PROPERTIES OF THE QCD VACUUM IN THE DUAL GINZBURG-LANDAU THEORY

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Recent studies of the lattice gauge theory strongly support the dual superconductor picture for the QCD vacuum in the confinement phase in the 't Hooft abelian gauge [1], and show abelian dominance and monopole dominance for the confinement and dynamical chiral-symmetry breaking (DχSB) [2]. The dual Ginzburg-Landau (DGL) theory is an infrared effective theory of QCD based on dual Higgs mechanism in the abelian gauge [3, 4], and is described by the diagonal gluon A^μ, the dual gauge field B^μ, and the QCD-monopole field χ_α ($\alpha = 1, 2, 3$) [3].

In the QCD-monopole condensed vacuum $|\chi_\alpha|^2 \equiv v^2$, the dual gauge field acquires a mass of $m_B = 4\pi\sqrt{3}v/e$ and propagates as a massive vector field [3]. On the other hand, the nonperturbative gluon propagator takes the form $D_{\mu\nu}(p) = -d(p^2)\ T_{\mu\nu}(p)\ /p^2 - \alpha_{\rm e}\ L(p)_{\mu\nu}\ /p^2$, where $T_{\mu\nu} = g_{\mu\nu} - L_{\mu\nu}$ and $L_{\mu\nu} = p_\mu p_\nu/p^2$ is the transverse and the longitudinal tensor, respectively. In the DGL theory, $d(p^2)$ is given as

$$d(p^2) = \frac{1}{3}\left\{\frac{4m_B^2}{(p^2+m_B^2)}\cdot\frac{p^2+a^2}{a(a+\sqrt{a^2+p^2})} + \frac{2p^2}{p^2+m_B^2} + 1\right\} \tag{1}$$

with the infrared cutoff parameter a corresponding to the hadron size introduced in relation with the polarization effect of light quarks [4]. The first and the second term, $d_{\rm conf}(p)$ and $d_{\rm Y}(p)$ is related to the linear confinement and Yukawa part in the static quark potential, respectively. The third term, $d_{\rm C}(p)$ does not contribute to the static potential.

First, we discuss DχSB in the quark sector physics. Our group showed the essential role of QCD-monopole condensation to DχSB by solving the Schwinger-Dyson (SD) equation [3, 4]. For further investigation, we calculate the effective potential to extract the contribution of the confinement effect energetically. Using the nonperturbative gluon propagator in the DGL theory, the effective potential is expressed as [5]

$$\begin{aligned} V_{\rm eff}[M(p^2)] &= -2N_{\rm c}N_{\rm f}\int\frac{d^4p}{(2\pi)^4}\{\ln(\frac{p^2+M^2(p^2)}{p^2}) - 2\frac{M^2(p^2)}{p^2+M^2(p^2)}\} \\ &+ N_{\rm f}(N_{\rm c}-1)\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}e^2\frac{M(p^2)}{p^2+M^2(p^2)}\frac{M(q^2)}{q^2+M^2(q^2)}D_{\mu\mu}(p-q) \end{aligned} \tag{2}$$

in the improved ladder approximation [6]. The first term is the quark-loop contribution. The second term is divided into three parts ($V_{\rm conf}$,$V_{\rm Y}$,$V_{\rm C}$) corresponding to the decomposition of $D_{\mu\mu}$ in Eq.(1).

Fig.1 shows $V_{\rm eff}[M(p^2)]$ as a function of the infrared quark mass $M(0)$ using the quark-mass ansatz which is consistent with the renormalization group analysis of QCD [6]. The parameters $e = 5.5$, $m_B = 0.5$ GeV and $a = 85$ MeV are chosen so as to reproduce the inter-quark potential and the flux-tube radius $R \simeq$0.4 fm [3]. One finds a nontrivial minimum at $M(0) \simeq$0.4 GeV in $V_{\rm eff}[M(p^2)]$. Fig.2 shows the separate contribution $V_{\rm quark}$, $V_{\rm conf}$, $V_{\rm Y}$ and $V_{\rm C}$. The effective potential is mainly lowered by $V_{\rm conf}$. Since the lowering of the effective potential contributes to DχSB, the main driving force of DχSB is brought by the confinement part $D^{\rm conf}_{\mu\mu}(p)$ in the nonperturbative gluon propagator, which means monopole dominance for DχSB [2].

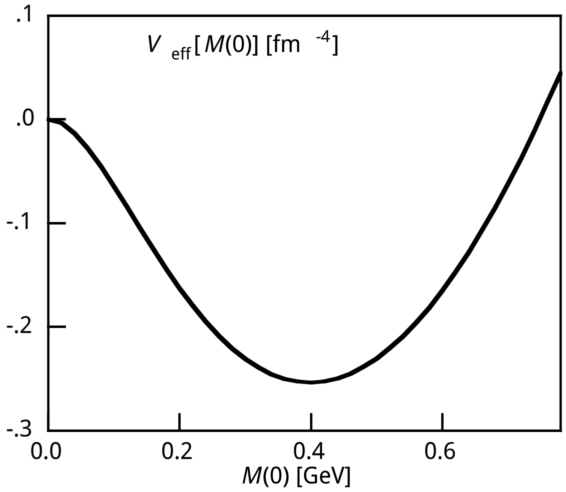

Fig.1 : The effective potential $V_{\rm eff}[M(p^2)]$ as a function of infrared quark mass $M(0)$.

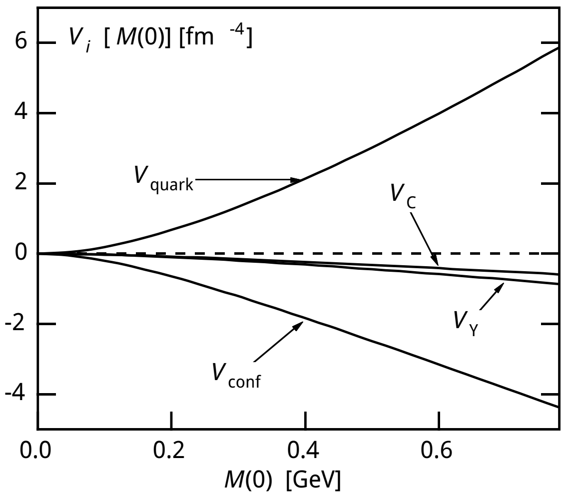

Fig.2 : The separation of the effective potential. The confinement part $V_{\rm conf}$ dominates $V_{\rm Y}$ and $V_{\rm C}$.

Next, we discuss the gluon sector physics. The observed feature of abelian dominance for confinement and DχSB indicates that the remaining off-diagonal gluon C^α_μ ($\sqrt{2}C^1_\mu = A^1_\mu + iA^2_\mu$, *etc.*), which behaves as charged matter field in the abelian gauge, is irrelevant to the low-energy physics. Here, we study the self-energy of the charged gluon using the SD equation for C^α_μ in the DGL theory. In the Hartree-type approximation (Fig.3), the SD equation for C^α_μ is expressed as

$$i\Delta^M_{\mu\nu} = i\Delta^0_{\mu\nu} + \frac{1}{2} i\Delta^M_{\mu\rho}\Gamma^{ab}_{\rho\sigma\tau\upsilon} iD^{ab}_{\sigma\tau} i\Delta^0_{\upsilon\nu}, \tag{3}$$

where the full propagator of C^α_μ is assumed to be

$$i\Delta^M_{\mu\nu} = -\frac{i}{k^2 - M^2}\left\{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right\} \tag{4}$$

in the Landau gauge, while the free one $i\Delta^0_{\mu\nu}$ and the vertex factor

$$\Gamma^{ab}_{\kappa\lambda\mu\nu} = -ie^2\varepsilon^a_\alpha\varepsilon^b_\alpha\{2g_{\kappa\nu}g_{\lambda\mu} - g_{\kappa\lambda}g_{\mu\nu} - g_{\kappa\mu}g_{\lambda\nu}\} \tag{5}$$

is obtained from the QCD Lagrangian.

Fig.3 : The SD equation for the off-diagonal gluon. The dashed lines with or without a blob represent full or free propagators of the charged gluon C^α_μ, respectively. The curly line denotes the nonperturbative propagator of the diagonal gluon.

The self-energy Σ defined by $\Sigma_{\rho\upsilon} \equiv \Gamma^{ab}_{\rho\sigma\tau\upsilon}\ D^{ab}_{\sigma\tau}/2 = g_{\rho\upsilon}\Sigma$ is related to the effective mass M of the charged gluon as $\Sigma = M^2$. To drop off the divergence from the perturbative part in $d(p^2)$, we replace $d(p^2)$ by $\tilde{d}(p^2) \equiv d(p^2; m_B) - d(p^2; 0)$ in the momentum integration in Eq.(3). Then, the self-energy is given by the integral

$$\Sigma = \frac{9e^2}{4} \int \frac{d^4p}{(2\pi)^4} \frac{\tilde{d}(p^2)}{p^2}. \tag{6}$$

Considering the reduction of the monopole condensate in the core region of the dual Abrikosov vortex [3, 5], we introduce a physical ultraviolet cutoff to the monopole condensate as $m_B(p) = m_B \theta(m_\chi - p)$, where $m_\chi = 1.21$ GeV is the mass of the monopole. We estimate the charged gluon mass as $M_{ch} = 1.31$ GeV, which suggests the irrelevance of the charged gluon for low-energy physics in the abelian gauge. This property is also seen in the lattice QCD [7].

Finally, we investigate the gluon condensate. With the assumption of abelian dominance for the gluon condensate $\langle G^a_{\mu\nu} G^{\mu\nu}_a \rangle \simeq \langle \vec{f}_{\mu\nu} \vec{f}^{\mu\nu} \rangle$, where $\vec{f}_{\mu\nu}$ is the abelian field strength tensor [3], we can estimate it within the DGL theory. Since the non-perturbative QCD vacuum is filled with condensed monopoles, it is convenient to proceed in the dual field formalism,

$$\begin{aligned} \langle \vec{f}_{\mu\nu} \vec{f}^{\mu\nu} \rangle &= -\langle {}^*\vec{f}_{\mu\nu} {}^*\vec{f}^{\mu\nu} \rangle \\ &= -2(g^{\alpha\beta} g^{\gamma\delta} - g^{\beta\gamma} g^{\delta\alpha}) \partial^x_\alpha \partial^y_\beta \langle \vec{B}_\gamma(x) \vec{B}_\delta(y) \rangle |_{x=y}, \\ &= 4(g^{\alpha\beta} g^{\gamma\delta} - g^{\beta\gamma} g^{\delta\alpha}) i \int \frac{d^4k}{(2\pi)^4} \frac{k_\alpha k_\beta}{k^2 - m_B^2} \left\{ g_{\gamma\delta} - \frac{k_\gamma k_\delta}{m_B^2} \right\} \end{aligned} \tag{7}$$

to obtain the expression

$$\langle \vec{f}^2 \rangle = 12i \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^2 - m_B^2(k)}. \tag{8}$$

By subtracting the contribution of perturbative part, $\langle \vec{f}^2 \rangle_{m_B} - \langle \vec{f}^2 \rangle_{m_B=0}$, we obtain the value $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (344\text{MeV})^4$, which is comparable to the values obtained by other studies [8, 9].

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