TRIVIALITY OF THE VACUUM ENERGY DENSITY WITH AN INFRARED-VANISHING GLUON PROPAGATOR

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The vacuum energy density is generally badly divergent in quantum field theory (QFT), in particular QCD. The main problem thus is how to extract the finite part of the vacuum energy density which is relevant for the QCD vacuum quantum models under consideration. The main object in such kind of investigations is, of course, the full gluon propagator which is to be supposed to reflect the structure of the QCD vacuum. Some lattice calculations [1,2] suggest the smooth behaviour of the full gluon propagator in the IR domain, which is hard to relate to quark confinement and dynamical chiral symmetry breaking (DCSB). In Ref. [3] it is shown that vanishing at zero gluon propagator, the so-called Zwanziger-Stingle (ZS) gluon propagator (for references see the above mentioned paper [3]) neither confines quarks nor breaks chiral symmetry dynamically. Unfortunately, three particular-type expressions for the dressed-quark-gluon vertex free from ghost contributions was used in their investigation. The main purpose of this note is to calculate the vacuum energy density of Yang-Mills (YM)fields using the ZS gluon propagator. Complementing their main conclusion we will show that there is no nontrivial (negative anf finite) vacuum energy density in this case as well. It always remains positive and our investigation does not require the particular choice of the dressed-quark-gluon vertex, i. e. it is a general one.

Let us consider the full gluon propagator

$$iD_{\mu\nu}(q) = \left\{T_{\mu\nu}(q)d(-q^2,a) + aL_{\mu\nu}(q)\right\}\frac{1}{q^2}, \tag{1}$$

where a is a gauge fixing parameter ($a = 0$, Landau gauge) and $T_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu/q^2 = g_{\mu\nu} - L_{\mu\nu}(q)$. Evidently, its free (perturbative) counterpart is obtained by simply setting the full gluon form factor $d(-q^2,a) = 1$ in (1). The exact expression for the full gluon propagator is not known. Let us for simplicity approximate the full gluon propagator by the ZS Ansatz in the whole region, i. e. to put

$$d(-q^2,a) = \frac{(q^2)^2}{(q^2)^2 + \mu^4}, \tag{2}$$

where μ^4 is the mass scale parameter resposible for the non-perturbative dynamics in this model. When it is zero then the ZS gluon propagator (1-2) becomes the free perturbative one indeed.

In this note we are going to analytically establish completely quantum part of the vacuum energy density which is due to ZS (2) gluon propagator omitting all the renormalization group improvements for the UV region for simplicity. Let us start from the non-perturbative gluon part of the vacuum energy density which to leading order (log-loop level $\sim \hbar$) is given by the effective potential approach for composite operators [4] as follows

$$V(D) = \frac{i}{2}\int\frac{d^nq}{(2\pi)^n}Tr\{\ln(D_0^{-1}D) - (D_0^{-1}D) + 1\}, \tag{3}$$

where $D(q)$ is the full gluon propagator (1) and $D_0(q)$ is its free (perturbative) counterpart. Here and below the traces over space-time and color group indices are understood. The effective potential is normalized as $V(D_0) = 0$, i. e. the perturbative vacuum is normalized to zero. In order to evaluate the effective potential (3) we use the well-known expression, $Tr\ln(D_0^{-1}D) = 8 \times \ln det(D_0^{-1}D) = 8 \times 4\ln\left[\frac{3}{4}d(-q^2) + \frac{1}{4}\right]$. It becomes zero (in accordance with the above mentioned normalization condition) when the full gluon form factor is replaced by its free counterpart. This composition does not explicitly depend on the gauge choice.

Going over to four ($n = 4$) dimensional Euclidean space in (3) by introducing the appropriate dimensionless momentum variable as $x = q^2/\Lambda_{YM}^2$ and after doing some algebra, we finally obtain the following expression for the vacuum energy density, $\epsilon_g = V(D)$,

$$\Omega_g(x_0;b) = \frac{1}{\Lambda_{YM}^4}\epsilon_g = \frac{1}{8\pi^2}\Big\{[(8\ln 2)x_0^2 + (b^2 + 4x_0^2)\ln\frac{b^2 + x_0^2}{b^2 + 4x_0^2}\Big\}, \tag{4}$$

where we introduce the effective potential at a fixed scale, namely $\Omega_g(x_0;b)$. Here $x_0 = q_0^2/\Lambda_{YM}^2$ is the dimensionless UV cut-off while $b = \mu^2/\Lambda_{YM}^2$ and the constant $a = (3/4) - 2\ln 2$. Let us now investigate asymptotic properties of the effective potential (4) as a function of b and x_0. In the $b \to 0$, $x_0 \to \infty$ limits to-leading order, one has

$$\Omega_g(x_0;b) \sim_{b\to 0, x_0\to\infty} \frac{1}{8\pi^2}(3 - 2\ln 2)b^2, \tag{5}$$

which clearly shows that the effective potential as a function of b approaches zero from above and as a function of the UV cut-off itself x_0 goes to positive constant at infinity. Its asymtotics at $b \to \infty$ and $x_0 \to 0$ to-leading order can be easily calculated as

$$\Omega_g(x_0;b) \sim_{b\to\infty, x_0\to 0} \frac{1}{8\pi^2}(8\ln 2 - 3)x_0^2. \tag{6}$$

The effective potential $\Omega_g(x_0;b)$ (4) is monotonically increasing (up to some constants) function of both b and x_0, always remaining positive without extremes (maximums and minimums) and zeros. Our main conclusion is that in the above discussed model vacuum is trivial and that is why it is inconsistent with quark confinement and DCSB and vice versa.

References

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