Ground-state Three-quark Potential in SU(3) Lattice QCD

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The strong interaction in hadrons and nuclei is fundamentally governed by Quantum ChromoDynamics(QCD). For the deep understanding of the strong interaction, it would be desirable to extract strong interaction directly from QCD. However it still remains difficult to construct hadron physics based on QCD. For an important and fundamental example, it has not been successful to derive the quark-antiquark($\mathrm{Q\bar{Q}}$) potential, which is directly responsible for the meson properties. This is due to the strong coupling nature in the infrared region of QCD. For the infrared physics, we cannot apply the perturbative calculations, and hence the non-perturbative analysis is needed.

Lately, the lattice QCD calculation has been accepted as a powerful and reliable method for the non-perturbative analysis. For instance, the $\mathrm{Q\bar{Q}}$ potential ($V_{\mathrm{Q\bar{Q}}}$)has been well studied using lattice QCD[1, 2, 4], and has been found to be the form $V_{\mathrm{Q\bar{Q}}} = -\frac{A_{\mathrm{Q\bar{Q}}}}{r} + \sigma_{\mathrm{Q\bar{Q}}} r + C_{\mathrm{Q\bar{Q}}}$, where r denotes the distance between quark and antiquark. This potential can be split into two parts, the perturbative one-gluon-exchange term $\frac{A_{\mathrm{Q\bar{Q}}}}{r}$ and the non-perturbative confinement term $\sigma_{\mathrm{Q\bar{Q}}} r$. The linear confinement term in the long distance region supports the flux-tube picture. The one-dimensional flux-tube which has the tension $\sigma_{\mathrm{Q\bar{Q}}}$ links the quark and antiquark, and consequently the potential is proportional to r. The flux-tube picture is supported by the Regge trajectory of hadrons, the analysis of heavy quarkonia spectrum and the strong-coupling expansion of QCD. However, there is almost no reliable formula to represent the three-quark (3Q) potential (V_{3Q}) based on QCD. The three-quark potential, directly responsible for the baryon structure and properties[3], has been treated phenomenologically more than 20 years. We carry out the detailed study of the 3Q potential using SU(3) lattice at $\beta = 5.7$ (lattice unit $a \simeq 0.2$ fm), $\beta = 5.8$ ($a \simeq 0.14$ fm) and $\beta = 6.0$ ($a \simeq 0.1$ fm)[1, 2].

Theoretically, 3Q potential is considered to be described by a sum of the perturbative two-body term and the non-perturbative confinement term and to take a form as

$$V_{\mathrm{3Q}} = -A_{\mathrm{3Q}} \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{\mathrm{3Q}} L_{\mathrm{min}} + C_{\mathrm{3Q}}, \qquad (1)$$

where L_{min} denotes the minimal linking length of the flux-tube. Denoting the three side length of the 3Q triangle by a,b and c as shown in Fig.1, L_{min} is explicitly represented[1, 2] as

$$L_{\mathrm{min}} = \left[\frac{1}{2}(a^2+b^2+c^2) + \frac{\sqrt{3}}{2}\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)} \right]^{\frac{1}{2}}, \qquad (2)$$

when any angle of the 3Q triangle does not exceed $2\pi/3$. With one angle larger than $2\pi/3$, $L_{\mathrm{min}} = (a+b+c) - \max(a,b,c)$.

The static 3Q potential can be obtained from the 3Q Wilson loop. The 3Q Wilson loop is a color current in the four-dimensional Euclidean space-time, which is defined in a gauge-invariant manner as $W_{\mathrm{3Q}} \equiv \frac{1}{3!}\varepsilon_{abc}\varepsilon_{a'b'c'} U_1^{aa'} U_2^{bb'} U_3^{cc'}$, $\quad U_k = P\exp\{ig\int_{\Gamma_k} dx^\mu A_\mu(x)\} \in$

SU(3). Here, P denotes the path-ordered product along the path Γ_k in Fig.2. The 3Q potential is extracted as $V_{3\mathrm{Q}} = -\lim_{T\to\infty} \frac{1}{T} \ln\langle W_{3\mathrm{Q}}\rangle$. Practically, there are some difficulties in obtaining ground-state potential. We take enough large T to reduce the excited-state contaminations. However, $\langle W_{3\mathrm{Q}}\rangle$ also decreases exponentially and we cannot obtain the numerical signals. The previous studies of the 3Q potential using lattice QCD contained these contaminations and were not reliable. To avoid this difficulties, we adopt the smearing technique to enhance the ground-state overlap[1, 2, 4].

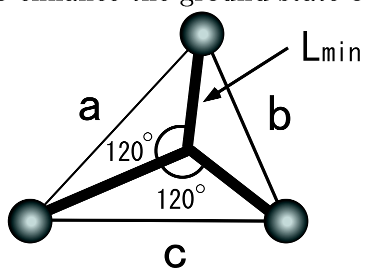

Figure 1: The flux-tube configuration of the 3Q system.

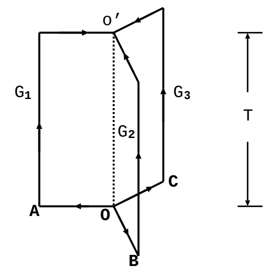

Figure 2: The 3Q Wilson loop.Γ denotes the path linking $\mathcal{O}$ and $\mathcal{O}'$.

In the following table, we show the fitted parameters, which are determined by fitting the lattice data by the form in Eq.1. By this best fit parameters, 3Q potential data can be reproduced with accuracy better than a few %.

	σ (GeV/fm)	A	C (lattice unit)
3Q ($\beta = 5.7$)	0.832(15)	0.1331(66)	0.9182(213)
$\mathrm{Q\bar{Q}}$ ($\beta = 5.7$)	0.890(25)	0.2793(116)	0.6203(161)
3Q ($\beta = 5.8$)	0.818(6)	0.1304(17)	0.9326(53)
$\mathrm{Q\bar{Q}}$ ($\beta = 5.8$)	0.890(21)	0.2580(159)	0.6081(182)
3Q ($\beta = 6.0$)	0.811(7)	0.1363(11)	0.9590(35)
$\mathrm{Q\bar{Q}}$ ($\beta = 6.0$)	0.890(12)	0.2768(24)	0.6374(30)

We proved remarkable features numerically based on QCD[1, 2].

- Perturbative color factor at the short distance ($A_{3\mathrm{Q}} \simeq \frac{1}{2} A_{\mathrm{Q\bar{Q}}}$)
- Universality of the string tension ($\sigma_{3\mathrm{Q}} \simeq \sigma_{\mathrm{Q\bar{Q}}}$)
- Consistency of the constant C in the di-quark limit ($C_{3\mathrm{Q}} \simeq \frac{3}{2} C_{\mathrm{Q\bar{Q}}}$)

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