**Semiclassical Trace Formulas in Terms of Phase Space Path Integrals**

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The semiclassical trace formula developed by Gutzwiller [1] is one of the most important tools to study the spectrum of non-integrable systems. Shell fluctuations of quantum many-body systems like nuclei are related to short classical periodic orbits in mean fields by this formula [2]. However, the original derivation by Gutzwiller is very complicated, and the canonical invariance of the formula (especially the Maslov index) is unclear.

In this report, we explain geometrical structure of this formula following our recent works [3, 4]. We start with the phase space path integral of the partition function:

$$Z(T) = \int \mathcal{D}\boldsymbol{p}\mathcal{D}\boldsymbol{q} \exp\left[\frac{i}{\hbar}\oint(\boldsymbol{p}d\boldsymbol{q} - H\,dt)\right]. \tag{1}$$

The density of states is obtained as the Fourier-Laplace transformation of the partition function:

$$\rho(E) = -\frac{1}{\pi}\mathrm{Im}g(E + i\epsilon), \tag{2}$$

$$g(E) \;=\; \frac{1}{i\hbar}\int_0^\infty dT e^{iET/\hbar} Z(T). \tag{3}$$

We obtain the semiclassical trace formula by applying the stationary phase approximation to the path integral.

The stationary condition leads to the Hamiltonian equation of motion, and the periodic boundary condition of the path integral (1) has the effect of closing the paths. Therefore the the partition function is approximated by a sum over classical periodic orbits:

$$Z(T) = \sum_{p.o} K \exp\left[\frac{i}{\hbar}R\right]. \tag{4}$$

Here, $R = \oint \boldsymbol{p}d\boldsymbol{q} - H dt$ is the classical action of the periodic orbit, and $K$ is the contribution of the quadratic fluctuation around the orbit:

$$K = \int \mathcal{D}\boldsymbol{x} \exp[i\delta^2 R[\boldsymbol{x}(t)]], \tag{5}$$

$$\boldsymbol{x}(t) = \frac{1}{\sqrt{\hbar}}(\delta\boldsymbol{q}, \delta\boldsymbol{q}). \tag{6}$$

The Maslov index appears as the phase factor of this quadratic path integral. Since this is a kind of Fresnel integral, the phase is determined by the signs of diagonal elements of the quadratic form $\delta^2 R$.

The main idea of our works is that the Maslov index of the periodic orbit is determined by the linearized symplectic flow around the orbit. The set of displacement vectors $\{\boldsymbol{x}(t)\}$ is considered to be a vector bundle over $S^1$, and the flow around the orbit define a connection (Fig. 1). Actually the quadratic path integral (5) can be written as a gauge theory

$$K = \int \mathcal{D}\boldsymbol{x} \exp\left[\frac{i}{2}\boldsymbol{x}^T \not{D}\, \boldsymbol{x}\right]. \tag{7}$$

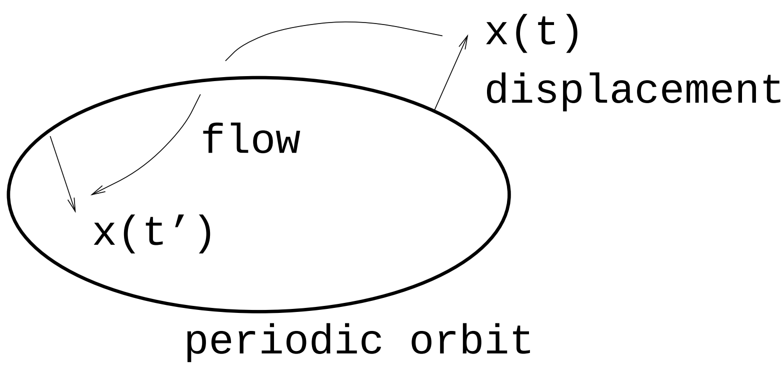


Figure 1: The set of displacement vectors is regarded as a fibre bundle, and Hamiltonian symplectic flow around the orbit defines the connection. The structure group of this space is $Sp(2n, R)$, where $n$ is the number of degrees of freedom of the system.

Here,

$$\not{D} = JD, \tag{8}$$

and $D$ is the covariant derivative

$$D = \frac{d}{dt} + A(t) = \frac{d}{dt} + JH''(t). \tag{9}$$

$J$ is a $2n \times 2n$ matrix which defines the symplectic product:

$$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}. \tag{10}$$

Mathematically speaking, our problem is the classification of connections on the fibre bundle. However, the quadratic path integral around the periodic orbit is not invariant to all canonical transformations. If the canonical transformation is topologically non-trivial, the path integral may change the sign. The toplogically non-trivial transformations do exist because the fundamental group of $Sp(2n, R)$ is non-trivial:

$$\pi_1(Sp(2n, R)) = Z. \tag{11}$$

This is essentially the same as the global anomalies of the gauge field theories [5, 6]. Therefore we regard two connections as equivalent if they are connected by a topologically trivial canonical transformation, and classify the connections by this equivalence relation.

This anomaly can be seen even in simple harmonic oscillators. For the details, see [4].

## References

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