Casimir scaling in Abelian-projected SU(3) gluodynamics

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The observation of Casimir scaling is an important argument in any discussion of the virtues of different QCD vacuum models as far as the respective confinement mechanism is concerned. Taken literally, the Casimir scaling suggests that the potential at intermediate distances between static charges in different representation are proportional to the eigenvalues $C^{(2)}(D)$ of the quadratic Casimir operator T^aT^a in the respective D dimensional representation, such that $F_{D_1}(r)/F_{D_2}(r) = C^{(2)}(D_1)/C^{(2)}(D_2)$ at all distances. This property is obvious only for the one-gluon exchange component of the static force. Although there is no asymptotically linearly rising potential for the higher representations, at intermediate distances a string tension can be defined which enters $F_D(r)$ as a constant part. For example, this property suggests that the ratio of string tensions of adjoint to fundamental charges in SU(3) gauge theory, respectively, would be $\sigma_{\rm adj}/\sigma_{\rm fund} = 9/4 = 2.25$.

Recently, as a contribution to the discussion of competing confinement mechanisms, Ref. [1] appeared where the string tensions of the fundamental and higher representations have been calculated in pure SU(3) lattice gauge theory, and the ratio was obtained nearly equal to 2, already rather close to 9/4. In Ref. [2] Bali has studied the ratios of entire interaction potentials (including Coulomb and constant terms in addition to the linear term) also for quenched SU(3) gauge theory, and in the case of adjoint and fundamental charges the ratio turned out to be very close to 9/4. All detailed (microscopic) mechanisms of confinement find it hard to explain the Casimir scaling, while it appears more natural from the point of view of the semi-phenomenological stochastic vacuum model [3]. If the confinement mechanism is described by center vortices, approximate Casimir scaling for the potential can be achieved by introducing a finite thickness of the vortex, as demonstrated for the case of SU(2) lattice gauge theory [4], although the original center vortex picture gives a strictly vanishing potential for pairs of charges which transform trivially under Z_N center of the gauge group.

For the dual superconductor scenario of confinement [5, 6], practically realized in the form of the dual Ginzburg-Landau (DGL) theory [7], one tends to believe that it would be difficult to accommodate Casimir scaling in this framework. Indeed, in the Abelian projection scheme [8] for SU(3) gluodynamics the long range forces are transmitted only by "diagonal gluons" which couple to charges only via T^3 and T^8. This makes it hard to understand why the Casimir scaling should hold in Abelian projected gluodynamics. For example for the ratio between adjoint and fundamental forces one would naively expect the Abelian ratio equal to 3.

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As far as the derivation of the DGL theory is based on the Abelian projected gluodynamics, this seems to be unavoidable in the DGL theory, too. Moreover, considering the DGL theory just at a phenomenological level, it would be sufficient to restrict it exclusively to mesonic, baryonic, glueball [9] and perhaps to exotic states, and it would be inappropriate to apply it to the so-called gluelump bound states made of infinitely heavy adjoint charges.

We have found however that the DGL theory is rather promising to discuss the Casimir scaling problem without extra effort [10]. To see this, we recall that the DGL theory represents the mesonic string as degenerate $R-\bar{R}$, $B-\bar{B}$ and $G-\bar{G}$ colored states. In the same spirit it is natural to represent gluelump strings as stretching out between pairs of adjoint charges, each of them being made out of quark and antiquark as $B\bar{G}-\bar{B}G$, $G\bar{R}-\bar{G}R$, and $R\bar{B}-\bar{R}B$ states. Thus, it is rather a string formed by two pairs with their respective Dirac strings superposed. In the Bogomol'nyi limit one directly gets the ratio $\sigma_{\rm adj}/\sigma_{\rm fund}$ using the manifest Weyl invariant formulation of the DGL theory [11, 12]. In this limiting case the ratio is equal to 2, reflecting the presence of two independent color-electric Dirac strings inside the adjoint flux tube. For more higher representations we have also found that the ratio is simply equal to the sum of Dynkin index of corresponding representations. Entering the type-II dual superconductor parameter range, the ratio increases, while decreasing towards the type-I region. Our studies show that the ratio of string tensions depends only on the ratio between dual vector and monopole mass, via $\kappa = m_\chi/m_B$. It has been conjectured that the ratio 9/4 is reproduced in a certain type-II vacuum around $\kappa = 5$.

References

[1] S. Deldar, Phys. Rev. **D62**, 034509 (2000).

[2] G. S. Bali, Phys. Rev. **D62**, 114503 (2000).

[3] V. I. Shevchenko and Y. A. Simonov, Phys. Rev. Lett. **85**, 1811 (2000).

[4] M. Faber, J. Greensite, and S. Olejnik, Phys. Rev. **D57**, 2603 (1998).

[5] Y. Nambu, Phys. Rev. **D10**, 4262 (1974).

[6] S. Mandelstam, Phys. Rept. **23C**, 245 (1976).

[7] T. Suzuki, Prog. Theor. Phys. **80**, 929 (1988).

[8] G. 't Hooft, Nucl. Phys. **B190**, 455 (1981).

[9] Y. Koma, H. Suganuma, and H. Toki, Phys. Rev. **D60**, 074024 (1999).

[10] Y. Koma, E.-M. Ilgenfritz, H. Toki, and T. Suzuki, Phys. Rev. **D64**, 011501 (R) (2001).

[11] Y. Koma and H. Toki, Phys. Rev. **D62**, 054027 (2000).

[12] Y. Koma, E.-M. Ilgenfritz, T. Suzuki, and H. Toki, Phys. Rev. **D64**, 014015 (2001).