# SOME TOPOLOGICAL ASPECTS OF CHIRAL QCD

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Our model of the QCD ground state is based on the existence and importance of the nonperturbative $q^{-4}$ *quantum* excitations of gluon field configurations in the true QCD vacuum [1]. On the other hand, it is based on solutions to the quark and ghost Schwinger-Dyson (SD) equations in the infrared (IR) which should be complemented by the investigation of the corresponding Slavnov-Taylor (ST) identities. Such sigular behaviour of the full gluon propagator in the IR requires the introduction of a small IR regulation parameter $\epsilon$, in order to define the initial equations in the IR by the dimensional regularization method within the distribution theory [1]. This yeilds the regularization expansion for the above mentioned strong IR singularity as follows (Euclidean space)

$$(q^2)^{-2+\epsilon} = \frac{\pi^2}{\epsilon}\delta^4(q) + finite\ terms, \qquad \epsilon \to 0^+ \tag{1}$$

and the terms of order $\epsilon$ are not shown here for simplicity. Because of this, the quark propagator and other Green's function become dependent, in general, on this IR regulation parameter $\epsilon$, which is to be set to zero at the end of computations, $\epsilon \to 0^+$. For the sake of brevity, this dependence is always understood but not indicated explicitly. In this case, it is necessary to develop a method for the extraction of the IR finite (regularized) Green's functions in QCD. The IR finiteness of the Green's functions means that they exist as $\epsilon \to 0^+$. For this purpose, we have worked out a renormalization program in order to cancel all IR nonperturbative divergences, which makes it possible to explicitly show that all Green's functions are IR multiplicative renormalizable (MR). On the other hand this leads to a close set of equations in the quark sector. We first approximate the exact quark SD equation by its deep IR (confinement) piece resonably assuming that precisely this term is mainly responsible for nonperturbative effects in QCD in particular quark confinement. Introducing the appropriate dimensionless variables, the quark SD equation for the quark propagator $iS(p) = \hat{p}A(p^2) - B(p^2)$, becomes

$$xA' = -(2+x)A - 1 - m_0 B, \qquad 2BB' = -3A^2 + 2(m_0 A - B)B, \tag{2}$$

where $A, B \equiv A(x), B(x)$. and the prime denotes the derivative with respect to the Euclidean dimensionless momentum variable, $x$. In the chiral limit $m_0 = 0$, it can be solved exactly and solutions are

$$A(x) = x^{-2}(1 - x - e^{-x}), \qquad B^2(x) = 3e^{-2x}\int_x^{x_0} dx' e^{2x'} A^2(x'), \tag{3}$$

where $x_0$ is an arbitrary constant of integration. It is easy to see that obtained solution for the quark propagator is regular at zero point, has no pole-type singularities (indicative of confinement) and corresponds to dynamical breakdown of chiral symmetry (quark mass generation) since the chiral symmetry violeting solution ($m_0 = 0, A(x) \neq 0, B(x) \neq 0$) is only allowed.

The effective potential method for composite operators [2] allows one to investigate the vacuum of QCD since in the absence of external sources it is nothing but the vacuum energy density. The quark part of the vacuum energy density evaluated at the log-loop level and expressed in terms of the above mentioned dimensionless variables is

$$\epsilon_q = \frac{3}{8\pi^2} p_0^4 x_0^{-2} \int\limits_0^{x_0} dx\, x\, \Big\{\ln(x[xA^2(x) + B^2(x_0,x)]) + 2xA(x) + 2\Big\}, \qquad (4)$$

where we identified the UV cut-off with $x_0$ in order to avoid vacuum instability which may otherwise occur because of the unphysical branch point at $x = x_0$ in the solution for B-function, see (3). Here and below $p_0$ is the mass scale parameter determining the scale of nonperturbative dynamics in QCD. In the same way may be evaluated the gluon part of the vacuum energy density using the deep IR $q^{-4}$ behaviour of the full gluon propagator. It should be also properly regularazed at the expence of ghost degrees of freedom. The final expression is

$$\epsilon_g = \frac{1}{\pi^2} p_0^4 x_0^{-2} \times \Big\{\frac{9}{2}\ln(1 + \frac{x_0}{3}) - \frac{3}{4}x_0 - \frac{1}{2}x_0^2 \ln(1 + \frac{3}{x_0}) - (1 + \frac{a}{2})x_0^2\Big\}, \qquad (5)$$

and $a = -0.6363$. The total vacuum energy within our approach becomes $\epsilon_t = \epsilon_g + N_f \epsilon_q$, where we introduce dependence on number of different quark flavors, $N_f$. The vacuum energy density is important in its own right as the main characteristics of the nonperturbative vacuum of QCD. Futhermore it assists (through the trace anomaly relation) in estimating such an important phenomenogical parameter as the gluon condensate. The final expression is (in the chiral limit)

$$\langle \bar{G}^2 \rangle \equiv \langle \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \rangle \equiv \langle 0| \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^a_{\mu\nu} |0\rangle = 4\epsilon_t = 4(\epsilon_g + N_f \epsilon_q), \qquad (6)$$

where $\epsilon_q$ and $\epsilon_g$ are given by (4) and (5), respectively. This expression determines the gluon condensate in the strong coupling limit since we can not use (like in instanton calculus [3]) the weak coupling limit for the $\beta$-function. One of the main characteristic of the QCD vacuum is the topological density operator (topological susceptibility) in gluodynamics $\bar{\chi}_t = -\lim_{q\to 0} i \int d^4x\, e^{iqx} \langle 0|T\Big\{q(x)q(0)\Big\}|0\rangle$, where $q(x)$ is the topological charge density, defined as $q(x) = \frac{\alpha_s}{8\pi} G(x)\tilde{G}(x) = \frac{\alpha_s}{8\pi} G^a_{\mu\nu}(x)\tilde{G}^a_{\mu\nu}(x)$ and $\tilde{G}^a_{\mu\nu}(x) = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}(x)$ is the dual gluon field strenght tensor. The topological susceptibility can be related to the nonperturbative gluon condensate via the low energy theorem in gluodynamics as follows $\lim_{q\to 0} i \int d^4x\, e^{iqx} \langle 0|T\Big\{\frac{\alpha_s}{8\pi} G\tilde{G}(x) \frac{\alpha_s}{8\pi} G\tilde{G}(0)\Big\}|0\rangle = \xi^2 \langle \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \rangle$, where $\xi = 4/3b$ [4]. Using our values for the nonperturbative gluon condensate (6) the two-point function of the topological susceptibility can be calculated easily in the chiral limt

$$\bar{\chi}_t = -\xi^2 \langle \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \rangle = -(2\xi)^2 \epsilon_g. \qquad (7)$$

The topological susceptibility (7) asists in the resolution of the $U(1)$ problem (the large mass of the $\eta'$ meson) within the Witten-Veneziano formula for the mass of the $\eta'$ meson [5]. Within our notations it is expressed as follows ( in the chiral limit)

$$F_\pi^2 m_{\eta'}^2 = \frac{8N_f}{N_c^2} \bar{\chi}_t = 2N_f \chi_t. \qquad (8)$$

In the numerical evaluation of the above formula, it is necessary to put, of course, $N_f = N_c = 3$. Our numerical calculations have been done at the scale of the $\rho$ meson mass which a standard scale-setting scheme in lattice approach, i. e. seting $p_0 = M_\rho = 768.5\ MeV$. The constant of integration $x_0$ was determined from the bounds for the pion decay constant in the chiral limit $87.2 \leq F_\pi^o \leq 93.3\ (MeV)$ [1]. This requires $1.181 \leq x_0 \leq 1.337$. Results of our calculations are presented in Table. Thus our values of the topological susceptibility in

| $F_\pi^0$ (MeV) | | 87.2 | 93.3 |
|---|---|---|---|
| $\epsilon_{\rm q}$ (GeV$^4$) | | 0.00113 | 0.00148 |
| $\epsilon_{\rm g}$ (GeV$^4$) | | $-0.02399$ | $-0.03144$ |
| $\epsilon_{\rm ZME}$ (GeV$^4$) | $N_{\rm f} = 1$ | $-0.02286$ | $-0.02996$ |
| | $N_{\rm f} = 2$ | $-0.02173$ | $-0.02848$ |
| | $N_{\rm f} = 3$ | $-0.02060$ | $-0.02700$ |
| $\langle \bar{G}^2 \rangle$ (GeV$^4$) | $N_{\rm f} = 0$ | 0.09597 | 0.12577 |
| | $N_{\rm f} = 1$ | 0.09145 | 0.11985 |
| | $N_{\rm f} = 2$ | 0.08692 | 0.11392 |
| | $N_{\rm f} = 3$ | 0.08240 | 0.10800 |
| $\chi_{\rm t}^{1/4}$ (MeV) | | 158.2 | 169.3 |
| $m_{\eta'}$ (MeV) | | 703.2 | 752.4 |

the chiral limit nicely saturate its phenomenological value $\chi^{phen} = (180.36)^4\ MeV$ as well as the experimental mass of the $\eta'$ meson, namely $m_{\eta'}^{exp} = 957.77\ MeV$. Also our values for the vacuum energy density and the gluon condensate are by one order of magnitude larger than instanton liquid model (in various modifications)of the QCD vacuum [3] can provide at all since our contributions are the contributions in the strong coupling limit while instantons are weak coupling limit phenomenon.

## References


[1] V. Gogohia, H. Toki and T. Sakai, in preparation

[2] J.M.Cornwall, R. Jakiw and E. Tomboulis, Phys.Rev., **D10**, 2428 (1974)

[3] T. Schafer and E. Shuryak, hep-ph/96104

[4] I.Halperin and A.Zhnitsky, hep-ph/9711398, hep-ph/9803301

[5] E.Witten, Nucl.Phys., **B156**, 269 (1979):
G.Veneziano, Nucl.Phys., **B159**, 213 (1979