The P^2 Expansion Method for the Bethe-Salpeter Approach and its Application to Scalar Meson

T. Sakai,[a] K. Kusaka,[b] and H. Toki[a,b]

[a] *Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan*
[b] *Institute for Chemical and Physical Research (RIKEN), Wako, Saitama 351-0106, Japan*

The coupled approach with Dyson-Schwinger (DS) and Bethe-Salpeter (BS) equations is a useful tool for non-perturbative investigations of properties of mesons, i.e. mass spectra, their structures and so on. However, there has been a technical problem in solving the BS equations for mesons. We need quark propagator in the time-like region to solve the BS equations directly, while what we can know by solving the DS equation for quarks is only the quark propagator in the space-like region. As a result of this difficulty, we have to make some extrapolation from calculations for space-like momenta to time-like momenta. There have been two methods to do such extrapolations. One is to extend the quark propagator to the whole complex plane of momentum space. Another is to extrapolate the fictitious eigenvalue of the BS equation from the space-like region to the point where the eigenvalue becomes one.

The third method we proposed[1] is similar to the latter. The idea is to expand all the ingredients in the BS equation by the total momentum the system carries, P^2, which can be done systematically. Then we solve the series of expanded BS equations order by order, and obtain the coefficients of the eigenvalue for each order term. Once we obtained the eigenvalue as a power series of P^2 up to the necessary order, the meson mass is obtained as the point which makes the eigenvalue one. This expansion may be justified especially for confined system like in QCD, because the eigenvalues in such systems cannot have the break-up thresholds. It is clearly contrasted with the well-known Wick-Cutkosky model[2].

In the previous work[1], we have adapted this P^2 expansion method to the BS equation in the ladder approximation for the π-family mesons, which have the parity $\Pi = -(-)^J$ and the charge conjugation $C = -\Pi$, using the gluon propagator of the dual Ginzburg-Landau (DGL) theory. From the calculation up to $O(P^2)$, the Regge trajectory of the meson mass spectrum is well reproduced. This success supports that the DGL is a good effective theory to describe the low energy hadron physics, in addition to the reproduction of the linear potential between heavy quarks and chiral properties such as constituent quark mass and pion decay constant[3].

Encouraged from these successes, we have investigated the scalar meson through the above expansion method. The BS amplitude χ of the scalar meson with $\Pi = +$ and $C = +$ is written up to order $O(P^2)$ as

$$\begin{aligned}\chi(p,P) &= S^0(p^2) - iP^0(p^2)\not{p} - \frac{1}{2}T^1(p^2)(\not{p}\not{P} - \not{P}\not{p}) + S^{2\mathrm{A}}(p^2)P^2 - iS^{2\mathrm{B}}(p^2)(p\cdot P)^2 \\ &\quad -iP^{2\mathrm{A}}(p^2)P^2\not{p} - iP^{2\mathrm{B}}(p^2)(p\cdot P)^2\not{p} - iP^{2\mathrm{C}}(p^2)(p\cdot P)\not{P}\end{aligned}$$

in the Euclidean metric. Here, the numbers in the superscripts of $S^0(p^2)$, $P^0(p^2)$, $T^1(p^2)$, $S^{2\mathrm{A}}(p^2)$, $S^{2\mathrm{B}}(p^2)$, $P^{2\mathrm{A}}(p^2)$, $P^{2\mathrm{B}}(p^2)$ and $P^{2\mathrm{C}}(p^2)$ denote the order of $\sqrt{P^2}$. Solving the BS equation up to $O(P^2)$, we have obtained the mass of the scalar meson and the above functions. To reduce the burden of the numerical calculation, Higashijima-Miransky approximation is used and the dependence of the gluon propagator on the angle variable is truncated to the lowest order in the same manner in Ref.[1].

In the chiral limit and using the gluon propagator of the DGL theory, we have obtained the scalar meson mass, 1.24 GeV. Jain and Munczek [4] give very low mass for the scalar meson, 0.59 GeV, although their calculation uses other expansion method for the BS amplitude, i.e. by Tschebyshev polynomials:

$$\begin{aligned} \chi_{\rm S} &= \chi_{\rm S0} + \not{p}\chi_{\rm S1} + \not{P}\chi_{\rm S2} + [\not{p},\not{P}]\chi_{\rm S3}, \\ \chi_{{\rm S}i} &= \sum_n \chi_{{\rm S}i}^{(n)}(p^2,P^2)T^{(n)}(\hat{p}\cdot\hat{P}). \end{aligned} \tag{1}$$

This large deviation between estimated meson masses may be ascribed to the difference in the infrared part of the gluon propagator, but the sufficiency of the order of expansion also should be checked. In Fig.1, the functions in the zeroth order BS amplitude, $S^0(p^2)$ and $P^0(p^2)$, are shown. $S^0(p^2)$ has a node at the point $M^2(p^2) = p^2$, where $M(p^2)$ is the mass function of quark. The behavior of $S^0(p^2)$ is very similar to the corresponding function $\chi_{\rm S0}$ in eq.(1).

Once the BS amplitudes of the relevant mesons are given, investigation of meson decay process like $M_1 \to M_2 M_3$ is possible[5], where M_1, M_2 and M_3 denote some mesons. That will be done in the near future.

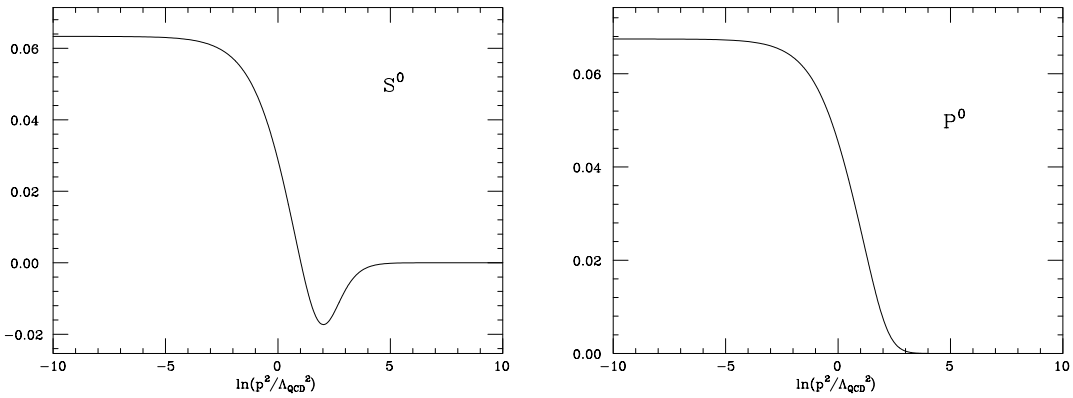

Figure 1: The functions S^0 and P^0 are shown as functions of $\ln(p^2/\Lambda_{\rm QCD}^2)$. The magnitude is in arbitrary scale, but the relative magnitude between S^0 and P^0 is correctly given, being made dimensionless with the help of $\Lambda_{\rm QCD}$.

References

[1] K. Kusaka, T. Sakai and H. Toki, Prog. Theor. Phys. **101** (1999) 721.
[2] G.C. Wick, Phys. Rev. **96** (1954) 1124; R.E. Cutkosky, *ibid.* 1135.
[3] H. Suganuma, S. Sasaki and H. Toki, Nucl. Phys. **B435** (1995) 207.
S. Sasaki, H. Suganuma and H. Toki, Prog. Theor. Phys. **94** (1995) 373.
[4] P. Jain and H.J. Munczek, Phys. Rev. **D48** (1993) 5403.
[5] P.C. Tandy, Prog. Part. Nucl. Phys. **39** (1997) 117.