Virtual charm contribution in polarized deep inelastic scattering

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We would like to discuss the role of the charm content of the light hadrons in deep–inelastic scattering with polarized charged leptons on polarized nucleons. In this case there are no charmed hadrons both in the initial and the final states, which means that a charm may give a contribution only through virtual processes.

The quark spin Δq is defined as

$$\Delta q \, 2m_N \, s^\mu = \langle p,s|\overline{q}\gamma^\mu\gamma_5 q|p,s\rangle, \tag{1}$$

where m_N and s^μ are the mass and the spin of the nucleon, respectively. On the other hand,

$$\Delta q \;=\; \int_0^1 dx \;\left(q_R(x) - q_L(x)\right),$$

where $q_{R(L)}(x)$ are quark distributions of chirally right-handed (left-handed) quarks in a polarized proton.

In all of the consideration of the nucleon spin the contribution of the charm was neglected completely. We may expect sizable value of Δc, since in this case we are dealing again with axial currents which may be strongly affected by the nonperturbative gluon fields.

In the Euclidean QCD the axial anomaly in the light quark axial current in the chiral limit reads

$$\partial_\mu \psi_f^\dagger \gamma_5 \gamma_\mu \psi_f = -i\frac{g^2}{16\pi^2} G\widetilde{G}, \tag{2}$$

where $G^a_{\mu\nu}$ and $\widetilde{G}^a_{\mu\nu}$ are the gluon field strength operator with a being the color index and its dual, respectively, ψ_f is the light quark field ($f = u,d,s$) and g is the QCD coupling constant.

The situation with a heavy quark is very different, since we must take into account the contribution for the mass term. The divergence of the axial current of charmed quarks has a form:

$$\partial_\mu c^\dagger \gamma_\mu \gamma_5 c = -i\frac{g^2}{16\pi^2} G\widetilde{G} + 2m_c c^\dagger \gamma_5 c, \tag{3}$$

The main problem here is to calculate the contribution from the second term in (3). It is clear that this one is reduced to the problem of the calculation of the vacuum expectation value of the operator $2m_c c^\dagger\gamma_5 c$ in the presence of a gluon fields.

We consider here the first step – the integration over c-quarks. The resulting $\langle 2m_c c^\dagger\gamma_5 c\rangle$ must be a gauge invariant function of the gauge field A and therefore must be expressed through the gluon field strength tensor and their covariant derivatives [1, 2]. The matrix elements of such types of the operators are almost completely defined by nonperturbative topologically nontrivial (like instantons) contributions at least for the small virtualities flowing through these operators. This one can be clearly justified by the consideration of low-energy theorems for the various matrix elements of the gluon operator [3].

The results of our estimations of the divergence of the quark axial currents [2, 4] gives the following forms for $\Delta\Sigma$ and Δc,

$$\Delta\Sigma \, 2m_N \, \bar{p} i\gamma_5 p \;=\; \langle p,s|\frac{N_f g^2}{16\pi^2} G\widetilde{G}|p,s\rangle, \tag{4}$$

$$\Delta c\, 2m_N\, \bar{p} i\gamma_5 p \;=\; -\langle p,s|\frac{ig^3}{2^5 3\pi^2 m_c^2} f_{abc} G^a \tilde{G}^b G^c |p,s\rangle \tag{5}$$

(These formulas are defined in Minkowski space), where $\Delta\Sigma$ means a nucleon spin with light quarks. In the constituent quark model, $\Delta\Sigma = 1$. On the other hand it was shown that Skyrme soliton model gives zero for this quantity. Chiral quark-soliton model (see recent review of this model [5]) interpolate between these constituent quark and Skyrme models and give the value of $\Delta\Sigma$ in the range $\Delta\Sigma = 0.3 \sim 0.5$ [6, 7].

Chiral quark-soliton model [5] is essentially based on Diakonov-Petrov effective action [8], where all degrees of freedom are frozen except constituent quarks and pions. In that case, by restoring of the quark degrees of freedom in the above effective action, we find

$$\hat{Z}_N[\kappa_{2(3)}] = \int D\psi^\dagger D\psi DU \exp \int \psi^\dagger \left\{ i\hat{\partial} + iF^2 M U^{\gamma_5} \left(1 + \gamma_5(\kappa_{2(3)} f)\right)^{N_f^{-1}} \right\} \psi, \tag{6}$$

where $U^{\gamma_5} = \frac{1}{2M}((\Phi_+ + \Phi_-) + (\Phi_+ - \Phi_-)\gamma_5)$, $\Phi_\pm = M\exp(\pm i\phi)$ and the usual decomposition for the pions $\phi = \sum_1^3 \tau_i \phi_i$ may be used.

The calculation of the nucleon matrix elements of any combination of the gluon operators may be reduced to the differentiation over $\kappa_{2(3)}$ of the two-point correlator of the composite quark operators calculated by using $\hat{Z}_N[\kappa_{2(3)}]$ [4].

The ratio $\Delta c/\Delta\Sigma$ can be easily calculated in the same line as for $f_{\eta'}^{(c)}/f_{\eta'}^{(u)}$ in Ref. [4], which leads to

$$\Delta c/\Delta\Sigma \;=\; -\frac{12}{5\rho^2}\frac{1}{N_f 6 m_c^2} \;\sim\; -0.033. \tag{7}$$

With the use of the range of $\Delta\Sigma$ [6, 7], it means

$$\Delta c \;=\; -(0.01 \;\sim\; 0.016). \tag{8}$$

The quantity Δc in (8) is given at the scale of the size of the instanton. On the scale $\approx m_c$, we have

$$\Delta c(\mu \simeq m_c) \simeq 1.5\Delta c(\mu \simeq \rho^{-1}) \;=\; -(0.015 \;\sim\; 0.024). \tag{9}$$

Seeing this result, we conclude that the charm contribution is very small.

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