A Study on High Performance Quadrupole Magnet for an Accelerator

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In recent years, many synchrotron-type accelerators have been constructed not only for physics but also for technology and medical care. Under this condition, cost performance of a shnchrotron becomes required. One of the answer is a smaller synchrotron, which requires accelerator elements to develop qualitatively.

In a quadrupole magnet case, a thin magnet is strongly required. This request is, however, not so easy because edge effect to a quadrupole field must be considered when a quadrupole magnet is designed, which is not experienced. In other words, usual way to determine the shape of the pole of a quadrupole magnet, i.e., empirical way or two dimensional computer calculations, must be not enough. Therefore, as a start point of a magnetic pole shape, the solution of three dimensional Laplace's equation should be adopted[1]. Although the final shape of the pole may be a little bit changed because both of presence of iron and of magnetic hysterisis, it should be possible to determine the pole shape experimentally, if the shape at the start point(i.e. the solution of three dimensional Laplace's equation) is not far away from the best one.

The magnetic scalar potential, ϕ, for a quadrupole magnet is assumed as[1]

$$\phi(r,\theta,z) = -\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} r^n b_m \sin(m\theta) g_n(z). \tag{1}$$

Since the potential should be the solution of three dimensional Laplace's equation, i.e., $\triangle\phi = 0$, we got the following relationship,

$$g_n(z) = -\frac{g''_{n-2}(z)}{(n+2)(n-2)} (n \geq 3). \tag{2}$$

The equation to represent the pole shape of a quadrupole magnet is

$$\frac{1}{\sin(2\theta)} = -2\sum_{\mu=1}^{\infty} k(\mu) g_2^{(2\mu-2)}(z) \frac{r^{2\mu}}{R^2}, \tag{3}$$

where R is a bore radius at the center of the quadrupole magnet and

$$k(\mu) = \frac{(-1)^\mu}{4^{\mu-1}(\mu+1)!(\mu-1)!}. \tag{4}$$

The function $g_2(z)$ was adopted as

$$g_2(z) = \frac{1}{2}\frac{erf(\frac{z+z_0}{\sigma}) - erf(\frac{z-z_0}{\sigma})}{erf(\frac{z_0}{\sigma})}, \tag{5}$$

where $2z_0$ is a measure of the pole length and σ is a measure of the stray magnetic field extension.

Though right hand of eq.(3) includes infinite terms, we calculated it mathematically until $\mu = 5$. The result shows us that the outline of the pole shape (1)is similar to hyperbolic

curve at the center, (2)has some bulge compared to the hyperbolic curve in the intermediate region and (3) is different from the hyperbolic curve near the pole edge.

To ensure this conceptual idea, a normal-type quadrupole magnet (i.e. "thick" magnet) with widely adjustable edge piece was prepared. Before experiments, three dimensional magnetic field computational calculation by the TOSCA code[2] which includes an iron effect was carried out for two cases, i.e., (1)no further manufacturing of the edge pieces and (2)manufacturing based on eq.(3). The comments will be shown with the experimental results.

For a quadrupole magnet, so-called "GL product" is important, which have been directly measured by a moving search coil method in this study. Some preliminary GL product measurements were carried out with no change of the adjustable edge pieces. Experimental results are quite different from the calculations by the TOSCA code. It should be noticed that the calculational result in the case of the manufacturing based on eq.(2) is also different from the desirable results of the concept of the pole shape from the solution of three dimensional Laplace's equation, i.e., almost constant derivative of the GL product. Thus the calculations by the TOSCA code may be inadequate in this case by some reasons.

Now the edge pieces of the magnet with the shape based on the analytical solution of the three dimenstional Laplace's equation have been made and the magnetic field measurements are under way.

References

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[2] Vector Field Ltd., "OPERA-3d REFERENCE MANUAL", VF-09-96-D4