frac{d\sigma}{du} = \frac{8\pi r_0^2}{s - M_e^2} \left( \frac{u^2}{u_0^2} - \frac{u}{u_0} + \frac{1}{4}(1 + u + \frac{1}{1+u}) \right)$$

where $$u_0 = u_{\text{max}} = \frac{s - M_e^2}{M_e^2} = \frac{2E_\gamma^L}{M_e}$$

# Compton $\gamma\, e \rightarrow \gamma\, e$ scattering (results)

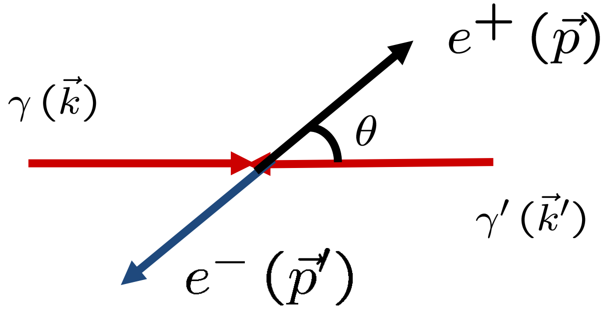



$\xi^2$

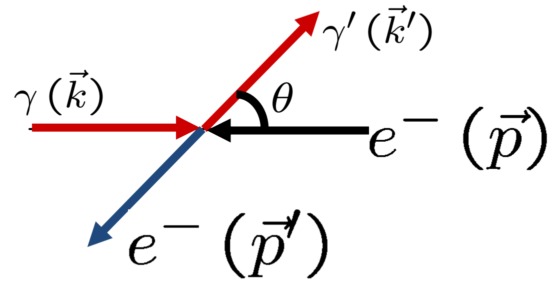
***Cross section has a sharp maximum at backward angles of outgoing (re-scattering) photons***

# Example 3: reaction $\gamma\gamma' \to e^+e^-$

***Breit-Wheeler process***




***Compton scattering (Klein-Nishina process)***



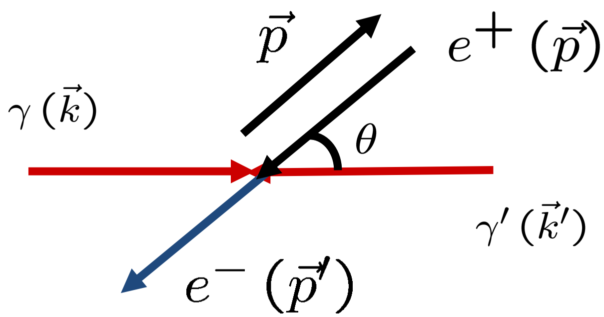

$$d\sigma^{B-W} = \frac{1}{32\pi s}\frac{|\vec{p}|}{|\vec{k}|}|T^{B-W}|^2\, d\cos\theta_{\widehat{kp}}$$

$$d\sigma^{K-N} = \frac{1}{32\pi s}[1]|T^{K-N}|^2\, d\cos\theta_{\widehat{kk'}}$$

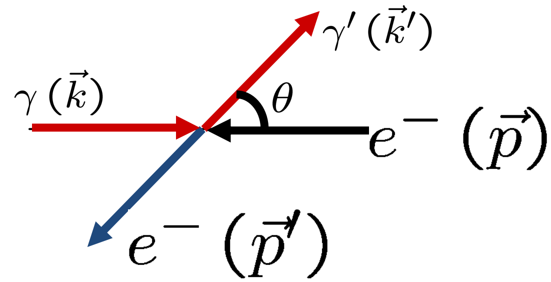
$$|T^{B-W}(p,p',k,k')|^2 = -|T^{K-N}(-p,p',k,-k')|^2$$

# Example 3: reaction $\gamma\gamma' \to e^+e^-$

***Breit-Wheeler process***




***Compton scattering (Klein-Nishina process)***




$$d\sigma^{B-W} = \frac{1}{32\pi s}\frac{|\vec{p}|}{|\vec{k}|}|T^{B-W}|^2\, d\cos\theta_{\widehat{kp}}$$

$$d\sigma^{K-N} = \frac{1}{32\pi s}[1]|T^{K-N}|^2\, d\cos\theta_{\widehat{kk'}}$$

$$|T^{B-W}(p,p',k,k')|^2 = -|T^{K-N}(-p,p',k,-k')|^2$$

$$u_p\bar{u}_p = \gamma\cdot p + M_e \to -\gamma\cdot p + M_e = -(\gamma\cdot p - M_e) = -v_p\bar{v}_p$$

***projection o.***

$$\boxed{\frac{d\sigma^{B-W}}{d\theta_{\widehat{kp}}} = \frac{4\pi r_0^2 M_e^2}{s}\left\{\frac{1}{4}\left(\frac{k\cdot p}{k\cdot p'} + \frac{k\cdot p'}{k\cdot p}\right) - \left(\frac{M_e^2}{2k\cdot p} + \frac{M_e^2}{2k\cdot p'}\right)^2 - \left(\frac{M_e^2}{2k\cdot p} + \frac{M_e^2}{2k\cdot p'}\right)\right\}.}$$

# $\gamma\gamma \to e^+e^-$ Breit-Wheeler process (continue)

***invariant variable***

$$u = \frac{(k \cdot k')^2}{4(k \cdot p)(k \cdot p')} = \frac{1}{1 - v^2 \cos^2\theta}$$

***where*** $v$ ***is velocity of electron (positron)*** $v = \frac{p}{E_p}$

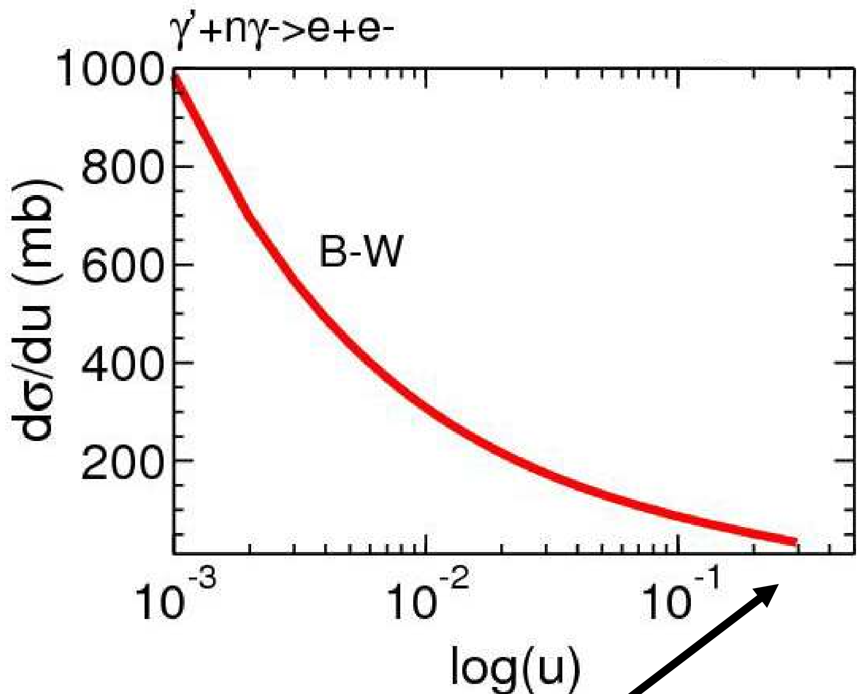
$$\frac{d\sigma^{B-W}}{du} = \frac{e^4}{8\pi s}\left\{\left(\frac{1}{\gamma^2}+1\right)u - \frac{1}{\gamma^4}u^2 - \frac{1}{2}\right\}\frac{1}{u^{3/2}\sqrt{u-1}}$$

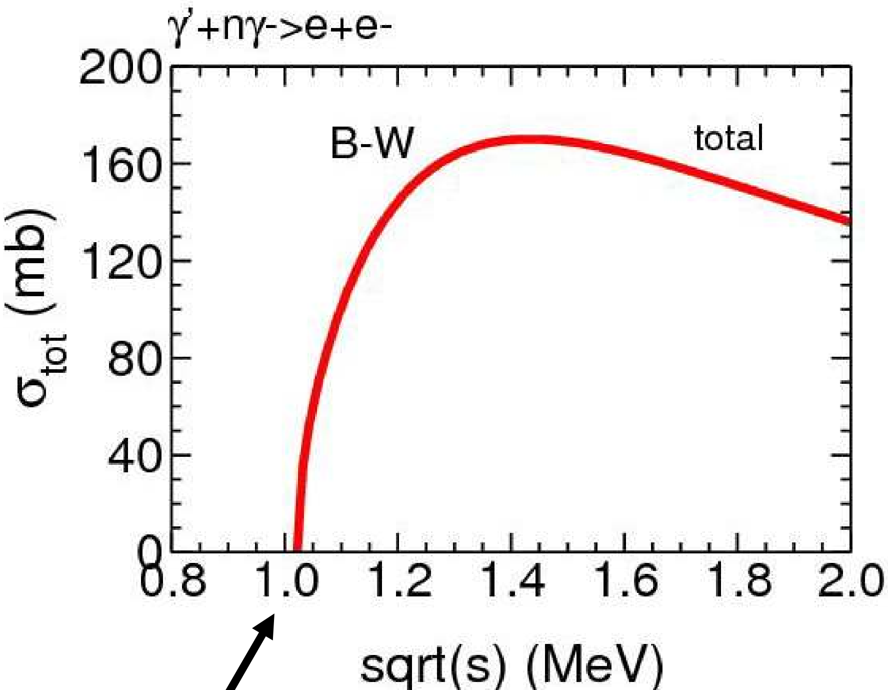
***where*** $\gamma^2 = 1/(1-v^2) = \frac{E_p^2}{M_e^2}$

***and*** $1 < u < u_{\text{max}},\ u_{\text{max}} = \gamma^2$

$$d\sigma_{\text{tot}}^{B-W} = \frac{e^4}{4\pi s\gamma^4}\left\{(2\gamma^4 + 2\gamma^2 - 1)\text{arc}\,\text{sh}(\sqrt{1-\gamma^2}) - \gamma(1+\gamma^2)\sqrt{1-\gamma 2}\right\}$$

## $\gamma\gamma \to e^+e^-$ *Breit-Wheeler process (continue)*




$$\log(u_{\mathrm{max}}) = 2\log\left(\frac{E_p}{M_e}\right)$$

***threshold*** $\sqrt{s_{\mathrm{thr.}}} = 2M_e$

# Example 4:

## *Dimuon production by laser - wakefield accelerated electrons*

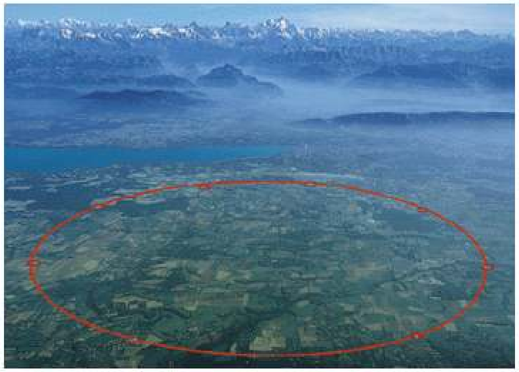
**A.T., B.Kampfer,and H. Takabe**
**Phys.Rev.ST Accel.and Beams 12 (2009)**

VOLUME 43, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 1979

**Laser Electron Accelerator**

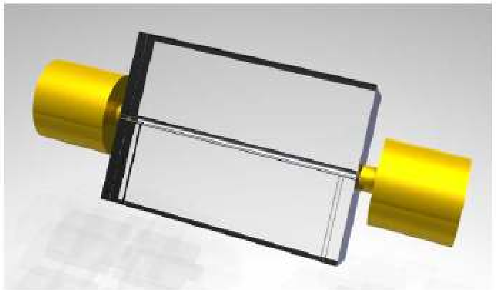
T. Tajima and J. M. Dawson
*Department of Physics, University of California, Los Angeles, California 90024*

An intense electromagnetic pulse can create a *wake* of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density $10^{18}$W/cm$^2$ shone on plasmas of densities $10^{18}$ cm$^{-3}$ can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.



- The acceleration gradient in a plasma accelerator is <u>much</u> bigger:
  - In a conventional particle accelerator acceleration gradient limited by electrical breakdown to:

$$E_z \approx 10 - 100 \text{ MV m}^{-1}$$



  - A plasma accelerator can reach acceleration gradients of order:

$$\boxed{E_z \approx 100 \text{ GV m}^{-1}}$$

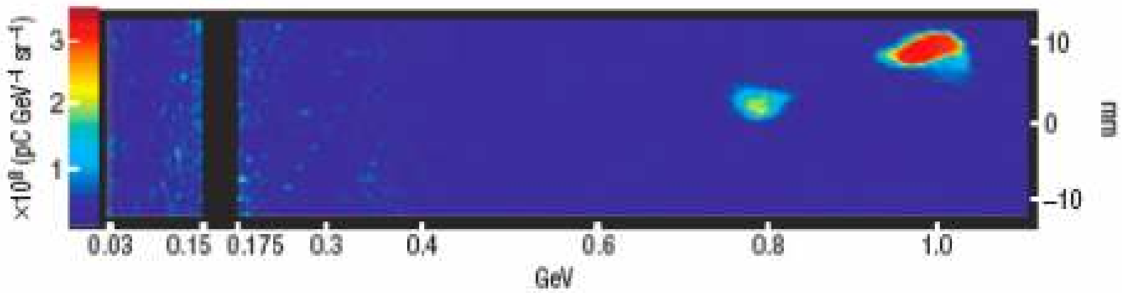
***i.e. 3 to 4 orders of magnitude larger***

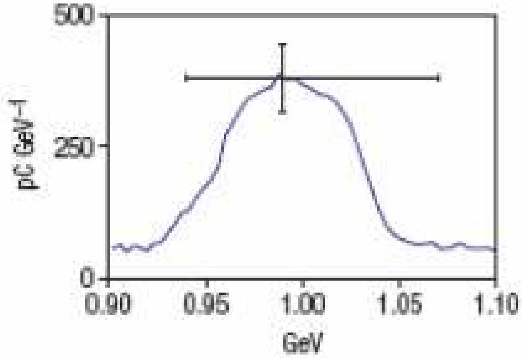
## *Leemans experiment* *[Lawrence Berkeley National Laboratory]*

High quality GeV Electron Beams from Laser-Plasma Accelerator

W. P. Leemans et al. *Nature Physics* **2** 696 (2006)

K. Nakamura et al. *Phys. Plasmas* **14** 056708 (2007)



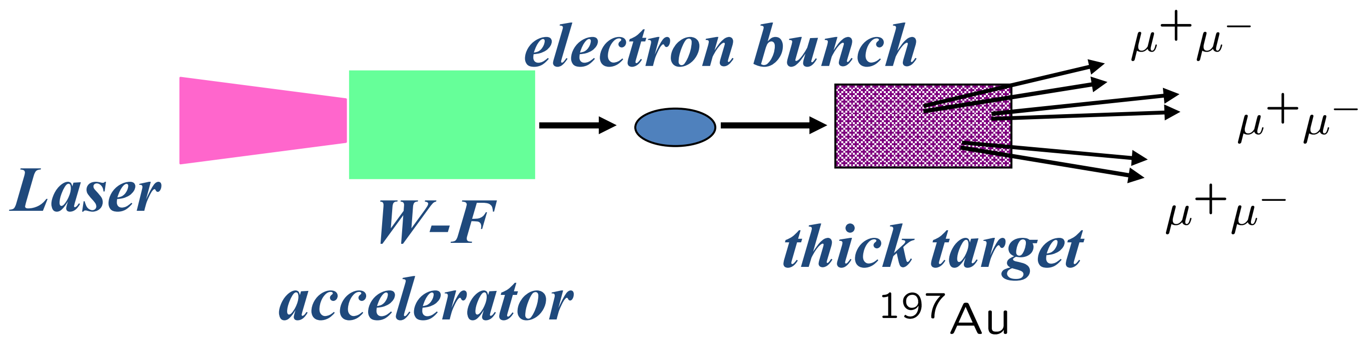

Experiment conditions:

- Capillary: 33 mm, 312 μm diam.
- Density $4.3 \times 10^{18}$ cm$^{-3}$
- Laser: 40 TW: 1.5 J, 37 fs

$E$ = (1.0 +/- 0.06) GeV
$\Delta E$ = 2.5% r.m.s
$\Delta\theta$ = 1.6 mrad r.m.s.
Q = 30 pC

In recent experiments with the Astra-Gemini laser by the Imperial College et al, 0.8 GeV beams were generated by relativistically-guided (200 TW) laser pulses [Kneip et al. *Phys. Rev. Lett.* **103**, 035002 (2009)]

**Our aim:** ***analysis of effectiveness of laser driven electrons for dimuon production in strong electric field of high-Z atoms***




## Motivation:

***(1) muons as a source of neutrino beams***

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \qquad\qquad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

*for studying neutrino oscillations*

***(2) studying different aspects of $\mu$-meson physics***

***(3) different problems of muon-nuclear physics***

# EM sources of muons

*Direct muons*

$$e + Z \to \mu^+ + \mu^- + e' + Z$$

*muons come from secondary interactions*

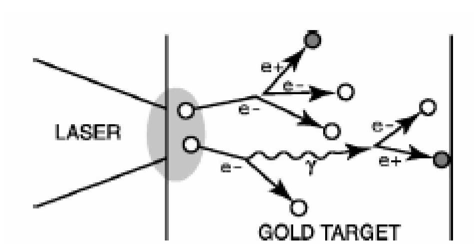
$$e + Z \to \gamma + e' + Z \longleftrightarrow$$ *photon bremsstrahlung*

$$\gamma + Z \to \mu^+ + \mu^- + Z$$

**Generally, we have to solve two problem**

**(1) elementary** $e(\gamma)Z \to \mu^+\mu^-$ **processes**

**(2) cascade processes (transport dynamics)**



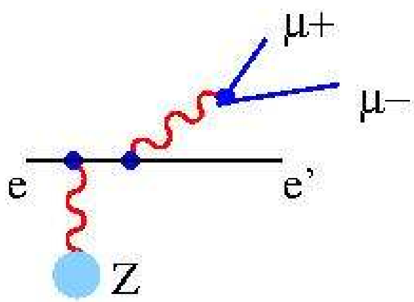

$$e + Z \to e^+ + e^- + e' + Z$$

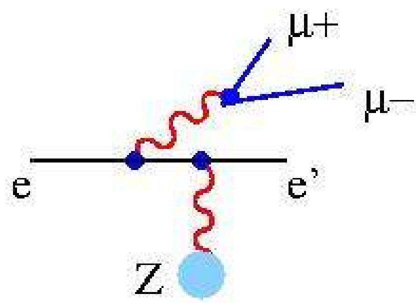
$$\gamma + Z \to e^+ + e - + Z$$

Nakashima&Takabe
*Phys. Plasmas,* 9,1505 (2002)

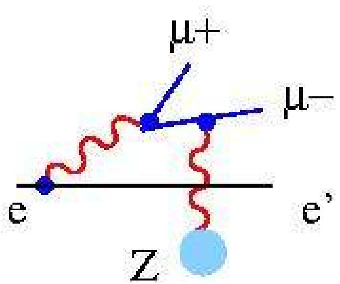
# EM processes (EM sources)

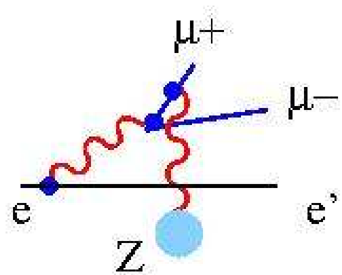
$$e \rightarrow \mu^{+}\mu^{-}$$




**Direct dimuon production in eA interaction**

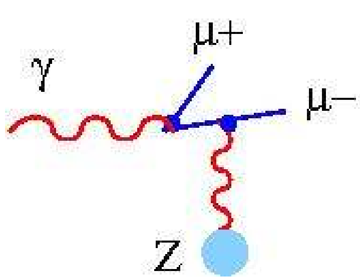


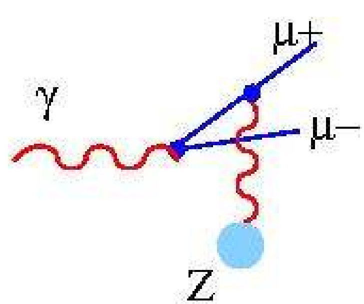




***Trident process***

**Dimuon production by secondary high energy photons**

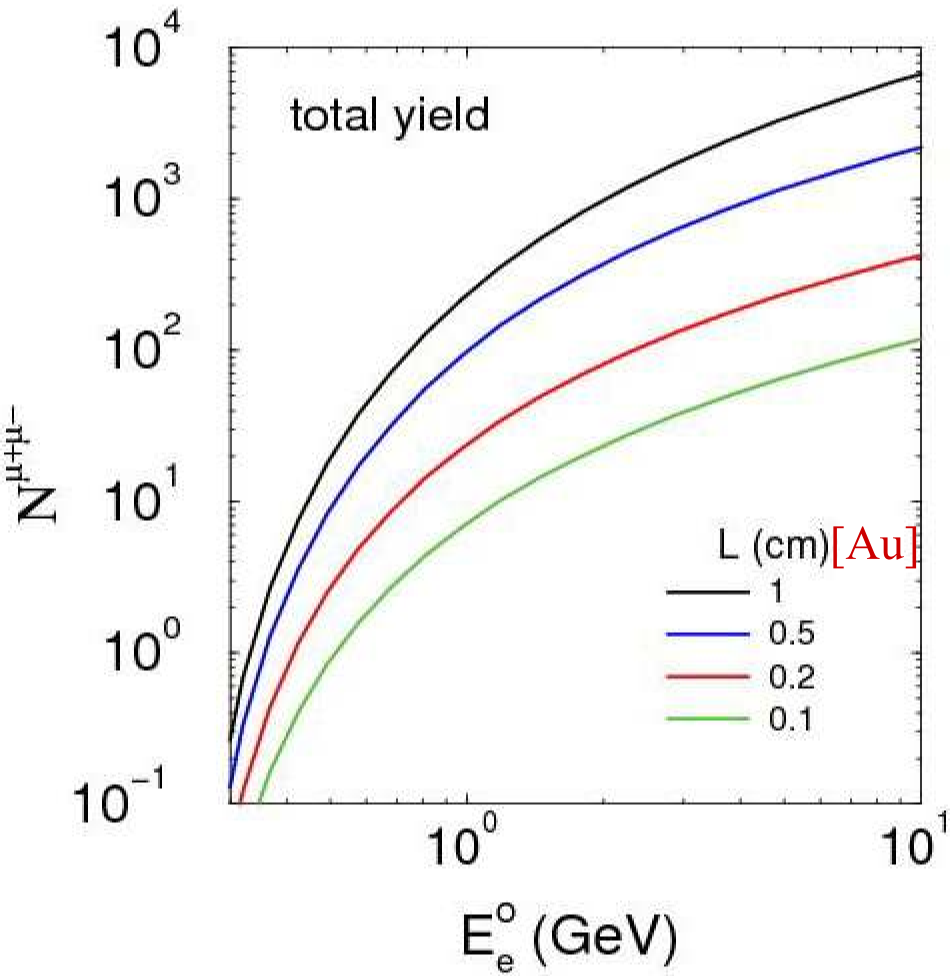
$$\gamma \rightarrow \mu^{+}\mu^{-}$$




***Bethe – Heitler process***

# *Total dimuon yield*




$$N_{\text{tot}}^{\mu^+\mu^-} \simeq N_{\gamma A}^{\mu^+\mu^-}$$

at large $E_e^0$ and $L$

The number of dimuons in eA interactions is about 200 and 6000 for $E_e$=1 and 10 GeV, respectively, for target thickness of 1 cm.

(for $1.25\times10^8$ electrons in pulse)

# Summary of Parts I+II:

Gauge Invariance is the basis for construction of coupled equations of QED

These equations predict electron spin, positron

Feynman rules give clear prescription for construction and evaluation of matrix elements for processes with photons and fermions (electrons and positrons)

We illustrate this with examples ….

# Part III. Non-Perturbative QED
# Electron in a strong EM field

# *Electron in a strong electromagnetic field*

**D.M. Volkov, Z. Phys. 94, 250 (1935)**

**Über eine Klasse von Lösungen der Diracschen Gleichung.**

1. Der Fall eines sinusoidalen Feldes. — 2. Lösung für den Fall, daß das äußere Feld aus polarisierten, in einer bestimmten Richtung fortschreitenden Wellen besteht, die ein *abzählbares* Spektrum nach Frequenz und Anfangsphasen haben.

$$(i\nabla\!\!\!/ - eA + m)\cdot(i\nabla\!\!\!/ - eA - m)\psi = 0,$$

***Second order Dirac equation***

$$[(i\nabla - eA)^2 - m^2 - i\frac{1}{2}F_{\mu\nu}\sigma^{\mu\nu}]\psi = 0,$$

***4-componenrt spinor***

***where*** $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ ***is EM field tensor***

$$\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu), \;\; \gamma_\mu \;\; - 4\times 4 \;\; \textit{Dirac matricies} \quad \gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \; \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

***Special case:***

$A = A(\phi)$ ***with*** $\phi = k \cdot x = \omega t - kz \longrightarrow$ *plane wave*

***and*** $A = (0, \vec{A}_\gamma)$ ***is four vector of electromagnetic field with the special part chosen as***

$$\vec{A}_\gamma = \vec{a}_x \cos(kx) + \vec{a}_y \sin(kx)$$

***with*** $|\vec{a}_x| = |\vec{a}_y| = a$

Transversality condition $\partial_\mu A^\mu = k_\mu A^{\mu\prime} = 0$ result in following equation

$$[-\partial^2 - 2ie(A\partial) + e^2A^2 - M_e^2 - ie(\gamma k)(\gamma A')]\psi = 0.$$

Then, we seek a solution in the form

$$\psi = \mathrm{e}^{-ipx}\, F(\phi), \quad \phi = kx$$

Using conditions

$$\partial^\mu F = k^\mu F', \quad \partial_\mu \partial^\mu F = k^2 F'' = 0, \qquad \text{since } k^2 = 0$$

we find the equation for the function *F(ϕ)*

$$2i(kp)F' + [-2e(pA) + e^2A^2 - ie(\gamma k)(\gamma A')]F = 0$$

the formal (exact) solution of this equation reads

$$F = \exp\left( -i \int_o^{kx} \left[ \frac{e}{(kp)}(pA(\phi')) - \frac{e^2A^2(\phi')}{2(kp)} \right] d\phi' + \frac{e(\gamma k)(\gamma A)}{2(kp)} \right) \frac{u}{\sqrt{2p_0}}$$

## (i) Volkov's solution and its properties

$$\psi_p = \underbrace{\left[1 + \frac{e(\gamma \cdot k)(\gamma \cdot A)}{2(k \cdot p)}\right] \frac{u_p}{\sqrt{2E_p}}}_{\textit{spinor modification}} \mathrm{e}^{-ip\cdot x} \cdot \underbrace{\mathrm{e}^{iS'(\phi)}}_{\textit{phase factor}}$$

***with*** $S'(\phi) = -\int_0^{kx} \left[\frac{e(p \cdot A(\phi'))}{(k \cdot p)} - \frac{e^2 A^2(\phi')}{2(k \cdot p)}\right] d\phi', \quad \phi = kx$

***when*** $\vec{A} \to 0$ ***or*** $(a_x, a_y \to 0)$

$$\psi_p \to \frac{u_p}{\sqrt{2E_p}} \mathrm{e}^{-ip\cdot x}$$

***Volkov solution → Dirac solution for free electron***

## *(ii) Properties of Volkov solution*

- ***effective "quasi" momentum***

$$\langle \psi^* (\hat{p}^\mu - eA^\mu)\psi\rangle \neq p^\mu - eA^\mu ==> q^\mu - eA^\mu$$

***where***

$$q^\mu \equiv p^\mu - \frac{e^2\bar{A}^2}{2(k\cdot p)}k^\mu = p^\mu + \frac{e^2a^2}{2(k\cdot p)}k^\mu = p^\mu + \frac{\xi^2 m_e^2}{2(k\cdot p)}k^\mu$$

$$\bar{A}^2 = -\frac{1}{2}(a_x^2 + a_y^2) = -a^2$$ ***with*** $$\xi^2 = \frac{e^2a^2}{m_e^2}$$ → ***"reduced" EM field intensity***

$$\boxed{q^\mu = p^\mu + \frac{e^2a^2}{2(k\cdot p)}k^\mu}$$

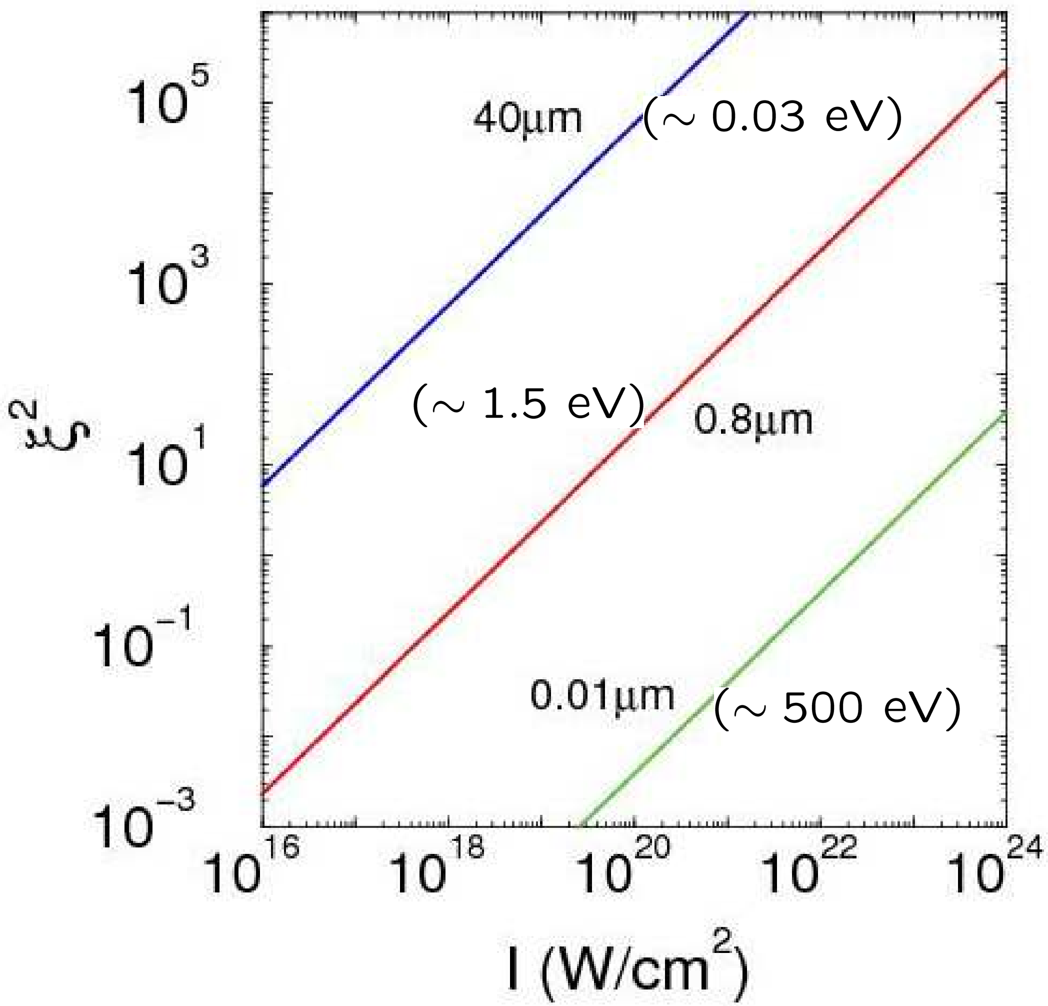
- ***effective electron mass***

$$q^2 = m_*^2 \equiv m_e^2\left(1 - \frac{e^2\bar{A}^2}{m_e^2}\right) = m_e^2\left(1+\xi^2\right)$$

$$\boxed{m_{e*}^2 = m_e^2(1+\xi^2)} \longrightarrow m_*^2 > m_e^2$$

***the "quasi-momentum" and the effective, dressed mass determine the momentum-energy conservation in processes with electrons***

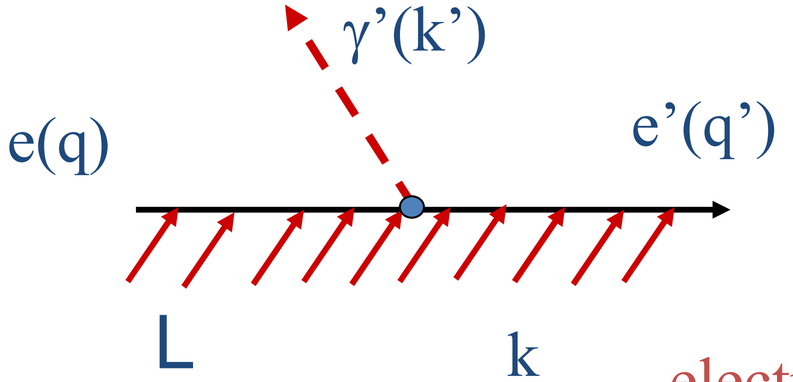
Dependance of reduced field strengh $\xi^2$ on laser pulse intensity $I$ at different wavelength $\lambda$

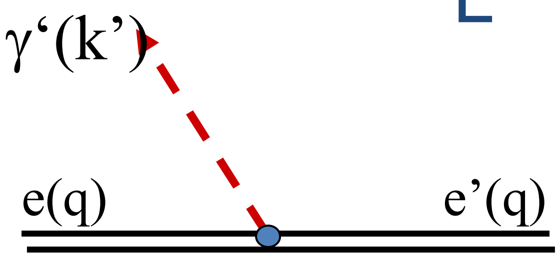



$$\xi^2 = \frac{\alpha \lambda_\gamma^2}{\pi M_e^2} I$$

# Photon Emission off an Electron in a Strong Electromagnetic Field

## *Emission of a photon by an electron in the field of a strong electromagnetic wave*




electron → "dressed" electron

with effective momentum $p \to q$

and mass $m_e^2 \to m_e^2(1+\xi^2)$

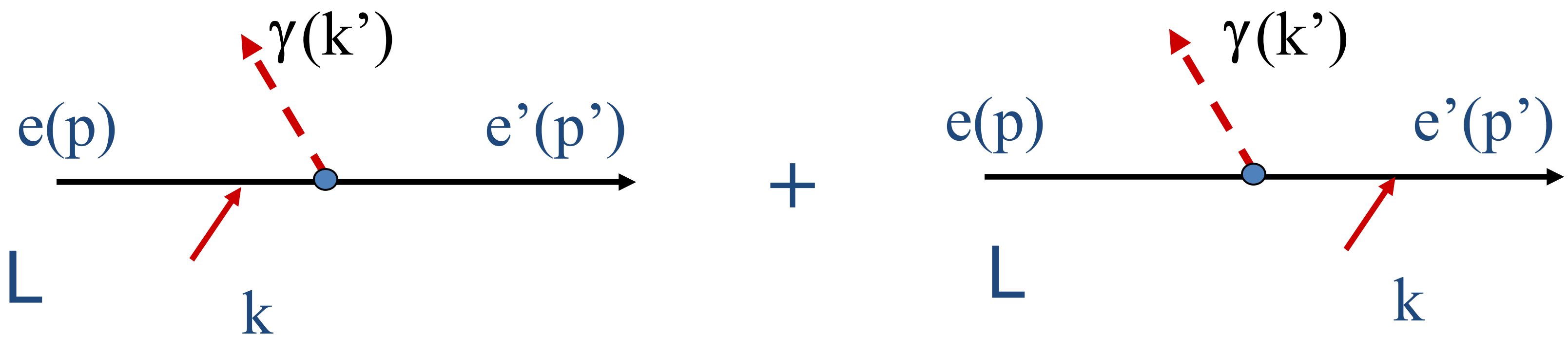
$$M_{fi} = -ie\int \psi_f^*(\gamma\cdot\varepsilon_f^*)\psi_i \mathrm{e}^{ik'x}\frac{d^4x}{\sqrt{2\omega'}}$$

***Interaction of electron with an external field is considered non-perturbatively (distorted waves)***

***Interaction of electron with outgoing photon is consider in first order of perturbation theory***

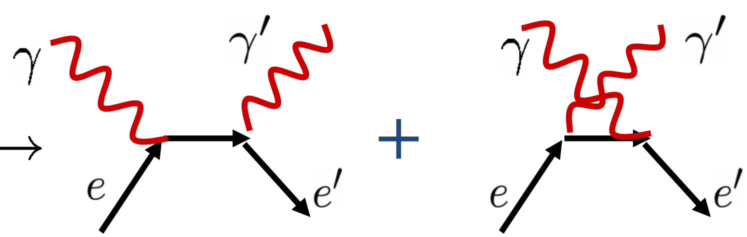
## *What we expect in the case of weak EM field with $\xi^2 \ll 1$ ?*

$$m_{e*}^2 = m_e^2(1+\xi^2) \rightarrow m_e^2 \quad \textit{and} \quad q^\mu = p^\mu + \frac{\xi^2 m_e^2}{2(k \cdot p)} k^\mu \rightarrow p^\mu$$




**Standard Compton scattering, described by the Klein-Nishina (K-N) equation**

$M^{\gamma e \rightarrow \gamma e} \rightarrow$




$$= e^2 \epsilon_\mu^*(\gamma')\epsilon_\nu(\gamma) \cdot [\overline{u}(e') M^{\mu\nu} u(p)] \times (2\pi)^4 \delta(p+k-p'-k')$$

$$M^{\mu\nu} = \gamma^\mu \frac{\gamma \cdot p + \gamma \cdot k + M_e}{2p \cdot k} \gamma^\nu - \gamma^\nu \frac{\gamma \cdot k - \gamma \cdot p' + M_e}{2p' \cdot k} \gamma^\mu$$

# *Structure of matrix element*

$$T_{fi} = -ie \int \psi_f^* (\gamma \cdot \varepsilon_f^*) \psi_i \mathrm{e}^{ik'x} \frac{d^4x}{\sqrt{2\omega'}}$$

$$\frac{\bar{u}_{p'}}{\sqrt{2E'_p}} \mathrm{e}^{-ip'\cdot x} \left[ 1 + \frac{e(\gamma \cdot A)(\gamma \cdot k)}{2(k \cdot p')} \right] \mathrm{e}^{iS'(k\cdot x, p'_e)}$$

*"non-perturbative" outgoing electron*

$$\left[ 1 + \frac{e(\gamma \cdot k)(\gamma \cdot A)}{2(k \cdot p)} \right] \mathrm{e}^{iS'(k\cdot x, p_e)} \frac{u_p}{\sqrt{2E_p}} \mathrm{e}^{-ip\cdot x}$$

*"non-perturbative" incoming electron*

$$\rightarrow \frac{-ie}{\sqrt{2q_0 2q'_0 2\omega'}} \int M(kx) \mathrm{e}^{-i(q-q'-k')x} d^4x$$

*with*

$$M(kx) = [..]_f \, \bar{u}_{p'} (\gamma \cdot \varepsilon_f^*) [..]_i \, u_p \mathrm{e}^{-i(S(kx) - S'(kx))}$$

In "K-N" Compton scattering γe→>γe, one has

$$M \sim \int M(k, k', p, p') \mathrm{e}^{-i(p+k-p'-k')x} d^4x$$
$$= (2\pi)^4 \delta^4 (p + k - p' - k') \, M(k, k', p, p')$$

$$\neq (2\pi)^4 \delta^4 (q + k - q' - k') \cdot M$$

## *Structure of matrix element (continuing)*

$$\frac{-ie}{\sqrt{2q_0 2q_0' 2\omega'}} \int M(kx) \mathrm{e}^{-i(q-q'-k')x} d^4x$$

Fourier series

$$M(kx) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{-in\,kx} M_n(k, k', q, q')$$

***The amplitude is a sum of infinite numbers of "partial harmonics"***

$$T_{fi} \;=\; -ie \sum_{n=-\infty}^{\infty} M_n \int \mathrm{e}^{-i(q+nk-q'-k')} d^4x$$

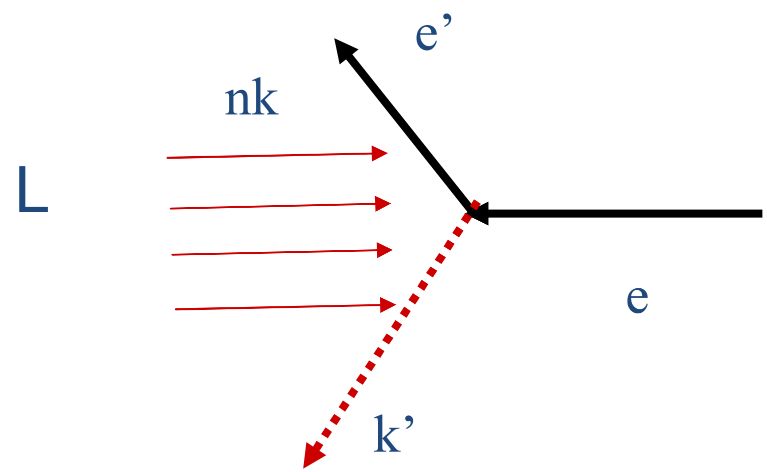
$$= \sum_n -ie\, M_n (2\pi)^4 \delta^4(q + nk - q' - k')$$

***Each harmonic describes absorption (emitting) of n photons of external field A with wave vector k and emitting of outgoing photon with the wave vector k' with corresponding conservation low***

# *Probability is a sum of partial contributions*

$$\boxed{dW = \sum_n dW^{(n)}}$$

$$dW(n) = \frac{1}{16\pi E_q} |T^{(n)}|^2 \frac{du}{(1+u)^2}$$



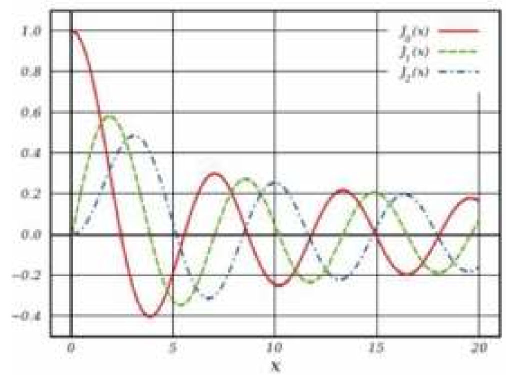
# *Partial contributions*

(Ritus & Nikishov, 1967)

$$dW_n = \frac{\alpha}{4E_q}\frac{du}{(1+u)^2}\left\{-4J_n^2(z) + \xi^2(2+\frac{u^2}{1+u})\left(J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)\right)\right\}$$

$$u = \frac{k\cdot k'}{k\cdot p'}, \qquad z = \frac{2n\xi}{\sqrt{1+\xi^2}}\sqrt{\frac{u}{u_n}\left(1-\frac{u}{u_n}\right)}$$

$$J_n(z) = \int_{-\pi}^{\pi} e^{i(n\phi - z\sin\phi)}\, d\phi; \quad \phi = kx$$



## *properties of partial contributions:*

$$u_{\max} = \frac{2n\omega_L(E_e+p)}{m_e^2(1+\xi^2)} \simeq \begin{cases} \dfrac{n\,\omega_\gamma^L}{m_e\sqrt{1+\xi^2}} & for\ E_e \simeq m_e \\[2ex] \dfrac{4nE_e\omega_\gamma^L}{m_e^2(1+\xi^2)} & for\ E_e \gg m_e \end{cases}$$

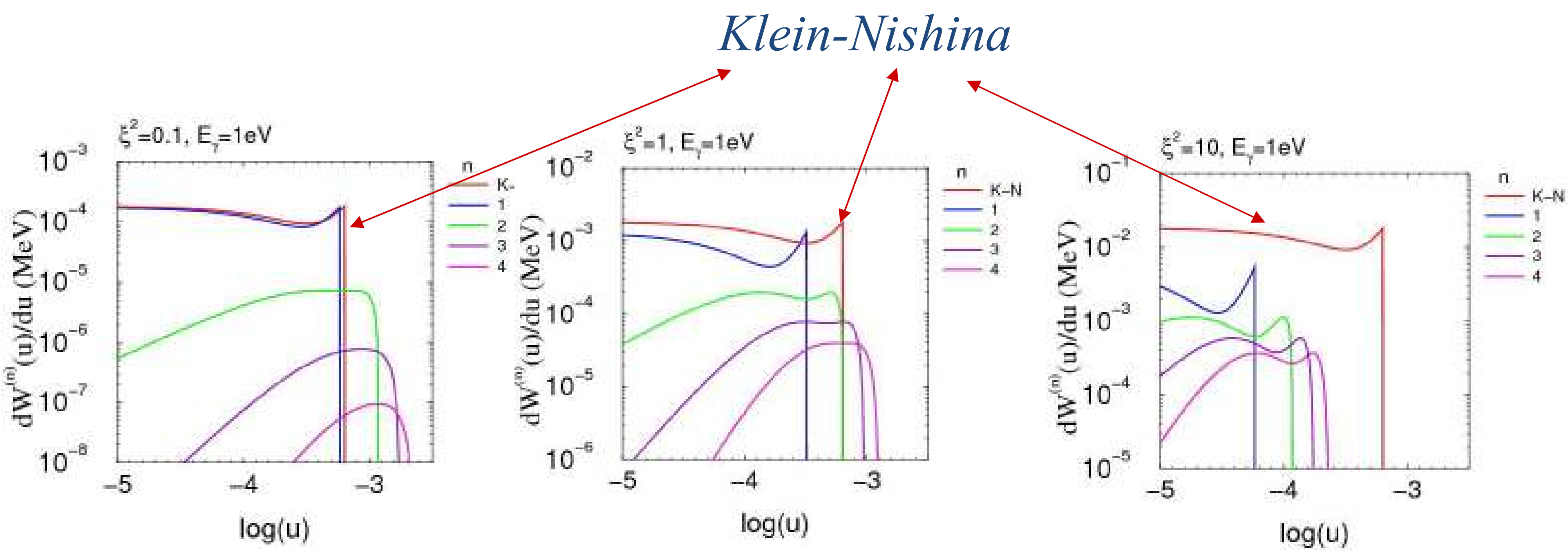
$m_*^2$

Kinematical limit (phase space) ***increases*** (n>0)

**2 effects:**
- *Electron can interact with a few photons simultaneously (“cumulative” effect)*
- *Dressed electron mass exceeds free electron mass*

Results in ***decrease*** of the phase space even for one photon absorption

# Photon emission in strong EM (results)




*At small field intensity* $\xi^2 \ll 1$ *effect of mass modification is small, "cumulative" effect is large*

*At large field intensity* $\xi^2 \gg 1$ *effect of mass modification is larger, than "cumulative" effect. However, the later one is also important.*

*At* $\xi^2 \geq 1$ *standard Klein-Nishina equation does not work even for* $n=1$.

A.T., B.Kampfer, H. Takabe, and A. Hosaka, *Phys.Rev.D,83, 053008, (2011)*

# Asymptotic solution

$$\sum_n \to \int_{n_{\min}(\xi)}^{\infty} dn \to \int_{-\xi/2}^{\infty} d\tau$$

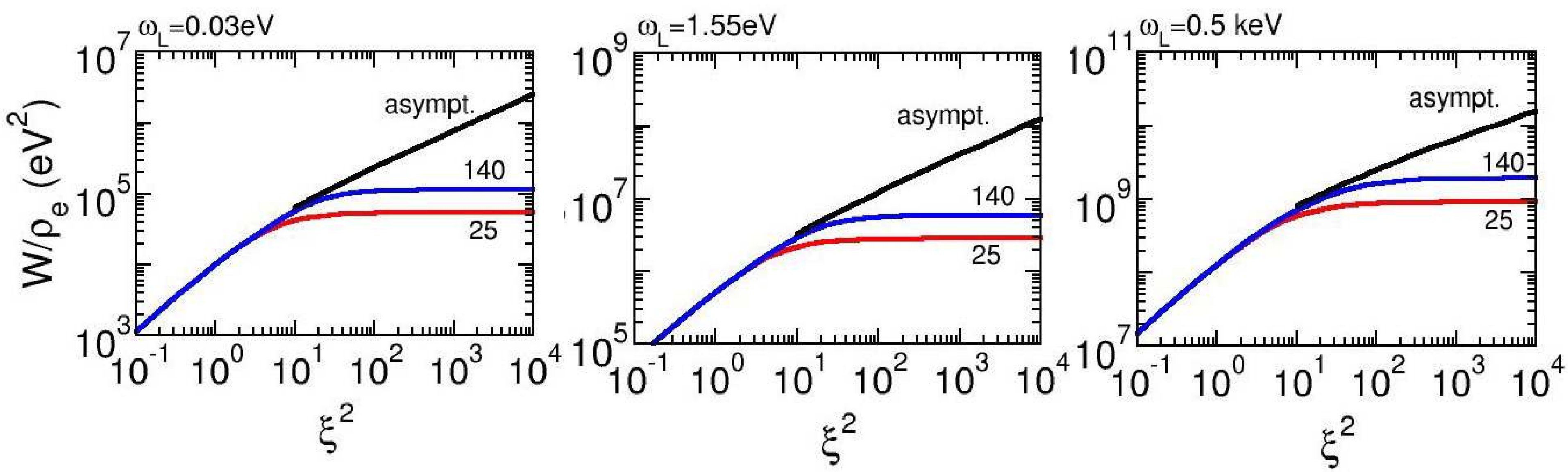
$$W^{Asympt.}(\xi,\chi) = \frac{\rho_e \alpha M_e^2}{\pi^2}\int_0^{\infty}\frac{\sqrt{t}\,du}{(1+u)^2}\int_{-\xi/2}^{\infty} d\tau\left[-\Phi^2(y) + [1+\frac{u^2}{2(1+u)}]\frac{1}{t}\left(y\Phi^2(y)-\Phi'^2(y)\right)\right] .$$

$$J_n^2(z) \to \frac{1}{\pi^2\xi^2 t}\Phi^2(y),$$

$$y = t(1+\tau^2),\;\; t = (\frac{u}{2\chi})^{\frac{2}{3}} .$$

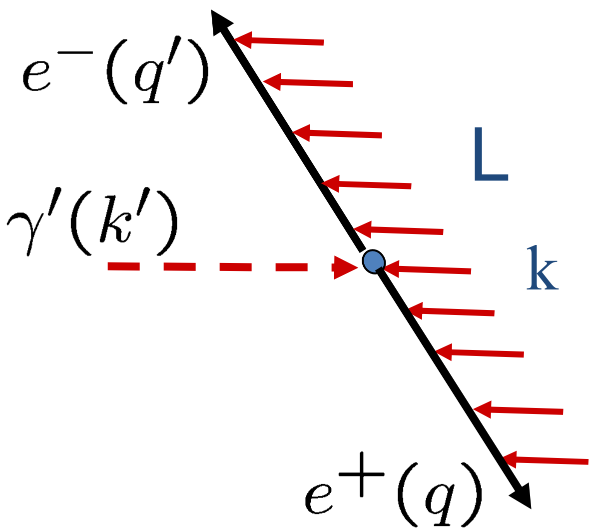
*where* $\tau$ *is an auxilary variable and* $\chi = \xi\frac{\kappa p}{M_e^2}$

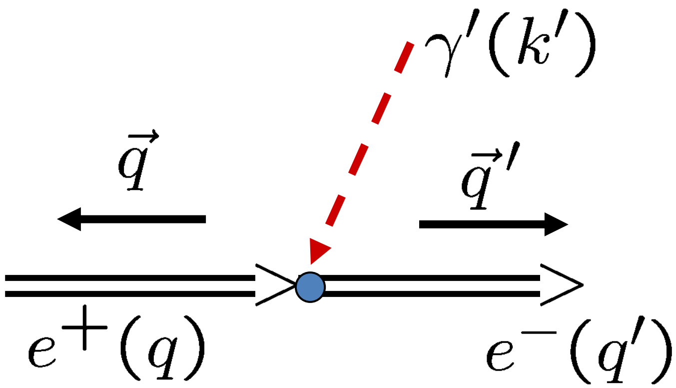
# Result:



# Electron-Positron Emission off a Photon in a Strong Electromagnetic Field

## ***Reaction*** $\gamma' + L(n\gamma) \rightarrow e^+ + e^-$ ***(Breit-Wheeler process)***



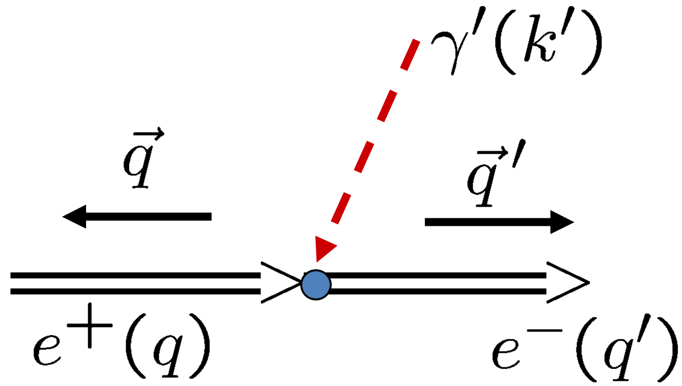

*electron/positron → "dressed" electron/positron*

with effective momentum $p \rightarrow q$
and mass $m_e^2 \rightarrow m_e^2(1+\xi^2)$

$$M_{fi} = -ie \int \psi^*_{e^-}(\gamma \cdot \varepsilon_{(}k'))\psi_{e^+}\mathrm{e}^{-ik'x}\frac{d^4x}{\sqrt{2\omega'}}$$

***Interaction of*** $e^+e^-$ ***with an external field is considered non-perturbatively***

***Interaction of*** $e^+e^-$ ***with outgoing photon is consider in first order of perturbation theory***




$$|M_{fi}^{(n)\,\text{B-W}}(q,q',k,k')|^2 = -|M_{fi}^{(n)\,\text{Compt.}}(-q,q',k,-k')|^2$$

***Probability of*** $e^+e^-$ ***creation is a sum of partial contributions***

$$dW = \sum_n dW^{(n)}$$

# *Probability is of partial contribution*

$$dW^{(n)} = \frac{e^2 M_e^2}{32\pi\omega'} \left\{ 2J_n^2(z) + \xi^2(2u-1)\left(J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)\right)\right\} \times \frac{du}{u^{3/2}(1-u)^{1/2}}$$

***invariant variable*** $\quad u = \frac{(k\cdot k')^2}{4(k\cdot p)(k\cdot p')} = \frac{1}{1-v^2\cos^2\theta}$

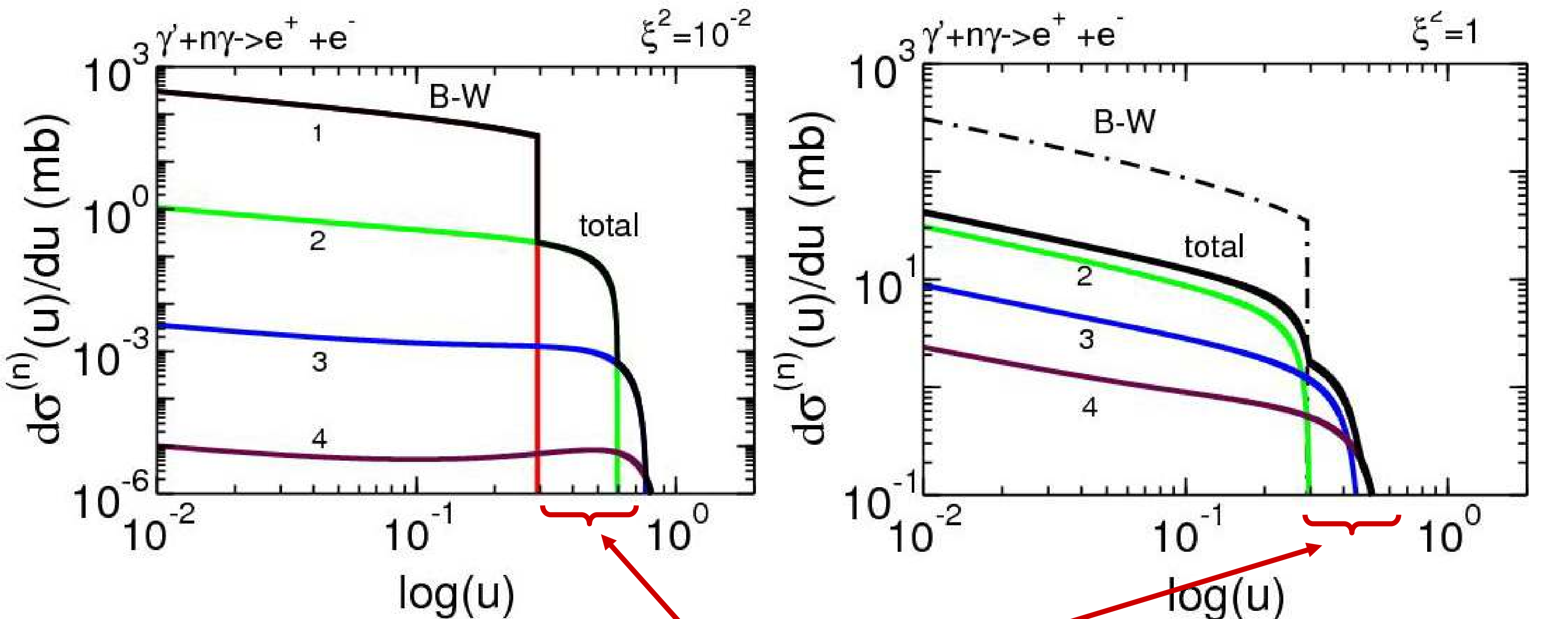
***where*** $v$ ***is velocity of electron (positron)*** $v = p/E_p$

$1 < u < u_{\max} = \frac{n}{n_{\min}}$ ***with*** $n_{\min} = \frac{\omega\omega'}{M_e^2} \longleftrightarrow (k' + n_{\min}k)^2 = 4M_e^2$

***argument of Bessel functions:*** $\quad z^2 = \frac{4n^2\xi^2}{1+\xi^2}\frac{u}{u_{\max}}\left(1 - \frac{u}{u_{\max}}\right)$

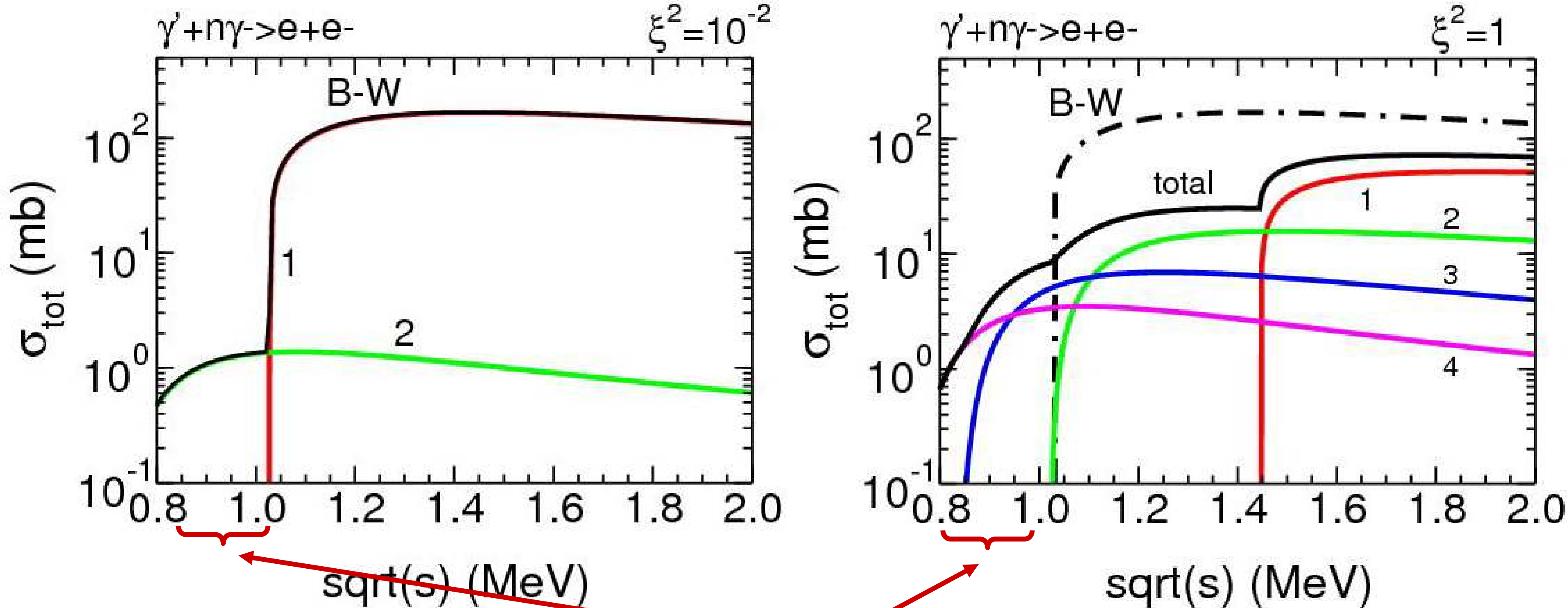
## *Differential cross section*

$$dW = 2\xi^2 \frac{\omega_\gamma M_e^2}{e^2} d\sigma$$




*Multi-photon subthreshold contributions*

# *Total cross section*



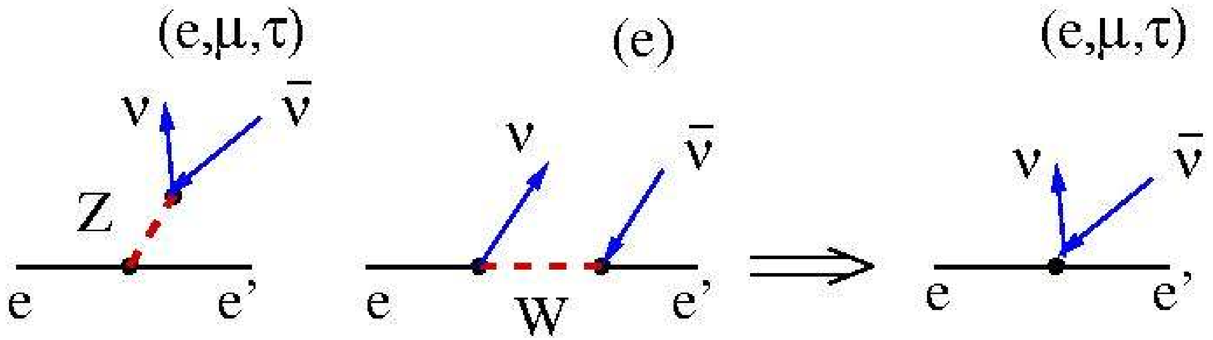

***Multi-photon subthreshold contributions***

$$s_{\mathrm{thr}} = \frac{4M_e^2\,(1+\xi^2)}{n}$$

# Neutrino Emission
# in a Strong Electromagnetic Field

**A.T., B.Kampfer, H. Takabe, and A. Hosaka, *Phys.Rev.D,83,*** *053008, (2011)*

## *Elementary* $e \to e' + \nu\bar{\nu}$ *vertices*




*in local limit with all* $p_i << M_B$

***effective interactions***

$$\mathcal{L}_{\text{eff}}^{(i)} = \frac{G_F}{\sqrt{2}} \left[ \bar{u}^e \gamma^\alpha (C_V^{(i)} - C_A^{(i)} \gamma_5) u^e \right] \cdot L_\alpha^{\nu(i)},$$

$$L_\alpha^{\nu(i)} = \left[ \bar{u}_{\nu_i} \gamma_\alpha (1 - \gamma_5) v_{\nu_i} \right] \ , \qquad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

***where***

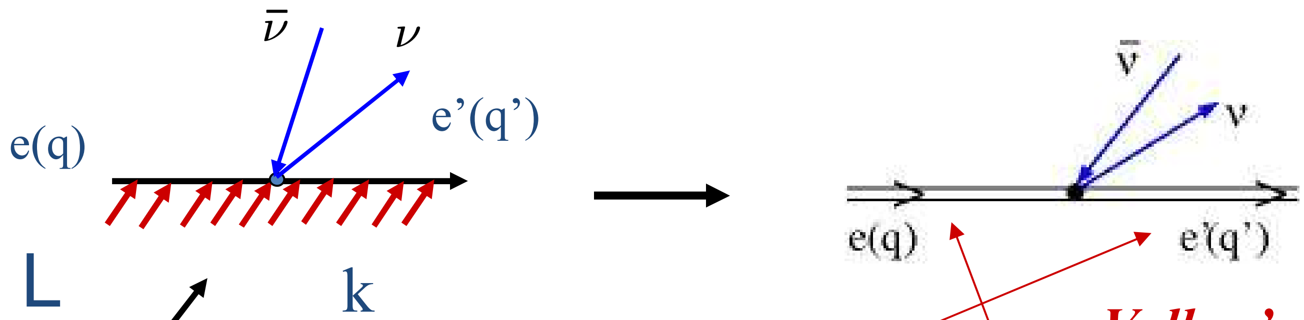
$$C_V^{(e)} = \frac{1}{2} + 2\sin^2\theta_W \ , \qquad C_V^{(\mu,\tau)} = -\frac{1}{2} + 2\sin^2\theta_W \ ,$$

$$C_A^{(e)} = \frac{1}{2} \ , \qquad C_A^{(\mu,\tau)} = -\frac{1}{2} \ ,$$

***and***

$$\sin^2\theta_W \simeq 0.23$$

# *Neutrino emission in external strong EM field*




***external field***

***Volkov’s states for electrons***

$$M^{(i)} = \frac{G_F}{\sqrt{2}} L_\alpha^{\nu(i)} \otimes \int [\bar{\psi}_{e^-}(x)\gamma_\alpha(C_V^{(i)} - C_A^{(i)}\gamma_5)\psi_{e^+}(x)]\mathrm{e}^{i(k_\nu + k_{\bar{\nu}})x}\frac{d^4x}{\sqrt{2E_\nu 2E_{\bar{\nu}}}}$$

$$(i = e, \mu\tau)$$

$$\frac{\bar{u}_{p'}}{\sqrt{2E'_p}}\mathrm{e}^{-ip'\cdot x}\left[1 + \frac{e(\gamma\cdot A)(\gamma\cdot k)}{2(k\cdot p')}\right]\mathrm{e}^{iS'(k\cdot x, p'_e)}$$

$$\left[1 + \frac{e(\gamma\cdot k)(\gamma\cdot A)}{2(k\cdot p)}\right]\mathrm{e}^{iS'(k\cdot x, p_e)}\frac{u_p}{\sqrt{2E_p}}\mathrm{e}^{-ip\cdot x}$$

***Difference/complication compare to the photon emission comes from***
***(1) four fermion vertices and***
***(2) vector + axial vector couplings*** $(\gamma_\alpha - \gamma_5\gamma_\alpha)\times(\gamma^\alpha - \gamma_5\gamma^\alpha)$

## *Structure of matrix element*

$$\int M(kx)\mathrm{e}^{-i(q-q'-Q)x}d^4x \qquad \textit{with} \qquad Q = k_\nu + k_{\bar{\nu}}$$

***Fourier series***

$$M(kx) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{-in\,kx}\, M_n(k, Q, q, q')$$

***The amplitude is a sum of infinite numbers of "partial harmonics"***

$$M_{fi} \;=\; \sum_{n=-\infty}^{\infty} M_n \int \mathrm{e}^{-i(q+nk-q'-Q)}d^4x$$

$$= \sum_n M_n(2\pi)^4\delta^4(q + nk - q' - Q)$$

***Each "harmonic" describes absorption (emitting) of* n *photons of external field* A *with wave vector* k *and emitting of outgoing neutrino pair with four-momentum* Q *with corresponding conservation low***

# *Emission probability*

$$dW = \sum_{n\geq 1}^{\infty} dW^{(n)}$$

*invariant mass of $\nu\bar{\nu}$ pair*

$$dW^{(n)} = R^{(n)} \frac{du\, dM_Q^2}{(1+u)^2}$$

***with*** $u = \frac{k\cdot Q}{k\cdot p'}$, ***where*** $Q = p_\nu + p_{\bar{\nu}}$

$M_Q^2 = Q^2$

$$R^{(n)} = F_V^{(n)} C_V^2 + F_A^{(n)} C_A^2 + 2\lambda F_I^{(n)} C_V C_A$$

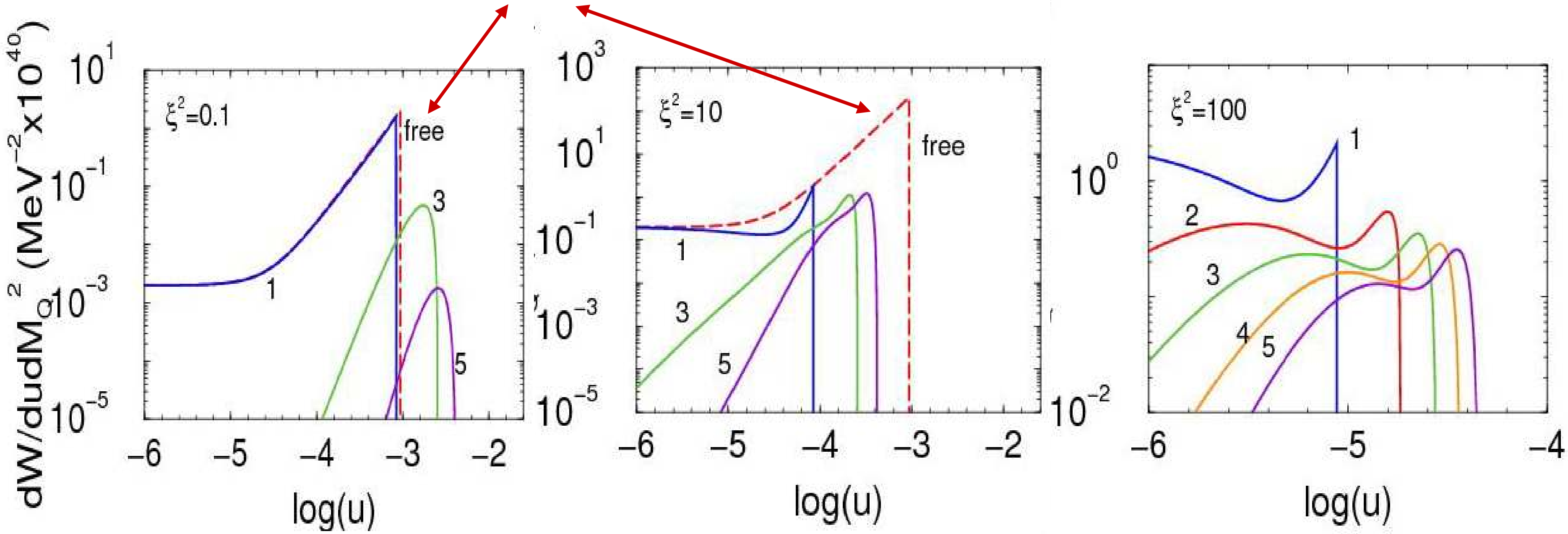
$$F_X^{(n)} = F_X^{(n)}(J_n(z),\, J_{n\pm 1}(z), \xi, u, M_Q^2)$$

$$z = \frac{2n\xi}{\sqrt{1+\xi^2}} \sqrt{\frac{u}{u_n}\left(1 - \frac{u}{u_n}\right) - \frac{1+u}{u_n} \frac{M_Q^2}{(1+\xi^2)M_e^2}}\,,$$

$$u_n = \frac{2n(k\cdot p)}{M_e^2(1+\xi^2)} \longrightarrow M_*^2$$

# *Differential emission probability*

$\gamma e \longrightarrow e \nu_e \bar{\nu}_e$




- ***Increase of*** $\xi^2$ ***at fixed* n *leads to decrease of kinematical limit (see case of* n=1*)***
- ***"Cumulative effect" – reactions with* n>1 *at fixed*** $\xi^2$ ***increases the phase space and the kinematical limit***
- ***In general, higher harmonics are not suppressed at large*** $\xi^2$
- ***Result is essentially non-perturbative even for small*** $\xi^2$

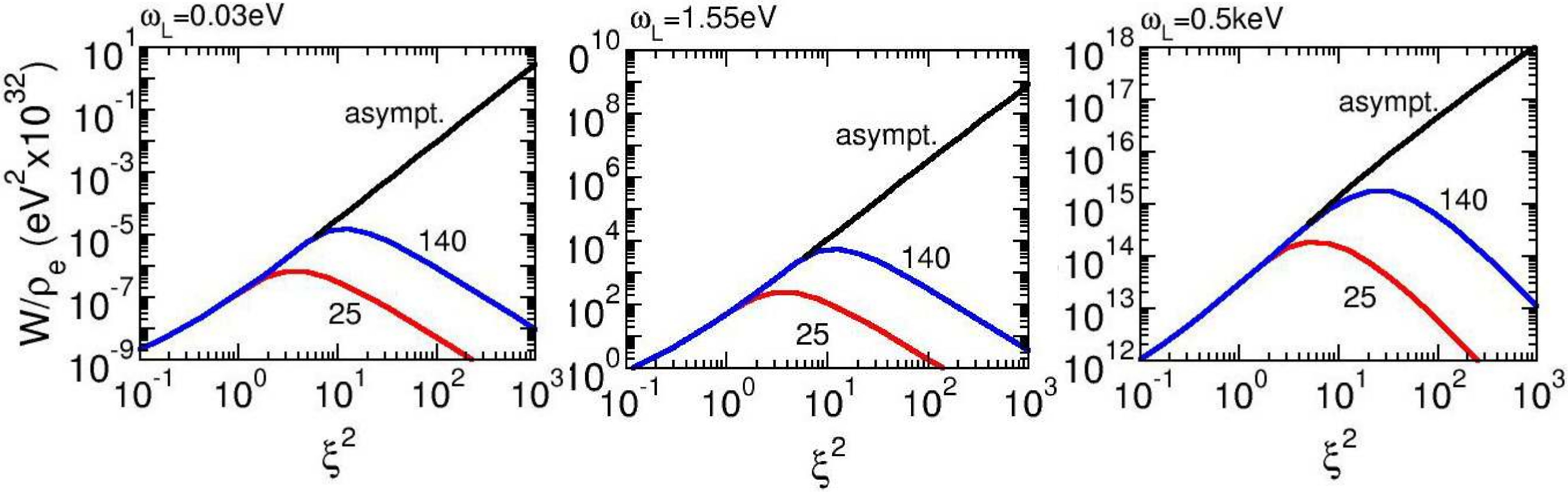
## *Region of large* $n,\ \xi^2$

***method of asymptotic summation***

$$\sum_n \to \int\limits_{n_{\min}(\xi)}^{\infty} dn$$

$$n_{\min} \sim \xi^3$$

$$J_n(z) \to \Phi(y); \qquad y = y(\xi, kp, n, M_Q^2)$$



# *Asymmetry of production of* $\nu_e$ *and* $\nu_\mu + \nu_\tau$

$$\mathcal{A}_{(e,\mu\tau)} = \frac{W_{(e)} - W_{(\mu+\tau)}}{W_{(e)} + W_{(\mu+\tau)}}.$$

$$\mathcal{A}_{(e,\mu\tau)} = \frac{C_V^{(-)} + C_A^{(-)} R_{AV}}{C_V^{(+)} + C_A^{(+)} R_{AV}},$$

***where***

$$C_{A,V}^{(+)} = C^{e2}_{V,A} + 2C^{\mu 2}_{V,A}$$
$$C_{A,V}^{(-)} = C^{e2}_{V,A} - 2C^{\mu 2}_{V,A}$$

***and*** $R_{AV} = \dfrac{h_A}{h_V}$

***For "elementary" or "free" process*** $\gamma + e \to e' + \nu\bar{\nu}$

$$h_V(\kappa) = -\frac{55}{144}\kappa^2 + \frac{77}{24}\kappa + \frac{733}{48} + \frac{25}{2\kappa} + \frac{1}{6(1+\kappa)} - \frac{1}{48(1+\kappa)^2}$$
$$+ \left(\frac{\kappa^2}{6} - \frac{\kappa}{2} - \frac{39}{4} - \frac{65}{3\kappa} - \frac{25}{2\kappa^2}\right) \ln(1+\kappa),$$

---

$$h_A(\kappa) = -\frac{55}{144}\kappa^2 + \frac{79}{24}\kappa - \frac{97}{16} - \frac{15}{2\kappa} + \frac{5}{3(1+\kappa)} - \frac{3}{16(1+\kappa)^2}$$
$$+ \left(\frac{\kappa^2}{6} + \frac{3\kappa}{2} - \frac{25}{4} + \frac{25}{3\kappa} + \frac{15}{2\kappa^2}\right) \ln(1+\kappa).$$

***with*** $\kappa = \dfrac{2(kp)}{M_e^2}$

---

at $\kappa \gg 1$, $h_V \simeq h_A$

$R_{AV} \simeq 1 \longrightarrow \mathcal{A}_{(e,\mu\tau)} \simeq +0.4$

at $\kappa \ll 1$:

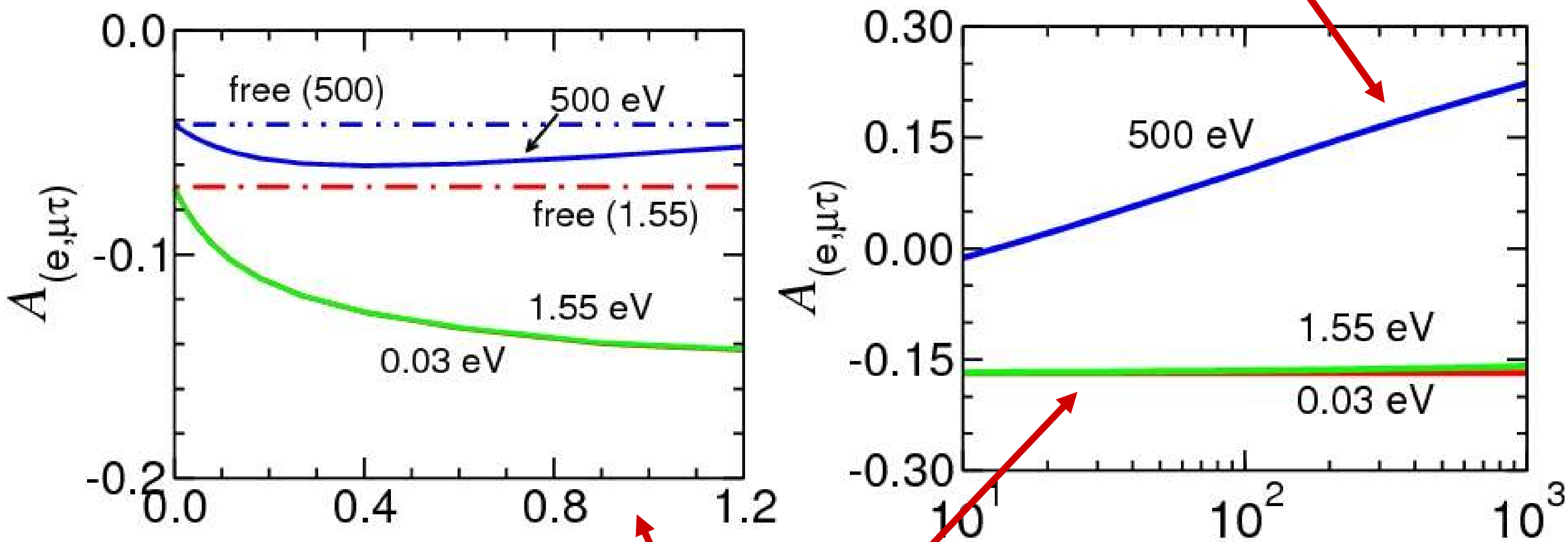
$$h_V(\kappa) \simeq \kappa^5(109\kappa^2 - 72\kappa + 36)/2520,$$
$$h_A(\kappa) \simeq \kappa^5(253\kappa^2 - 148\kappa + 60)/840.$$

$R_{AV} \simeq 5 \longrightarrow \mathcal{A}_{(e,\mu\tau)} \simeq -0.071$

# *Asymmetry of production of* $\nu_e$ *and* $\nu_\mu + \nu_\tau$

$$\mathcal{A}_{(e,\mu\tau)} = \frac{W_{(e)} - W_{(\mu+\tau)}}{W_{(e)} + W_{(\mu+\tau)}}.$$

*excess of* $\nu_e$



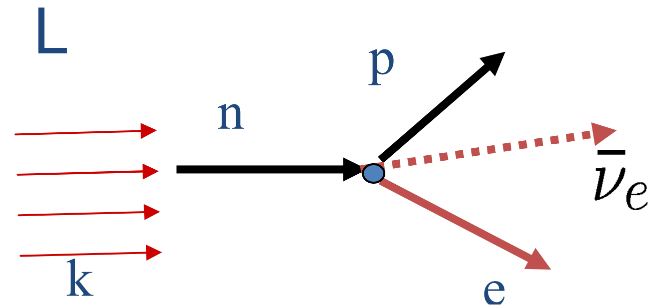

*small* $\xi^2$ $\xi^2$ *large*

*excess of* $\nu_\mu + \nu_\tau$

# Neutron Decay in a Strong Electromagnetic Field

# *Neutron decay in the field of a strong electromagnetic wave*

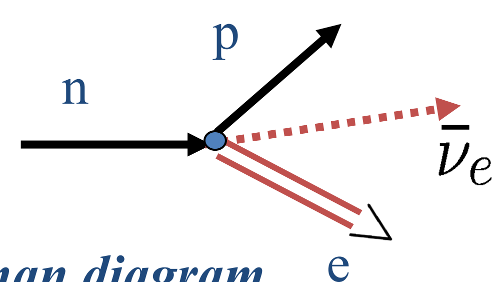
$$n \to p + e + \bar{\nu}_e$$




electron modification
$$\begin{cases} \psi^D \to \psi^V \\ p \to q \\ m_e^2 \to m_*^2 = m_e^2 + e^2a^2 = m_e^2 + m_e^2\xi^2 \end{cases}$$

proton modification
$$\begin{cases} p_\mu \to q_\mu = p_\mu + \dfrac{e^2a^2}{2\omega_\gamma M_p} k_\mu \sim p_\mu \\ M_p \to M_{p*} = \sqrt{M_p^2 + m_e^2\xi^2} \simeq M_p\left(1 + \dfrac{m_e^2}{M_p^2}\xi^2\right) \simeq M_p \\ \psi^D \to \psi^V \simeq \psi^D \end{cases}$$

neutron modification
$$\left\{ \psi^D \to \psi^V \simeq \psi^D \right.$$




***Feynman diagram***

# *Amplitude*

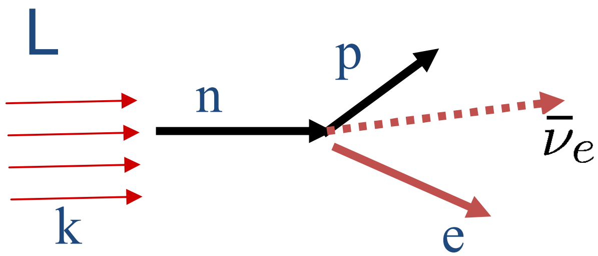
$$M_{fi} = \frac{G_F}{\sqrt{2}} \int [\bar{u}_p \gamma_\mu (1 - g_A \gamma_5) u_n] \otimes [\psi_e^* \gamma^\mu (1 - \gamma_5) v_{\bar{\nu}}] \, \mathrm{e}^{-i(p_n - p_p - p_{\bar{\nu}})x} \frac{d^4 x}{\sqrt{2E_n 2E_p 2E_{\bar{\nu}}}}$$

***main effect comes from electron mass modification***

$$\left.\begin{array}{c} m_{e*}^2 = m_e^2 + \xi^2 m_e^2 \\ m_{e*\mathrm{max}} \simeq M_n - M_p \end{array}\right\} \longrightarrow \quad \xi^2_{\mathrm{max}} = \frac{(M_n - M_p)^2}{m_e^2} - 1 \simeq 5.4$$

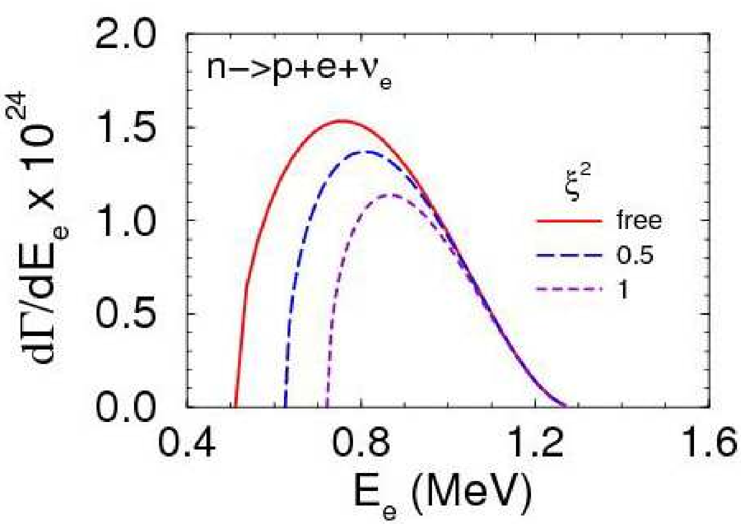
# Weak decay in the field of a strong electromagnetic wave

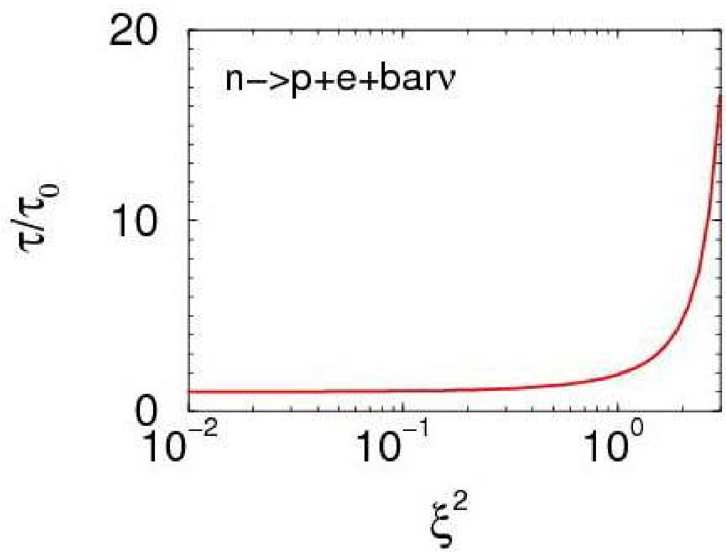
$$n \to p + e + \bar{\nu}_e$$




$$m_*^2 = m_e^2(1+\xi^2)$$

$$\xi_{\max}^2 = \frac{(M_n - M_p)^2}{m_e^2} - 1 \simeq 5.4$$



# Summary (Part III)

- *Strong electromagnetic fields modify basic/fundamental interactions and result in non-trivial nonlinear non-perturbative effects.*
- *Modification of kinematical limits because of (a) electron dressing and (b) coherent interactions with several photons (cumulative effect)*
- *Non-trivial dynamical effects, like* $\nu_e$ *and* $\nu_\mu + \nu_\tau$ *asymmetries, decay widths, … are discovered*
- *Powerful method for calculation of the emission probabilities at arbitrary* $\xi^2(I),\, \omega_\gamma,\, E_e$ *is elaborated*

# THE END

***Thank you very much for attention !***

# Backup

***Antiparticle of elementary particle corresponding to an ordinary particle, but having the opposite electrical charge and magnetic moment. Every elementary particle has a corresponding antiparticle; the antiparticle of an antiparticle is the original particle.***

***In a few cases, such as the photon and the neutral pion, the particle is its own antiparticle, but most antiparticles are distinct from their ordinary counterparts.***

Some peculiarities of antimatter

Preservation: ***magnetic traps***

Effectiveness of antimatter fuel is greatest compare to others sources

Fuel

$10^{10}$ greater than oil

$10^{4}$ greater than fission

$10^{2}$ greater than fusion

Cost

$\$300 \times 10^{9}$ per milligram

# Antiparticles

**1928** - prediction of positron (P. Dirac)

Discovery of antiparticles

**1932 – Positron** **cosmic rays (Anderson)**

**1955 – Antiproton** **pA collision BNL (Serge, Chamberlen)**

**1956 – Antineutron** **pA collision BNL (Cork)**

**1966 – Antideuteron** **PS CERN and BNL**

**1970 – Antihelium** **Pb+Pb collision at CERN**

**1998 – Antihydrogen** **LEAR CERN (Low Energy AP beam)**

**2011 – Antihydrogen (large amount)** **LEAR CERN**

## *estimation of the photon density* $\rho_\gamma$

$$\rho_\gamma = \frac{\langle \mathcal{E} \rangle}{\omega} = \frac{1}{2}\frac{\langle \vec{E}^2 + \vec{H}^2 \rangle}{\omega} = \frac{\langle \vec{E}^2 \rangle}{\omega} \qquad dW = \xi^2 \cdot \frac{\omega_\gamma M_e^2}{e^2} \cdot d\sigma$$

***energy density of EM field***

***circularly polarized photon field***

$$\vec{A}_\gamma = \vec{a}_x \cos(\omega t - kz) + \vec{a}_y \sin(\omega t - kz)$$

$$\vec{E} = -\frac{\partial \vec{A}_\gamma}{\partial t} = \omega(\vec{a}_x \sin(\omega t - kz) - \vec{a}_y \cos(\omega t - kz))$$

$$\langle \vec{E}^2 \rangle = \omega^2 a^2; \qquad a^2 = a_x^2 = a_y^2$$

$$\frac{1}{V_\gamma} = \frac{\mathcal{E}}{\omega} = \omega \cdot a^2 = \frac{\omega M_e^2 \xi^2}{e^2} \quad \textit{with} \quad \xi^2 = \frac{e^2 a^2}{M_e^2}$$

***reduced intensity of EM field***

$$\boxed{dW = \xi^2 \cdot \frac{\omega_\gamma M_e^2}{e^2} \cdot d\sigma}$$