Color Confinement in the Multi-Instanton System

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The QCD vacuum is composed of the gluon field interacting in a highly complicated way and there appear various non-perturbative phenomena, color confinement, dynamical chiral symmetry breaking and the $U_A(1)$ anomaly. It is hard to understand these properties from the perturbative point of view due to the strong coupling nature. It is noted that there are two nontrivial topological objects, *instanton* and *monopole* in the QCD vacuum. It is believed that these objects are independently important for understanding the non-perturbative properties. An instanton appears as a classical and non-trivial solution in the Yang-Mills theory corresponding to the homotopy group, $\pi_3(SU(N_c)) = Z_\infty$. Instantons are important for the phenomena related to the $U_A(1)$ anomaly and chiral symmetry breaking [1, 2]. On the other hand, QCD is reduced to an abelian gauge theory with QCD-monopoles after performing the abelian gauge fixing [3]. QCD-monopoles appear as the topological defects corresponding to the nontrivial homotopy group $\pi_2(SU(N_c)/U(1)^{N_c-1}) = Z_\infty^{N_c-1}$. Many analytical and numerical studies indicate that monopole condensation plays an essential role on color confinement and chiral symmetry breaking [4]-[6].

Until now, there has been no evidence that the instanton has anything to do with color confinement. However, recent studies [6]-[15] suggest that instantons directly relate to monopoles, whose condensation provides an interpretation of the confinement mechanism. It has been found that there exists a strong correlation between instantons and monopoles in the abelian projected theory of QCD, both in the analytical and lattice frameworks [6]-[13]. In the maximally abelian gauge, a monopole loop seems to be localized around the center of an instanton [11, 12]. We studied the multi-instanton system in terms of monopole condensation numerically [14, 15]. When the instanton density is high and instantons overlap each other, there appears an very long and highly complicated monopole loop. This monopole loop covers the entire physical volume $\mathbf{R}^4$ in the multi-instanton system [14, 15]. On the other hand, by the lattice QCD simulation, such long and complicated monopole loop is observed as a signal of monopole condensation in the confinement phase, which includes many topological excitations like instanton and anti-instanton [13]. These results indicate that instantons would play a relevant role on the confinement mechanism via the promotion of monopole condensation.

We study the multi-instanton system in order to clarify the relation of instantons with color confinement. The single-instanton configuration with the size ρ and the center z in the singular gauge is expressed as

$$A_\mu^I(x;z,\rho,O) = \frac{i\tau^a \rho^2 O^{ab} \bar{\eta}^b_{\mu\nu}(x-z)_\nu}{(x-z)^2[(x-z)^2+\rho^2]}, \tag{1}$$

where O^{ab} denotes the SU(2) color orientation matrix and $\bar{\eta}^a_{\mu\nu}$ the 't Hooft symbol. The anti-instantons configuration $A_\mu^{\bar{I}}$ is obtained by replacing $\bar{\eta}^a_{\mu\nu}$ with $\eta^a_{\mu\nu}$.

The multi-instanton ensemble is characterized by the size distribution, the randomness of the color orientation and the instanton density [14, 15]. The size distribution has to follow the one loop result, $f(\rho) \sim \rho^{b-5}$ with $b = 11N_c/3$, in the ultraviolet region. For the large size, the ordinary instanton liquid models suggest that the size distribution behaves as $f(\rho) \sim 1/\rho^5$ due to the infrared repulsive force [2]. Therefore, we assume the size distribution as $f(\rho) = 1 \ / \ [\ (\frac{\rho}{\rho_1})^5 + (\frac{\rho_2}{\rho})^{b-5} \ ]$. Here, the size parameters ρ_1 and ρ_2 should satisfy the normalization condition, $\int_0^\infty d\rho f(\rho) = 1$. The maximum of the distribution is fixed to the standard probable instanton size $\rho_0 = 0.4$ fm. The multi-instanton configurations are approximated as the sum of instanton and anti-instanton solutions,

$$A_\mu(x) = \sum\nolimits_k A^I_\mu(x; z_k, \rho_k, O_k) + \sum\nolimits_{\bar{k}} A^{\bar{I}}_\mu(x; z_{\bar{k}}, \rho_{\bar{k}}, O_{\bar{k}}). \tag{2}$$

We generate the ensemble of pseudo-particles with random color orientations O_k and random centers z_k. The instanton sizes ρ_k are randomly taken according to the size distribution $f(\rho)$. In this model construction, the interaction among these pseudo-particles would be included effectively in spite of simple sum ansatz.

Then, we introduce a lattice on such the multi-instanton configuration and define the link variables on each lattice site as $U_\mu(s) = \exp(iaA_\mu(s))$. We consider the the SU(2) Wilson loop, $W[C] \equiv \mathrm{trP} \exp[\ i \oint_C A_\mu dx_\mu \ ] = \mathrm{tr} \prod_C U_\mu(s)$, which is related with the static quark potential, $V(R) = -\ln\langle W(R,T)\rangle/T$. In order to smooth out the ultraviolet noise, we use the smeared link variable,

$$\tilde{U}_\mu \equiv \frac{U_\mu + \sum_{\nu \neq \pm\mu} U_\nu(s) U_\mu(s+\hat{\nu}) U^\dagger_\nu(s+\hat{\mu})}{\| \ U_\mu + \sum_{\nu \neq \pm\mu} U_\nu(s) U_\mu(s+\hat{\nu}) U^\dagger_\nu(s+\hat{\mu}) \ \|}, \tag{3}$$

instead of U_μ in the actual calculation, since such replacement never changes the string tension but only to reduce the errors.

We take the standard instanton density $(N/V)^{\frac{1}{4}} = 1 \ \mathrm{fm}^{-1} = 200$ MeV [1, 2] with the equal numbers of instantons and anti-instantons; $N_I = N_{\bar{I}} = N/2$. We adopt the lattice with the total volume as $V = (3.0 \ \mathrm{fm})^4$, and take the different lattice-spacing $a = 0.150$ fm and $a = 0.125$ fm on the 20^4 and 24^4 lattices, respectively. In the long-range region, the Wilson loop obeys the area law, which indicates existence of the linear confining potential between quark and anti-quark [16]. As shown in Fig.1, the static quark potential $V(R)$ increases in the infrared region, and is approximately proportional to the inter-quark distance R. By comparing the two different lattice-spacing cases, above properties are expected to hold in continuum limit. These results indicate that there exists the confining force between the quark-antiquark pair in the multi-instanton system.

We calculate the static potential for various instanton densities by using the 20^4 lattice with $a = 0.150$ fm. As the instanton density becomes smaller, the slope of the potential tends to decrease monotonously. We shall measure the Creutz ratio $\chi(R,T)$ in order to extract the string tension. If the logarithm of the Wilson loop can be approximated as $-\ln\langle W(R,T)\rangle = \sigma RT + m(R+T) + \mathrm{const}$, the Creutz ratio relates with the lattice string tension as

$$\chi(R,T) \equiv \frac{\langle W(R,T)\rangle \langle W(R-1,T-1)\rangle}{\langle W(R,T-1)\rangle \langle W(R-1,T)\rangle} = \exp(-\sigma_{lat}). \tag{4}$$

The dimensionless lattice string tension σ_{lat} provides the physical string tension σ_{phys} with the lattice-spacing a as $\sigma_{phys} = \sigma_{lat}/a^2$. It is remarkable that the string tension is obtained as $\sigma \simeq 1$ GeV/fm at usual instanton density about 200 MeV, which agrees with the string tension of

the QCD vacuum. As shown in Fig.2, the string tension decreases as the the instanton density becomes smaller. The finite temperature lattice QCD indicates that instantons and anti-instanton decrease drastically near the critical temperature. Therefore, the deconfinement phase transition would be caused by the disappearance of instantons. Our numerical study suggests that instantons play an substantial role on the confinement properties directly.

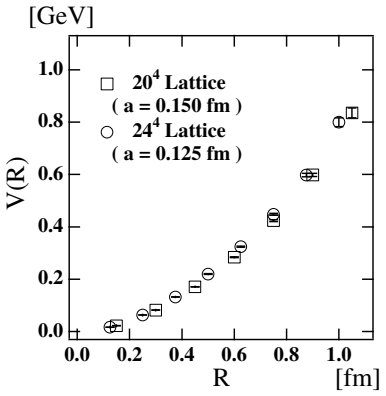

Figure 1: The static potential in the multi-instanton system.

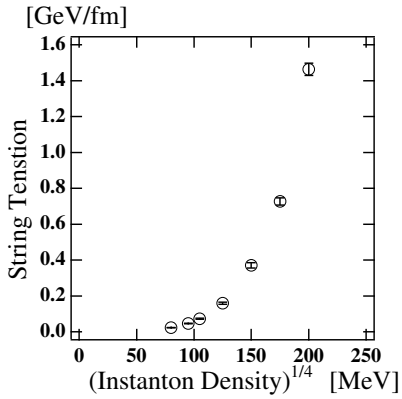

Figure 2: The string tension versus the instanton density.

References

[1] D. Diakonov and V. Petrov, in *Nonperturbative Approaches to QCD* (PNPI,1995) 239.

[2] For a recent review see T. Schäfer and E. Shuryak, hep-ph/9610451.

[3] G. 't Hooft, Nucl. Phys. **B190** (1981) 455.

[4] H. Suganuma, S. Sasaki and H. Toki, Nucl. Phys. **B435** (1995) 207.

[5] O. Miyamura, Phys. Lett. **B353** (1995) 91; Nucl. Phys. **B**(Proc.Suppl.)**42** (1995) 538.

[6] O. Miyamura and S. Origuchi,*Color Confinement and Hadrons* (World Scientific,1995)235.

[7] H. Suganuma, H. Ichie, S. Sasaki and H. Toki, *Color Confinement and Hadrons* (1995) 65.

[8] S. Thurner, H. Markum and W. Sakuler, *Color Confinement and Hadrons* (1995) 77.

[9] H. Suganuma, A. Tanaka, S. Sasaki and O. Miyamura, Nucl. Phys. **B** (Proc.Suppl.) **47** (1996) 302.

[10] M. Chernodub and F. Gubarev, JETP Lett.**62** (1995) 100.

[11] A. Hart and M. Teper, Phys. Lett. **B371** (1996) 261.

[12] R. Brower, K. Originos and C. Tan, Phys. Rev. **D55** (1997) 6313.

[13] H. Suganuma, S. Sasaki, H. Ichie, H. Toki and F. Araki, *Frontier '96* (World Scientific) 177.

[14] M. Fukushima, A. Tanaka, S. Sasaki, H. Suganuma, H. Toki and D. Diakonov, Nucl. Phys. **B** (Proc.Suppl.) **53** (1997) 494.

[15] M. Fukushima, S. Sasaki, H. Suganuma, A. Tanaka, H. Toki and D. Diakonov, Phys. Lett. **B399** (1997) 141.

[16] M. Fukushima, A. Tanaka, H. Suganuma, H. Toki and S. Sasaki, Nucl. Phys. **B** (Proc. Suppl.) **63** (1998) 513.