Effective Mass Generation of Off-diagonal Gluons and Abelian Dominance in the Maximally Abelian Gauge in QCD

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Based on the dual-superconductor picture [1], we study the properties of gluons in QCD in maximally abelian (MA) gauge using the SU(2) lattice QCD Monte Carlo simulation [2-5]. In the MA gauge, QCD reduces into the abelian gauge theory including the monopoles [6-8]. In the MA gauge, diagonal gluon components behave as neutral gauge fields like photons, and off-diagonal gluon components behave as charged matter fields on the residual abelian gauge symmetry. In the MA gauge, the diagonal part of the gluon field has been demonstrated to play a dominant role to the nonperturbative quantities like confinement and chiral symmetry breaking. On the other hand, the off-diagonal part of the gluon field does not contribute to the long-range phenomena. This is called the abelian dominance [7]. The abelian dominance is confirmed by the recent lattice QCD simulation. However, why the off-diagonal gluon in the MA gauge is irrelevant to the infrared physics is not understood yet.

We study essence of abelian dominance for NP-QCD in the MA gauge in terms of the generation of the effective mass $m_{\rm ch}$ of the off-diagonal gluon by the MA gauge fixing [2-4]. In the SU(2) QCD partition functional, the mass generation of the off-diagonal gluon $A_\mu^\pm \equiv (A_\mu^1 \pm iA_\mu^2)/\sqrt{2}$ in the MA gauge is expressed as

$$\begin{aligned} Z_{\rm QCD}^{\rm MA} &= \int DA_\mu \exp\{iS_{\rm QCD}[A_\mu]\}\delta(\Phi_{\rm MA}^\pm[A_\mu])\Delta_{\rm PF}[A_\mu] \\ &= \int DA_\mu^3 \exp\{iS_{\rm eff}[A_\mu^3]\}\int DA_\mu^\pm \exp\{i\int d^4x\ m_{\rm ch}^2 A_\mu^+ A_-^\mu\}\mathcal{F}[A_\mu], \end{aligned} \tag{1}$$

with $\Phi_{\rm MA} \equiv [\hat{D}_\mu,[\hat{D}^\mu,\tau_3]]$, the Faddeev-Popov determinant $\Delta_{\rm FP}$, the abelian effective action $S_{\rm eff}[A_\mu^3]$ and a smooth functional $\mathcal{F}[A_\mu]$.

In the Euclidean QCD, the MA gauge is defined by minimizing $R_{\rm off}[A_\mu] \equiv e^2\int d^4x A_\mu^+(x)A_-^\mu(x)$. In the MA gauge, the off-diagonal gluon components are forced to be as small as possible by the SU(2) gauge transformation [2, 4, 5]. In the MA gauge, the SU(2) gauge symmetry is reduced into $\mathrm{U(1)}_3^{\rm local}\times \mathrm{Weyl}^{\rm global}$, where the global Weyl symmetry is the subgroup of SU(2) relating the permutation of the bases in the fundamental representation [2, 5].

To investigate the off-diagonal gluon mass $m_{\rm ch}$, we study the Euclidean gluon propagator $G_{\mu\nu}^{ab}(x-y) \equiv \langle A_\mu^a(x)A_\nu^b(y)\rangle$ in the MA gauge, using the SU(2) lattice QCD [2-5]. As for the residual $\mathrm{U(1)}_3$ gauge symmetry, we impose the $\mathrm{U(1)}_3$ Landau gauge fixing to extract most continuous gauge configuration under the MA gauge constraint and to compare with the continuum theory. The continuum gluon field $A_\mu^a(x)$ is extracted from the link variable as $U_\mu(s) = \exp(iaeA_\mu^a(s)\frac{\tau^a}{2})$. Here, the scalar-type gluon propagator $G_{\mu\mu}^a(r) \equiv \sum_{\mu=1}^4\langle A_\mu{}^a(x)A_\mu{}^a(y)\rangle$ is useful to observe the interaction range of the gluon, because it depends only on the four-dimensional Euclidean distance $r \equiv \sqrt{(x_\mu-y_\mu)^2}$.

We show in Fig.1(a) $G_{\mu\mu}^3(r)$ and $G_{\mu\mu}^{+-}(r) \equiv \sum_{\mu=1}^4\langle A_\mu^+(x)A_\mu^-(y)\rangle = \frac{1}{2}\{G_{\mu\mu}^1(r)+G_{\mu\mu}^2(r)\}$ in the MA gauge using the SU(2) lattice QCD with $2.2 \le \beta \le 2.4$ and the various lattice

size ($12^3 \times 24$, 16^4, 20^4). Since the massive vector-boson propagator with the mass M takes a Yukawa-type asymptotic form as $G_{\mu\mu}(r) \sim \frac{M^{1/2}}{r^{3/2}} \exp(-Mr)$, the effective mass $m_{\rm ch}$ of the off-diagonal gluon $A_\mu^\pm(x)$ can be evaluated from the slope of the logarithmic plot of $r^{3/2} G_{\mu\mu}^{+-}(r) \sim \exp(-m_{\rm ch} r)$ as shown in Fig.1(b). The off-diagonal gluon $A_\mu^\pm(x)$ behaves as the massive field with $m_{\rm ch} \simeq 1$ GeV in the MA gauge for $r \gtrsim 0.2$ fm.

We perform also the standard mass measurement for the off-diagonal gluon in the similar manner to the hadron mass measurement in the lattice QCD [5]. We calculate $\Phi_\mu^\pm(\tau) \equiv \int d\vec{x} A_\mu^\pm(\vec{x}, \tau)$, and measure the temporal correlation of $\Gamma_{\mu\mu}^{+-}(\tau) \equiv \langle \Phi_\mu^+(\tau) \Phi_\mu^-(0) \rangle$ in the MA gauge with the $U(1)_3$ Landau gauge. We obtain the off-diagonal gluon mass $m_{\rm ch} \simeq 1$ GeV again from the slope of the logarithmic plot of $\Gamma_{\mu\mu}^{+-}(\tau)$ as the function of the temporal distance τ in the lattice QCD with $2.3 \leq \beta \leq 2.35$ with $16^3 \times 32$ and $12^3 \times 24$, as shown in Fig.1(c).

Thus, essence of infrared abelian dominance in the MA gauge can be physically interpreted with the effective off-diagonal gluon mass $m_{\rm ch}$ induced by the MA gauge fixing . Due to the effective mass $m_{\rm ch} \simeq 1$ GeV, the off-diagonal gluon $A_\mu^\pm$ can propagate only within the short range as $r \lesssim m_{\rm ch}^{-1} \simeq 0.2$ fm, and cannot contribute to the infrared QCD physics in the MA gauge, which leads to abelian dominance for NP-QCD [2-5].

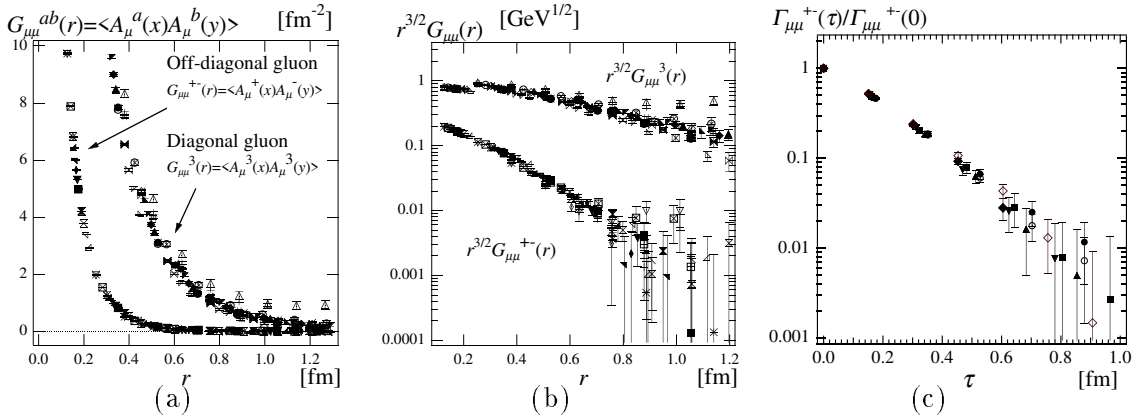

Figure 1: (a) The scalar-type gluon propagator $G_{\mu\mu}^a(r)$ as the function of the 4-dimensional distance r in the MA gauge in the SU(2) lattice QCD with $2.2 \leq \beta \leq 2.4$ and various lattice size ($12^3 \times 24$, 16^4, 20^4). (b) The logarithmic plot for the scalar correlation $r^{3/2} G_{\mu\mu}^a(r)$. The off-diagonal gluon propagator behaves as the Yukawa-type function, $G_{\mu\mu} \sim \frac{\exp(-m_{\rm ch} r)}{r^{3/2}}$. (c) The logarithmic plot of the temporal correlation $\Gamma_{\mu\mu}^{+-}(\tau) \equiv \langle \Phi_\mu^+(\tau) \Phi_\mu^-(0) \rangle$ as the function of the temporal distance τ in the lattice QCD with $2.3 \leq \beta \leq 2.35$ with $16^3 \times 32$ and $12^3 \times 24$. From the slope of the dotted lines in (b) and (c), the effective mass of the off-diagonal gluon $A_\mu^\pm$ can be estimated as $m_{\rm ch} \simeq 1$ GeV.

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