RELATIVISTIC EQUATION OF STATE FOR SUPERNOVAE AND NEUTRON STARS

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Supernova explosion is a spectacular stellar phenomenon, which may lead to the formation of a neutron star or black hole. In order to study the supernova explosion and neutron star properties, we need the equation of state (EOS) of nuclear matter in the wide density and temperature range with various proton fractions, which appear during the collapse and the explosion. There have been several EOS worked out so far for supernova study [1, 2, 3]. Most of them are constructed based on the non-relativistic Skyrme Hartree-Fock (SHF) framework. Although some of these EOS are carefully constructed, there are many places where the EOS are discontinuous or not available, and the supernova simulations stop there due to the lack of the completeness of the EOS table.

Recently, there is a tremendous development in the description of nuclei and nuclear matter in terms of the relativistic many body theory [4, 5, 6, 7, 8]. The relativistic mean field (RMF) theory was demonstrated to be successful in describing the properties of both stable and unstable nuclei. Hence, it is very important to study the properties of the EOS in the RMF theory which is best supported by both the theoretical and experimental information currently available.

We construct the EOS of nuclear matter in the wide range of the baryon mass density ($\rho_B = 10^5 \sim 10^{15.5} g/cm^3$), temperature ($T = 0 \sim 100 MeV$), and proton fraction ($Y_p = 0.01 \sim 0.5$). We have to work out consistent calculations for high density nuclear matter, inhomogeneous nuclear matter and finite nuclei under the wide range of density, proton fraction and temperature. For this purpose, we study the properties of dense matter with both homogeneous and inhomogeneous distribution in the RMF framework adopting the parameter set TM1 for the RMF lagrangian

$$\begin{aligned}
\mathcal{L}_{RMF} &= \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu} - e\gamma_\mu\frac{1-\tau_3}{2}A^\mu\right]\psi \\
&+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
&-\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\
&-\frac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.
\end{aligned} \tag{1}$$

We use the Thomas-Fermi approximation to describe inhomogeneous nuclear matter [9]. We assume that the inhomogeneous matter is composed of a lattice of spherical nuclei. At finite

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temperatures, a proton gas and neutron gas, as well as α-particles, may be present outside the nuclei. We minimize the free energy of the system with respect to some parameters in this approximation. The free energy density contributed by baryons is given as

$$f = (E - T S) / a^3, \tag{2}$$

where a is the lattice constant, the energy per cell E can be written as

$$E = E_{bulk} + E_s + E_C. \tag{3}$$

The bulk energy E_{bulk} and the entropy per cell S are calculated by

$$E_{bulk} = \int_{cell} \varepsilon_{RMF} (n_n (r) , n_p (r) , n_\alpha (r)) d^3 r, \tag{4}$$

$$S = \int_{cell} s_{RMF} (n_n (r) , n_p (r) , n_\alpha (r)) d^3 r, \tag{5}$$

where ε_{RMF} and s_{RMF} are the energy density and the entropy density in the RMF theory. We adopt the same expression for the surface energy term E_s and the Coulomb energy term E_C as in reference [9].

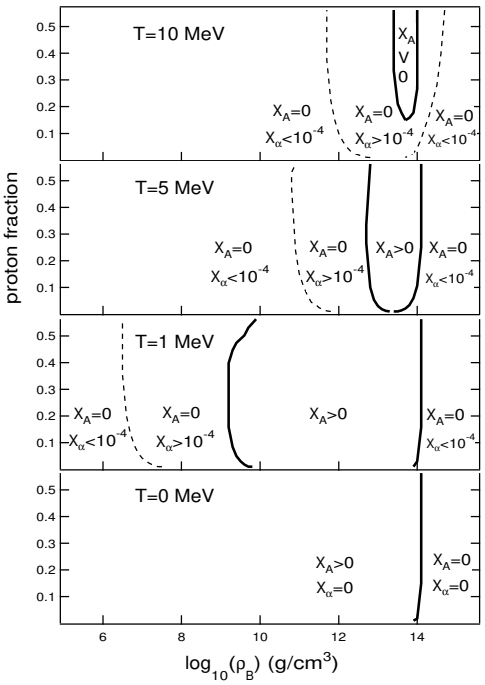

Figure 1: The configurations of nuclear matter at $T = 0, 1, 5$ and $10 MeV$. X_α and X_A are α-particle fraction and heavy nuclei fraction, respectively.

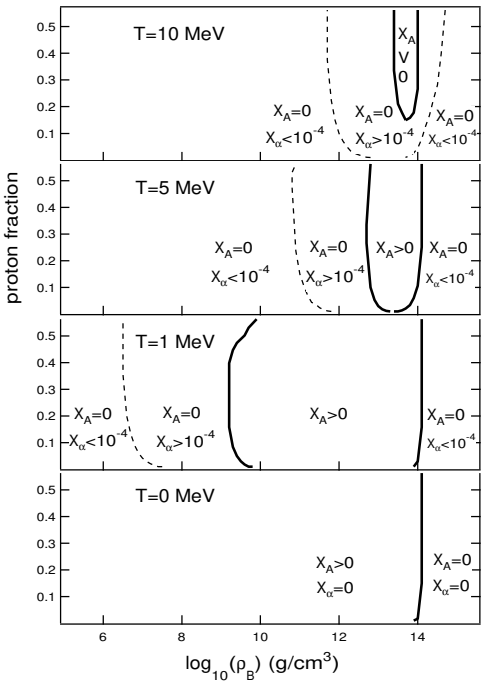

Figure 2: The pressure as a function of the baryon mass density ρ_B with the proton fraction $Y_p = 0.01, 0.1, 0.2, 0.3, 0.4,$ and 0.5 at $T = 0, 1, 5,$ and $10 MeV$.

We perform the minimization of the free energy for each density, temperature and proton fraction in the wide range necessary for the astrophysical use. We treat the uniform matter and non-uniform matter consistently using the same RMF theory. Hence, all the resulting thermodynamical quantities are consistent and smooth in the whole range in this set of the EOS. In Fig.1 we show the configurations of nuclear matter at $T = 0, 1, 5$ and

$10MeV$. For extremely low density and finite temperature range, the thermodynamically favorable state is uniform nucleon gas with less α-particles. The α-particle fraction X_α increases as the density increases. Heavy nuclei exist in the region denoted by $X_A > 0$. When the density is higher than $\rho_B \sim 10^{14} g/cm^3$, the heavy nuclei are dissociated. We show in Fig.2 the pressure as a function of ρ_B with various Y_p at $T = 0, 1, 5$ and $10MeV$.

This relativistic EOS of nuclear matter is designed for the use of supernova simulations and the neutron star calculations in the wide density and temperature range with various proton fractions. We have adopted the RMF theory with the non-linear σ and ω terms, which was demonstrated to be successful in describing the properties of both stable and unstable nuclei as well as reproducing the similar self-energies in the RBHF theory [6, 8]. The relativity plays an essential role in describing the nuclear saturation and the nuclear structure, it also brings some distinctive properties in the EOS comparing with the case in the non-relativistic framework [10, 11]. Therefore, it is very interesting and important to study the astrophysical phenomena such as supernova explosion, neutron star cooling and neutron star merger using the relativistic EOS through extensive comparison with the previous studies with the non-relativistic EOS.

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