Systematics in the masses of excited baryons and deformed oscillator quark model

M. Takayama,[a] A. Hosaka,[b] and H. Toki[a]

[a]*Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan*
[b]*Numazu College of Technology, Numazu, Shizuoka 410-0022, Japan*

Until now about 80 baryons are observed and their masses, spins and paritis are known. There are sufficient data to perform a systematic study from these experimental data. In this report, we study masses of all known flavor $SU(3)$ baryons.

For the flavor octet and decouplet baryons in the ground state, the Gell-Mann-Okubo mass formula is well known. This formula implies that the flavor dependence of masses of ground state baryons is explained by the mass difference of the masses of constituent quarks. We apply this idea to all baryons including excited states and attempt to find hidden systematics in the masses of flavor $SU(3)$ baryons.

First, we classify baryon excited states according to the representations of $SU(2)_s \times SU(3)_f$. We measure excitation masses from the ground states which belong to the same spin-flavor multiplets as the corresponding excited states. This normalization is an extension of the Gell-Mann-Okubo mass formula and it would remove the major part of flavor dependence of the masses of excitated baryons. For the multiplets which do not have their ground states, we assume two kinds of mass shifts of 200 MeV. One is for the multiplets which are assigned as spin quartet. The value of 200 MeV is extracted from $\Sigma(1385) - \Sigma(1189)$ or $\Xi(1530) - \Xi(1317)$. The origin of this mass shift is considered to be the spin-spin interaction. The other is for the multiplets which are assigned $L = 0$ and spin 1/2 with a spatially mixed-symmetric orbital wave function. This would be caused by a δ-function type interaction in order to split the spatially symmetric and mixed-symmetric states. After these manipulations, one can find very simple systematics in the masses of excited baryons as shown in Fig 1.

Further investigation of this plot tells us more information about properties of baryons. For example, since the spin degeneracy are found, we can describe the behavior of baryons by orbital angular momentum L rather than total spin$J$, where $J = L + S$. Level spacing structure in terms of L is similar to rotational band of deformed nuclei as suggested by Toki, Baduri [1, 2] for N and Λ resonance.

In the present work we consider that the deformation of baryons is particularly important to explain baryon masses and examine this possibility using the deformed oscillator quark model(DOQ). The DOQ model is an effective non-relativistic quark model with a deformed harmonic oscillator potential [3, 4]. The DOQ Hamiltonian takes the following form

$$H_{DOQ} = \sum_{i=1}^{3} \left(\frac{p_i^2}{2m_i} + \frac{1}{2} m_i \left(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2 \right) \right) - H_{c.m.}. \quad (1)$$

Here m_i are the masses of constituent quarks. We take $m_i \sim 300$ MeV for u,d and s quarks for simplicity. To investigate the effect of deformation exclusively, we do not consider other interactions such as spin or isospin dependent ones. The system of the Hamiltonian(1) has been worked out in [2, 3, 4]. The eigenvalues of (1) are given by $E_{int} = (N_x+1)\omega_x + (N_y+1)\omega_y + (N_z+1)\omega_z$, where N_x, N_y, N_z denotes oscillator quanta for oscillator excitations. For a given set of (N_x, N_y, N_z), we perform minimization of the energy E_{int} with respect to the oscillator parameters under the volume conservation condition $\omega_x\omega_y\omega_z = \omega^3 = const.$ We found excited states are strongly deformed while ground states are spherical. These deformed states break rotational symmetry, which can be recovered by rotating (cranking) the deformed states as known in nuclear physics [5]. The resulting masses of rotational excitations are given by

$$E^L_{DOQ} = E_{int} + \frac{L(L+1)}{2\mathcal{I}} - \frac{\langle L^2 \rangle}{2\mathcal{I}} \quad . \tag{2}$$

The second term represents the rotational energy and the third term is the energy subtraction due to angular momentum fluctuation. We fix the oscillator parameter ω_0 =644MeV to reproduce the empirical energy difference between ground state and the first $1/2^+$ excited state in $^2 8_S$ multiplet [3].

We compare the DOQ spectrum (2) with empirical data in Fig 1. We emphasize that we plot all known baryons which have three or four stars in Particle Data Group [6] and that almost all states fall into the DOQ systematics. The remarkable agreement between the experimental and the DOQ spectrum is a strong basis which supports the spatial deformation of excited baryons.

It is highly important that not only masses but also various transition processes should be investigated to further confirm this intereting possibility of deformed baryons.

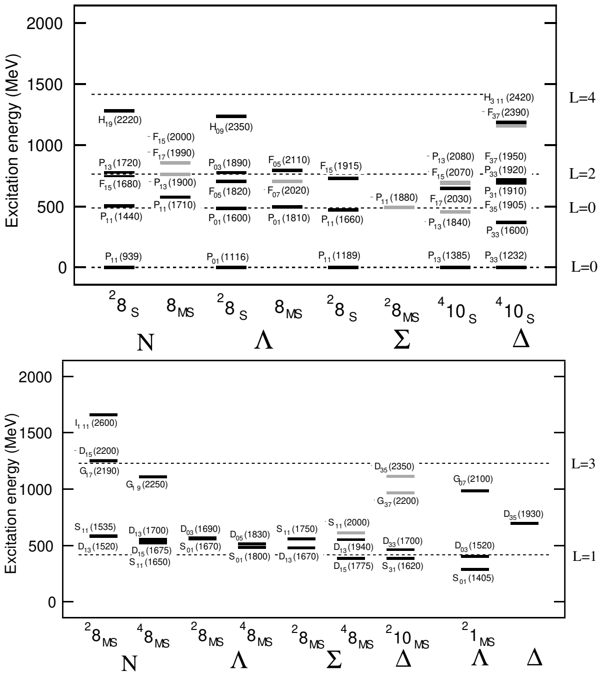

Fig. 1 : Observed mass spectrum of even and odd parity baryons as compared with the DOQ predictions (dashed lines). All states which have three or four stars in Particle Date Table [6] and some of one or two star states are plotted. The masses are measured from the ground state of each spin-flavor multiplet. For the channels without ground state, we shift masses 200 MeV downward which is caused by the spin-spin interaction. Further details are described in the text.

Fig. 1

References

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