COLOR CONFINEMENT AND FLUX-TUBE PHYSICS BASED ON DUAL GINZBURG-LANDAU THEORY

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1 Flux-tube solution in the DGL theory

Recent studies of lattice QCD in the maximally abelian (MA) gauge suggest that the QCD vacuum can be regarded as the dual version of superconductor [1, 2]: the QCD-monopole is condensed and the dual Meissner effect can be occurred in the QCD vacuum like Cooper-pair condensation and the Meissner effect in the ordinary superconductor. This also implies quark confinement by the hadronic flux-tube formation through the dual Meissner effect. The flux-tube has a constant energy per unit length, *i.e.* the string tension, which leads the linear inter-quark potential. Based on the dual superconductor picture, one can construct the dual Ginzburg-Landau (DGL) theory as an effective theory of the nonperturbative QCD. The DGL lagrangian in the pure gauge is given as [3],

$$\mathcal{L}_{\rm DGL} = -\frac{1}{4}(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu)^2 + \sum_{a=1}^{3}[|(\partial_\mu + ig\vec{\alpha}_a \cdot \vec{B}_\mu)\chi_a|^2 - \lambda(|\chi_a|^2 - v^2)^2], \qquad (1)$$

where $\vec{B}_\mu$ and χ_a denote the dual gauge field and the monopole field, respectively, and $\vec{\alpha}_a$ is the root vector of SU(3) algebra. The DGL theory just has vortex-type solutions as hadronic flux-tubes like the Abrikosov vortex in the Ginzburg-Landau (GL) theory for superconductivity.

Now, we pay attention to the flux-tube degrees of freedom in terms of the nonperturbative phenomena. Clarification of the flux-tube physics may enable us to understand the properties of the complicated QCD vacuum. Therefore, we study the flux-tube solution in the DGL lagrangian (1) and apply it to the two topics which are closely related to the nonperturbative dynamics in the QCD vacuum. One is the quark-gluon-plasma (QGP) formation scenario in the ultra-relativistic heavy-ion collision as the multi-flux-tube system, and the other is the properties of the glueball as the flux-tube ring [4].

2 Multi-Flux-tube System and QGP Formation

In the ultra-relativistic heavy-ion collision, it is thought that the *multi-flux-tube system* would be realized just after the collision [5], and whether the QGP is formed or not after the pre-equilibrium stage depends on the energy deposition within the formation time $\sim$ 1 fm/c. To seek the possibility of the energy deposition in such a system, here, we pay attention to the flux-tube interaction. Using the DGL theory, we study the interaction energy of two typical two-body flux-tube systems. One is the flux-tube and flux-tube (F-F) system and the other is the flux-tube and anti flux-tube (F-A) system. These systems are fundamental systems to study the essence of the interaction between the flux-tubes. We solve equations of motion so-called GL equations numerically using finite difference method. Here, each system is supposed to have translational invariance along the z-axis.

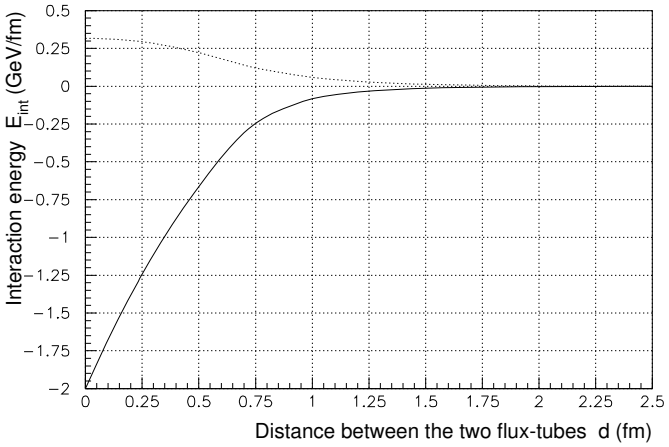

Fig.1: Interaction energy in the F-F system(dotted) and F-A system(solid) as a function of the flux-tube distance d.

The interaction energy of each system can be obtained as a function of distance between two flux-tubes shown in Fig.1, where the interaction energy is defined as,

$$E_{\rm int}(d) \equiv E_{\rm total}(d) - 2\times E_{\rm single\,flux-tube}, \tag{2}$$

where $E_{\rm total}(d)$ is a total energy of the system. These results have been calculated using the parameter set, $v=0.10$ GeV, $g=3.5$ ($e=4\pi/g=3.6$) and $\lambda=66$, which reproduces the string tension as $\sigma=1.0$ GeV/fm. Fig.1 shows that the interaction in the F-F system is repulsive, while the F-A system is strongly attractive. In the F-A system, flux-tube and anti flux-tube can annihilate by its strongly attractive interaction, and if such a process occurs, the large energy about 4 GeV/fm^3 = 2 GeV/fm $/(\pi\times(0.4\text{ fm})^2)$ is liberated (0.4 fm is a penetration depth of the flux-tube). Therefore we can conclude that this annihilation process can make a large contribution to the QGP formation.

3 Flux-tube ring and Glueball

Glueball is a fundamental particle of nonperturbative QCD, which is composed of the gluon field and considered it does not include a valence quark-antiquark pair. In the DGL theory this object corresponds to the no end flux-tube, that is, *flux-tube ring*. Therefore we can study the glueball properties by analyzing this flux-tube ring solution in the DGL theory.

We obtain the effective string tension as a function of the ring radius shown in Fig.2, where this value is defined as,

$$\tilde{\sigma}(R) \equiv E_{\rm cl}(R)/(2\pi R). \tag{3}$$

$E_{\rm cl}(R)$ is a classical energy of the ring and R is a ring radius, where the system is supposed to have rotational invariance along the z-axis and the parameter set is taken just same as Sec.2. We find the effective string tension is reduced as the ring radius is smaller, which suggests that the ring prefers to shrink at the classical level.

However, this ring solution would be stabilized against such a collapse by the quantum effect. Therefore we regard the flux-tube ring as a relativistic closed string with the effective

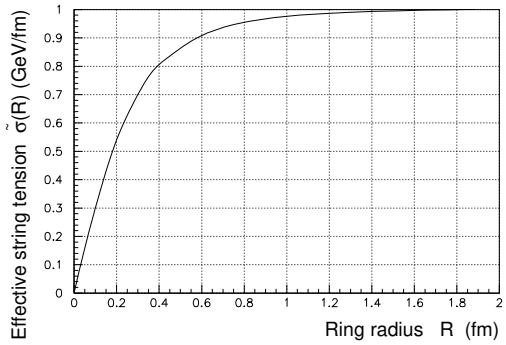

Fig.2: Effective string tension $\tilde{\sigma}$ as a function of ring radius R.

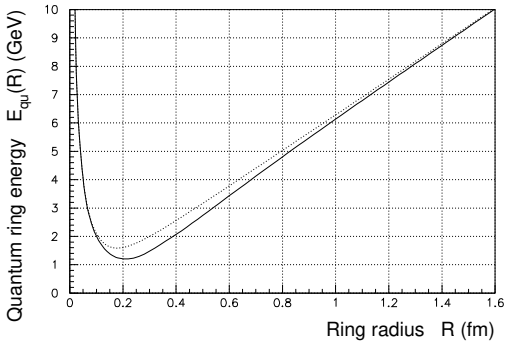

Fig.3: Ring energy with $\tilde{\sigma}$(solid) and σ(dotted) as a function of ring radius R.

string tension. In the string theory [6], we can introduce the quantum effect for the string using the Nambu-Goto (NG) action. By the parametrization of the ring as a circle with the radius R, we can obtain the Hamiltonian from the NG action, that is, the energy of the closed string as,

$$E_{\rm qu}(R) = \sqrt{P_R^2 + (2\pi\tilde{\sigma}(R)R)^2}, \tag{4}$$

where P_R is the canonical conjugate momentum of the coordinate R. Using the uncertainty relation $P_R \cdot R \geq 1$ as the quantum effect, we get the energy minimum 1.2 GeV at ring radius 0.22 fm shown in Fig.3. We consider this energy minimum and ring radius correspond to the lower limit of the mass and size of the glueball. Fig.3 shows that the effective string tension lowers the glueball mass. For more realistic calculation and to compare with the Lattice QCD results [7], we must consider the motion of the ring and extract the physical glueball state with definite quantum numbers " J^{PC} " using the angular momentum projection, which is now under consideration. In any case the DGL theory can provide a useful method for the study of the glueball.

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