Current quark mass effects on the chiral phase transition of QCD in the improved ladder approximation

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At zero temperature and zero baryon number density, the chiral symmetry is dynamically broken. It is generally believed, however, that at high temperature and/or density the QCD vacuum undergoes a phase transition into a chirally symmetric phase. This chiral phase transition plays an important role in the physics of neutron stars and the early universe and it may be realized in high-energy heavy-ion collisions. In this paper we study the phase structure of QCD using the improved ladder approximation. We calculate the Cornwall–Jackiw–Tomboulis (CJT) effective potential [1] at finite temperature and chemical potential and investigate the chiral phase transition. Note that in this report we fix the mass scale of our model by the condition of $\Lambda_{\rm QCD} = 1$, otherwise stated.

At $T = \mu = 0$, we derive the modified form of the Cornwall-Jackiw-Tombpulis effective potential [1] in the Landau gauge and in the improved ladder approximation with the Higashijima-Miransky approximation [2, 3] as follows

$$V = -2\int \frac{d^4 p_{\rm E}}{(2\pi)^4}\,\ln\frac{\Sigma^2(p_{\rm E})+p_{\rm E}^2}{p_{\rm E}^2} - \frac{2}{3C_2}\int dp_{\rm E}^2\,\frac{1}{\frac{d}{dp_{\rm E}^2}\mathcal{D}(p_{\rm E})}\left(\frac{d}{dp_{\rm E}^2}\Sigma(p_{\rm E})\right)^2,$$

where an overall factor $N_f N_c$ (the number of light quarks times colors) is omitted, $C_2 = (N_c^2-1)/(2N_c)$ is the quadratic Casimir operator for SU(N_c), $\mathcal{D}(p_{\rm E}) \equiv \bar{g}^2(p_{\rm E})/p_{\rm E}^2$ and $\Sigma(p_{\rm E})$ is a dynamical mass function of the quark. The extremum condition for V with respect $\Sigma(p_{\rm E})$ leads to the following differential equation which is equivalent to the Schwinger-Dyson equation:

$$\frac{\Sigma(p_{\rm E})}{\Sigma^2(p_{\rm E})+p_{\rm E}^2} = \frac{(4\pi)^2}{3C_2}\frac{d}{p_{\rm E}^2 dp_{\rm E}^2}\left(\frac{1}{\frac{d}{dp_{\rm E}^2}\mathcal{D}(p_{\rm E})}\frac{d\Sigma(p_{\rm E})}{dp_{\rm E}^2}\right)$$

apart from two boundary conditions.

At finite temperature and chemical potential, we use the imaginary time formalism:

$$\int\frac{dp_4}{2\pi}f(p_4) \to T\sum_{n=-\infty}^{\infty} f(p_4 = (2n+1)\pi T + i\mu)\ ,\ n \in \boldsymbol{Z},$$

and the following "real" functions for $\mathcal{D}_{T,\mu}(p_4,\boldsymbol{p})$ and $\Sigma_{T,\mu}(p_4,\boldsymbol{p})$.

$$\begin{aligned}\mathcal{D}_{T,\mu}(p_4,\boldsymbol{p}) &= \frac{2\pi^2 a}{\ln(\omega_n^2+\boldsymbol{p}^2+p_R^2)}\,\frac{1}{\omega_n^2+\boldsymbol{p}^2},\\ \Sigma_{T,\mu}(p_4,\boldsymbol{p}) &= m_R[\ln(\omega_n^2+\boldsymbol{p}^2+p_R^2)]^{-a/2}\\ &\quad+\frac{\sigma}{\omega_n^2+\boldsymbol{p}^2+p_R^2}[\ln(\omega_n^2+\boldsymbol{p}^2+p_R^2)]^{a/2-1},\end{aligned}$$

where ω_n is the fermion Matsubara frequency, m_R is a renormalization group invariant quark mass, σ is an order parameter of chiral symmetry, p_R is the infrared regularization parameter

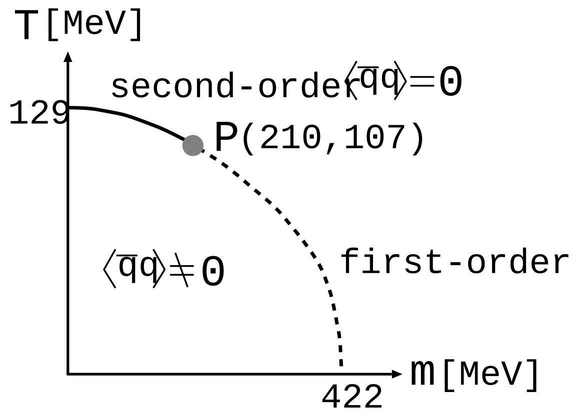

Figure 1: Schematic view of the phase diagram in the chiral limit.

and $a = 8/9$. We can determine the value of $\langle \bar{q}q \rangle$ through the relation $\langle \bar{q}q \rangle = -(3/2\pi^2 a)\sigma_{min}$, where σ_{min} is the location of the minimum of V.

In numerical calculations, we put $\ln(p_R^2) = 0.1$ and determine the value of $\Lambda_{\rm QCD}$ by the condition of $f_\pi = 93$ MeV at $T = \mu = 0$, $m_R = 0$. Then, we have $\Lambda_{\rm QCD} = 738$ MeV. Note that we use the Pagels–Stoker formula [4] for f_π. In the chiral limit, we have the second-order phase transition at $T_c = 129$ MeV, $\mu = 0$ and first-order one at $\mu_c = 422$ MeV, $T = 0$. The T-μ phase diagram is shown in Fig. 1 and the tricritical point 'P' is found at $(T_P, \mu_P) = (107, 210)$ MeV [5]. For $m(1\text{GeV}) = 7$ MeV case, the position of the critical end point is found

$$T_E \sim 95 \text{ MeV} \ , \quad \mu_E \sim 300 \text{ MeV}.$$

In this paper, we studied the current quark mass effects on the chiral phase transition of QCD at high temperature and density in the improved ladder approximation. We found the tricritical point on the phase diagram. The location of the tricritical point is approximately coincident with that found in the other models [6, 7]. We also studied the phase diagram with explicit chiral symmetry breaking and examined the position of the critical end point. The physics near the critical end point is interesting in the light of the high-energy heavy-ion collision experiments.

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