Color superconductivity at finite density and temperature with flavor asymmetry

O. Kiriyama, S. Yasui, and H. Toki

Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan

Quantum Chromodynamics (QCD) describes the interaction of quarks and gluons. Since QCD is asymptotically free, the coupling constant becomes small when either temperature or the chemical potential, namely the the Fermi momentum, is large. By Cooper theorem, Fermi surface has instability when an arbitrarily small attractive interaction between quarks presents. For quark–quark scattering, where the quarks are in the antitriplet $\bar{\mathbf{3}}$, one gluon exchange interaction is attractive and in the $\mathbf{6}$, it is repulsive. Thus we expect that, in the case of two quark flavors, the quark–quark condensate in $\bar{\mathbf{3}}$ channel is formed and the color superconductivity, spontaneous breakdown of the color gauge symmetry $SU(3)_C \to SU(2)_C$, occurs [1].

We consider the color superconductivity under the situation where a flavor asymmetry takes place. The color superconductivity at finite flavor asymmetry has been studied in Ref. [2, 3]. Especially, in Ref. [3], it has been argued that a mixed phase of the normal and the superconducting phase exists with a sufficiently large flavor asymmetry. In this paper, we study at finite temperature and discuss what extent does the mixed phase persist. Such an investigation has importance in studies of neutron stars and, of course, high-energy heavy-ion collisions at Alternating Gradient Synchrotron (AGS) and CERN Super Proton Synchrotron (SPS).

The study of the color superconductivity in QCD is nontrivial and, therefore, we choose a tractable model to describe the quark–quark condensate. The most familiar of such models is known as the NJL (-type) model. Particularly, in the leading order of the $1/N_c$ expansion, the resultant gap function is independent of momentum and it makes our analysis simple one. Moreover, the structure of the color superconducting phase depends on the number of the quark flavors. At extremely high density, where we can neglect the mass of s quark, the color-flavor locked phase appears [4]. However we expect, at relatively low density, there is a window of two–flavor color superconducting phase. Such being the case, we use the two–flavor, three–color NJL-type model.

The $SU(2)_L \times SU(2)_R$ chially symmetric Lagrangian [5] is as follows

$$\mathcal{L} = \bar{\psi} i\gamma \cdot \partial \psi + G_1 \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau} \psi)^2 \right].$$

The thermodynamical potential is obtained as

$$\begin{aligned} V(\Delta; T, \mu_B, \mu_I) &= \frac{\Delta^2}{4G_2} - 2\int^\Lambda \frac{q^2 dq}{2\pi^2} \Big\{ \epsilon_-(q) + \epsilon_+(q) \\ &+ T\ln\left(1 + \exp\left[-\beta(\epsilon_-(q) - \mu_I)\right]\right) + T\ln\left(1 + \exp\left[-\beta(\epsilon_-(q) + \mu_I)\right]\right) \\ &+ T\ln\left(1 + \exp\left[-\beta(\epsilon_+(q) - \mu_I)\right]\right) + T\ln\left(1 + \exp\left[-\beta(\epsilon_+(q) + \mu_I)\right]\right) \Big\} \\ &- 4\int^\Lambda \frac{q^2 dq}{2\pi^2} \Big\{ E_-(q) + E_+(q) \\ &+ T\ln\left(1 + \exp\left[-\beta(E_-(q) - \mu_I)\right]\right) + T\ln\left(1 + \exp\left[-\beta(E_-(q) + \mu_I)\right]\right) \\ &+ T\ln\left(1 + \exp\left[-\beta(E_+(q) - \mu_I)\right]\right) + T\ln\left(1 + \exp\left[-\beta(E_+(q) + \mu_I)\right]\right) \Big\}, \end{aligned}$$

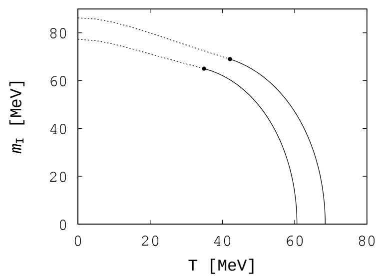

Figure 1: Phase boundaries in the T-μ_I plane for $\mu_B = 500$ MeV (upper line) and 400 MeV (lower line). The solid lines describe the transition of second order and the dashed line describe that of first order. Points indicate tricritical points.

where $\mu_B = (\mu_u + \mu_d)/2$, $\mu_I = (\mu_u - \mu_d)/2$,

$$\epsilon_\pm(q) = |\boldsymbol{q}| \pm \mu_B$$

are the gapless (color 3) quasi-particle energies relative to the (averaged) Fermi surface,

$$E_\pm(q) = (|\boldsymbol{q}| \pm \mu_B)\sqrt{1 + \frac{\Delta^2}{(|\boldsymbol{q}| \pm \mu_B)^2}}$$

are the gapful (color 1, 2) quasi-particle energies, G_2 represents the coupling strength in diquark channel and 3D cutoff Λ is introduced to regularize the divergent integrals. In this paper, we use $G_1 = 5.01\text{GeV}^{-2}$, $\Lambda = 0.65$ GeV and $G_2 = 3G_1/4$.

The phase boundaries in the T-μ_I plane for $\mu_B = 400,\ 500$ MeV are shown in Fig. 1. The most favorable condition for the color superconductivity, cold and dense quark matter, are at the core of neutron stars. However, the values of T, μ_B and μ_I accomplished in heavy-ion collisions at, for example, AGS and SPS, maybe reach into the mixed phase, since it lies up until $T \simeq 40$ MeV (see Fig. 1). Then, if the charged bubbles is formed, it may lead to observable effects, e.g., an obvious change of the π^-/π^+ ratio.

References

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