QUARK-PAULI EFFECTS IN LIGHT Λ HYPERNUCLEI

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There are various model descriptions for the Λ-nucleon(ΛN) interaction, which reproduce both nucleon-nucleon(NN) scattering data and few existing ΛN data. However, as the scarce of the experimental data on the ΛN interaction, the detailed character of each interactions are inconsistent by each of theoretical models. On the other hand, the binding energies of Λ-hypernuclei give one of the important pieces of information on the ΛN interaction. Our first aim is to make clear the differences between ^{1}S and ^{3}S forces in the ΛN interaction. Therefore, we investigate that how ΛN interaction is concerned with the binding energies of s-shell Λ hypernuclei, using through effective forces.

Furthermore, we are interested in the effect of the quark substructure of the baryon in nuclear phenomena. In hypernuclear calculation, though a Λ particle is distinguishable one from nucleons, they contain same quarks, i.e. u and d quarks. Then, all the u and d quarks included in $^5_\Lambda$He cannot occupy the lowest energy level at the same time, respectively. One of the purposes on the present study is to show that the Λ separation energy B_Λ of $^5_\Lambda$He is significantly reduced by taking into account the quark substructure of the baryons, thereby giving a clue to resolve at least partly the long standing problem of anomalously small binding energy of $^5_\Lambda$He.

As mentioned above, we are interested in the differences between the ^{1}S$_0$ and the ^{3}S$_1$ phase shifts, we use simple ΛN central potentials of the following type

$$V_{\Lambda N} = \left\{ V_R + \frac{1}{2}(1+P^\sigma)V_t + \frac{1}{2}(1-P^\sigma)V_s \right\} \left\{ \frac{1}{2}u + \frac{1}{2}(2-u)P^r \right\},$$

$$V_R = V_{0R}\mathrm{e}^{-\kappa_R r^2}, \quad V_t = -V_{0t}\mathrm{e}^{-\kappa_t r^2}, \quad V_s = -V_{0s}\mathrm{e}^{-\kappa_s r^2}. \tag{1}$$

The value of u was set equal to u=1.5, which is consistent with the F/B ratio of the Λp scattering data. The remained parameters are shown in table 1, which reproduce the A=3, 4 binding energies.

We assume that the spatial part of the s-shell hypernuclei interacting with the central potential can be well described by the wave function with total angular momentum L=0. We take the following correlated Gaussians

$$\varphi(\mathbf{x}) = \exp\left\{ -\frac{1}{2}\sum_{i,j=1}^{A-1} \mathbf{A}_{ij}\mathbf{x}_i \cdot \mathbf{x}_j \right\},$$

where $\mathbf{x}_i (i = 1, ..., A-1)$ stands for a set of independent relative coordinates. The matrix $\mathbf{A}_{ij}$ must be an $(A-1) \times (A-1)$ symmetric, positive-definite. It is noted that the non-diagonal elements of $\mathbf{A}_{ij}$ would be allowed some non-zero values. The trial function is given by

$$\Psi = \sum_{k=1}^{K} c_k \Phi_k,$$

with

$$\Phi_k = \mathcal{A}\left\{\varphi_k(\mathbf{x})\chi_k(JM)\eta_k(TM_T)\right\}, \tag{2}$$

where the operator $\mathcal{A}$ sntisymmetrize the nucleon coordinates, and $\chi_k(JM)$ and $\eta_k(TM_T)$ are spin and isospin functions, respectively. The basis elements are set up one by one by using the stochastic variational method[6].

In the baryonic picture where a baryon is treated as a structureless particle and the Λ can be distinguishable from the nucleon. However, when we take the quark substructure of the baryon into consideration, a special constraint has to be put on the wave function of ${}^{5}_{\Lambda}$He as seen below. Let us consider that a baryon is composite particle of three quarks and each quark moves in the 0s orbit of the harmonic-oscillator well with the size parameter β. When the five baryons constituting ${}^{5}_{\Lambda}$He overlap each other completely, the spatial part of the ${}^{5}_{\Lambda}$He is described as below:

$$\exp\left\{-\frac{1}{2\beta^2}\sum_{i=1}^{15}\boldsymbol{\xi}_i^2\right\} = \phi_{B_1}(123)\ldots\phi_{B_5}(13\cdot 14\cdot 15)\exp\left\{-\frac{3}{2\beta^2}\sum_{i=1}^{5}\mathbf{r}_i^2\right\}.$$

The wavefunction consisting of 15 quarks is reduced to the intrinsic parts $\phi_{B_1},...,\phi_{B_5}$ and center-of-mass parts of each baryons. The value of β can be estimated to be about 0.86 fm by requirement that reproduce the charge radius of the proton[7].

In such the situation, four nucleons of the ${}^{5}_{\Lambda}$He, when they are on top of one another, have already 6u and 6d quarks in the 0s orbit, so neither u nor d quark of Λ can take the same 0s orbit. This leads us to the conclusion that the five baryons of ${}^{5}_{\Lambda}$He are not allowed to take a configuration of $\exp\{-3/(2\beta^2)\sum_{i=1}^{5}\mathbf{r}_i^2\}$. To be more specific, the forbidden state is given by

$$\Phi_0 = \mathcal{A}\left\{\varphi_0\chi_0\eta_0\right\}$$

where

$$\varphi_0 = \exp\left\{-\frac{3}{2\beta^2}\sum_{i=1}^{5}(\mathbf{r}_i-\mathbf{R})^2\right\},$$

and $\mathbf{R}$ is the center-of-mass coordinate of the five baryons. For the trial function of ${}^{5}_{\Lambda}$He, the correlated Gaussian in (2) should be replaced by one which has no forbidden component:

$$\widetilde{\Phi}_k = \Phi_k - \Phi_0\frac{\langle\Phi_0|\Phi_k\rangle}{\langle\Phi_0|\Phi_0\rangle}.$$

As the NN interaction we use the Minnesota potential[3], which reproduce very well the ^{1,3}S phase shifts as well as the binding energies and sizes of the s-shell nuclei: The binding energies of ^{2}H, ^{3}H, ^{3}He, and ^{4}He are calculated to be 2.20, 8.38, 7.71, and 29.95 MeV, respectively. Table 2 compares the calculated B_Λ and E_x values with experiment. For A=3 and 4 systems, we see that the both B_Λ and E_x are rather well reproduced. In addition, the B_Λ value for ${}^{5}_{\Lambda}$He is found to be at most 0.5 MeV overbound when β=0.86 fm is employed. Figure **??** displays the B_Λ value of ${}^{5}_{\Lambda}$He as a function of β. It is seen that the B_Λ value does not change for $\beta \leq 0.6$ fm, decreases rather drastically beyond β=0.7 fm.

The maximum of S-wave phase shifts given by the potential of table 1 reach 31.5 degree for ^{1}S$_0$ and 19 degree for ^{3}S$_1$, respectively. We have calculated the S-wave effective-range parameters: The scattering lengths are a_s= -2.524 and a_t= -1.197 fm, and the effective ranges are r_s= 3.079 and r_t= 4.255 fm.

References

[1] Y. Fujiwara, C. Nakamoto, and Y. Suzuki, Phys. Rev. Lett. **76**, 2242 (1996); and Phys. Rev. **C54**, 2180 (1996).

[2] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. **D20**, 1633 (1979).

[3] D. R. Thompson, M. Lemere, and Y. C. Tang, Nucl. Phys. **A286**, 53 (1977).

[4] R. H. Dalitz, R. C. Herndon, and Y. C. Tang, Nucl. Phys. **B47**, 109 (1972).

[5] E. V. Hungerford and L. C. Biedenharn, Phys. Lett. B**142**, 232 (1984).

[6] K. Varga and Y. Suzuki, Phys. Rev. C**52**, 2885 (1995); K. Varga, Y. Ohbayasi, and Y. Suzuki, Phys. Lett. B**396**, 1 (1997).

[7] G. G. Simon, Ch. Schmitt, F. Borkowski, and V. H. Walther, Nucl. Phys. A**381**, 1980. M. McCord *et al.*, Nuclear Instruments and Methods in Physics Research B**56/57**, 496 (1991).

[8] A. Gal, Adv. Nucl. Phys. vol. **8**, ed. M. Baranger and E. Vogt, (Plenum Press, New York, 1975), p.1

[9] B. F. Gibson, Nucl. Phys. A**479**, 115c (1988); B. F. Gibson and D. R. Lehman, Phys. Rev. C**37**, 679 (1988).

Table 1: The ΛN potential parameters. The strengths are given in units of MeV and the ranges are in units of fm^{-2}.

V_{0R}	V_{0t}	V_{0s}	κ_R	κ_t	κ_s
200.0	109.8	121.3	1.638	0.7864	0.7513

Table 2: The Λ separation energies and excitation energies of s-shell hypernuclei in units of MeV. The size parameter β of the baryon is set equal to 0.86 fm. The value in parentheses is the energy obtained without the quark Pauli requirement.

	$B_\Lambda(^3_\Lambda\mathrm{H})$	$B_\Lambda(^4_\Lambda\mathrm{H})$	$B_\Lambda(^4_\Lambda\mathrm{He})$	$B_\Lambda(^5_\Lambda\mathrm{He})$
Exp.	0.13±0.05	2.04±0.04	2.39±0.03	3.12±0.02
Cal.	0.18	2.23	2.22	3.57(4.98)

	$E_x(^4_\Lambda\mathrm{H}^*)$	$E_x(^4_\Lambda\mathrm{He}^*)$
Exp.	1.04±0.04	1.15±0.04
Cal.	1.14	1.13

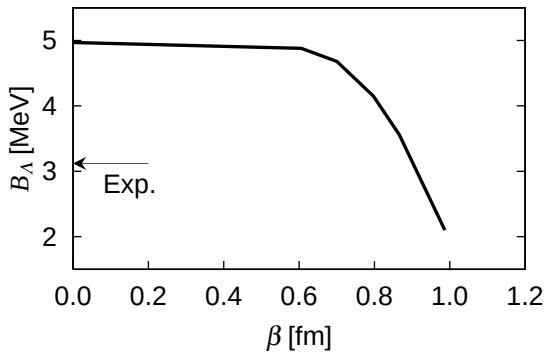

Figure 1: The Λ separation energy B_Λ of $^5_\Lambda$He as a function of the size parameter β of three-quark baryons.