Masses and electromagnetic decays of exited baryons in a deformed oscillator quark model

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Although it is believed that hadron physics is described by QCD, it is very difficult at the present time to understand them directly from QCD because of its non-perturbative nature at low energy. Here we would like to study baryon properties in a phenomenological way based on experimental data and using an effective model. In particular, we investigate excited baryons since it is useful to study excited states as well as the ground state in order to understand vacuum structure of QCD. Progress in experimental situation is also important. Detailed measurements of hadron properties are planed at facilities such as TJNAL, COSY and SPring8 at RCNP. We believe that an intimate collaboration between theory and experiment is extremely important to understand the complex system of hadrons.

It has been observed that there is a pattern similar to a rotational band in baryon spectra [1, 2]. This implies that baryon excited states may be regarded as rotational excitations of deformed intrinsic states. In order to model this, we construct an effective non-relativistic quark model with a deformed harmonic oscillator potential [3]. Our hamiltonian takes the following form

$$H_{DOQ} = \sum_{i=1}^{3} \left(\frac{p_i}{2m_i} + \frac{1}{2} m_i \left(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2 \right) \right) - H_{c.m.}. \tag{1}$$

Here m_i are masses of constituent quarks. We take $m_i \sim 300$ MeV for u,d and s quarks for simplicity. To investigate the effect of deformation exclusively, we do not consider other interactions such as spin or isospin dependent ones. The system of the hamiltonian(1) has been worked out in [1, 2, 3, 5]. The eigenvalues are given by

$$E_{int} = (N_x + 1)\omega_x + (N_y + 1)\omega_y + (N_z + 1)\omega_z \quad , \tag{2}$$

where N_x, N_y, N_z denotes oscillator quanta for oscillator excitations. For a given set of (N_x, N_y, N_z), we perform minimization of the energy E_{int} with respect to the deformation parameter under the volume conservation condition $\omega_x\omega_y\omega_z = \omega^3 = const.$ We found deformed states for excited states while a spherical state for the ground state. These deformed states break rotational symmetry, which can be recovered by rotating (cranking) the deformed states as we do in nuclear physics [5]. The resulting masses of rotational excitations are given by

$$E_l = E_{int} + \frac{l(l+1)}{2\mathcal{I}} - \frac{< l^2 >}{2\mathcal{I}} \quad . \tag{3}$$

The second term represents the rotational energy and the third term is the energy subtraction due to angular momentum fluctuation. In order to compare the theoretical masses with experimental data, we fix the oscillator parameter ω_0 =607MeV [3].

In Fig. 1 we see that theoretical masses compare very well with experimental data for N and Δ excitations. In particular, the agreement of the first $1/2^+$ excited state(Roper) is remarkable as well as the tendency of increasing level spacing in the positive energy excitations.

If such a deformation is sufficiently developed, we expect evidences not only in masses but also in other physical quantities such as transition amplitudes. From experiences of nuclear physics, we expect that electromagnetic decay amplitudes are useful to investigate. We study the matrix element $\langle f|e\vec{A}\cdot\vec{J}|i\rangle$, where $|i\rangle$ is an excited state of either N or Δ and $|f\rangle$ is the ground state N. For the current $\vec{J}$, we adopt the simple non-relativistic current

$$\vec{J} = \frac{1}{2m}\left[u_f^\dagger(i\overleftarrow{\nabla} - i\overrightarrow{\nabla})u_i + i\overrightarrow{\nabla}\times(u_f^\dagger\vec{\sigma}u_i)\right] \quad . \tag{4}$$

and the photon field with the helicity p can be written in terms of multipole fields,

$$\vec{A}_p = -p\sqrt{2\pi}\sum_l i^l\sqrt{2l+1}(\vec{A}_{lp}^{(M)} + ip\vec{A}_{lp}^{(E)}) \quad . \tag{5}$$

Calculated matrix elements are compared with experimental data of helicity amplitudes. One of unique points of our study is to compute matrix elements both in helicity and multipole basis. For an illustration we present results in multipole representations :

$$\begin{aligned}
\mathcal{M}(Ml) &\equiv \left\langle f\left|e\vec{A}_l^{(M)}\cdot\vec{J}\right|i\right\rangle \\
&= \frac{1}{2m}\left(-\sqrt{\frac{l}{2l+1}}\left\langle f\left\|[Y_{l+1}\sigma]^l\right\|i\right\rangle j_{l+1} + \sqrt{\frac{l+l}{2l+1}}\left\langle f\left\|[Y_{l-1}\sigma]^l\right\|i\right\rangle j_{l-1}\right) \quad , (6) \\
\mathcal{M}(El) &\equiv \left\langle f\left|e\vec{A}_l^{(E)}\cdot\vec{J}\right|i\right\rangle \\
&= i\frac{\sqrt{l(l+1)}}{m}\langle f'\|Y_l\|i\rangle\frac{j_l(kr)}{kr} - \frac{ik}{2m}\left\langle f\left\|[Y_l\sigma]^l\right\|i\right\rangle j_l \quad . \qquad (7)
\end{aligned}$$

Helicity amplitudes $A_{1/2}, A_{3/2}$ can be obtained by suitable linear combinations of $\mathcal{M}(El)$ and $\mathcal{M}(Ml\pm 1)$.

In Tables 1 and 2 we have summarized the results for helicity amplitudes of N and Δ excitations. The first column as indicated by DOQ is for the present results and the second column as indicated by KI for the results of the spherical case [4]. Calculated results are then compared with experimental data listed in the third column. The first $1/2^+$ excited state (Roper) corresponds to ${}^2S'_S$ for N and to ${}^4S'_S$ for Δ. We see that for many excited states the effect of deformation is rather mild and calculated amplitudes are in rough agreement with data. There is, however, one exception, that is the Roper. In this channel, as the deformation is increased, the magnitude of $A_{1/2}$ increases, but with the wrong sign. The sign problem of the Roper has been known already in the conventional quark model. One resolution was proposed by taking into account relativistic corrections [7]. Furthermore, coupling effect of the quark model state with mesonic open channels should be important [8], which has not been fully investigated.

For future progress, we would like to study various transitions not only of electromangnetic but also of strong and weak processes. In the DOQ model, intraband transitions as well as interband transitions are interesting to be investigated. Furthermore, we would like to study dynamical origin of spatial structure of baryons using more fundamental QCD effective theories.

Table 1: Helicity amplitudes of excited N(proton).

		$A_p^{1/2}$			$A_p^{3/2}$			
		DOQ	KI	Exp	DOQ	KI	Exp	
$1/2^+$	$^2S'_S$	109	22.6	-68± 5	–	–	–	$P_{11}(1440)$
	$^2S_{MS}$	-14.6	-15.9	+5±16	–	–	–	$P_{11}(1710)$
$1/2^-$	$^2P_{MS}$	151	156	74±11	–	–	–	$S_{11}(1535)$
	$^4P_{MS}$	0	0	48±16	–	–	–	$S_{11}(1650)$
$3/2^+$	^2D_S	70.9	111	52±39	-23.5	-36.7	-35±24	$P_{13}(1720)$
$3/2^-$	$^2P_{MS}$	24.8	25.6	-23±9	138	143	163±8	$D_{13}(1520)$
	$^4P_{MS}$	0	0	-22±13	0	0	0±19	$D_{13}(1700)$

Table 2: Helicity amplitudes of excited Δ.

		$A_p^{1/2}$			$A_p^{3/2}$			
		DOQ	KI	Exp	DOQ	KI	Exp	
$1/2^-$	$^2P_{MS}$	43	46	19±16	–	–	–	$S_{31}(1620)$
$3/2^+$	$^4S'_S$	59	22	-20± 29	102	38	1± 22	$P_{33}(1600)$
$3/2^-$	$^2P_{MS}$			116±17			77±28	$D_{33}(1700)$

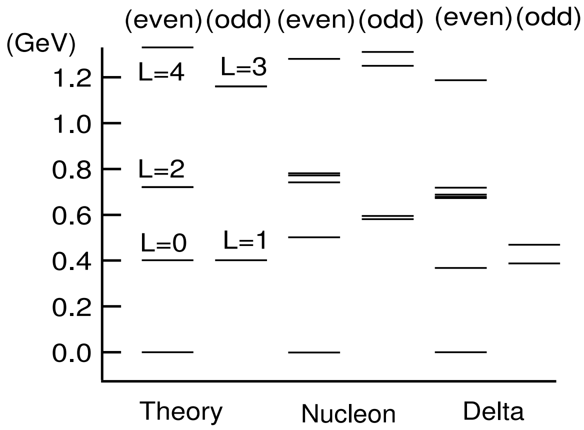

Figure 1: Mass spectrum of excited baryons in units of GeV. The masses are measured from the ground states of each spin- flavor quantum number. In the theoretical spectrum L for each state denotes the orbital angular momentum. The experimental spectrum of N and Δ are also shown in this figure [6].

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