Magnetic moment of a relativistic three-body system

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Non-relativistic constituent quark models (NRCQM) have been employed widely for the study of baryon properties, such as spectrum, form factors, and transition amplitudes. A typical success of the NRCQM is the magnetic moment of baryons. The magnetic moment in the NRCQM is just a sum of the magnetic moments of each constituent quark. Even in the simplest approach, where wave function has spin-flavor SU(6) symmetry and loosely binding limit (the nucleon mass M_N is three times of the constituent quark mass m_q), one could obtain the magnetic moment of proton and of neutron which is consistent with the experimental data within 15 % error (see Table 1). On the other hand, it is reported that the absolute values of the nucleon magnetic moment in the RCQM are too small.

In the RCQM [1, 2], the nucleon wave function is written as the products of three individual single-quark wave functions. When the center of mass motion should be removed from the wave function to obtain the nucleon spectrum, the Peierls-Yoccoz(PY) method has been usually adopted. In this method, one can get a state with a good total momentum. As we can see in Table 1, the values of the magnetic moment in their models are too small compared with the empirical one.

It is natural to think that the relativistic description is more reliable for hadrons, in particular for spin dependent observables such as magnetic moment since the magnetic moment is closely related to spin, whose origin is spinor structure of fermion in a relativistic formulation. Then, this result seems to suggest that the success of the NRCQM is just accidental and baryon is not provided as a simple three-quark composite system but more complicated object. However, it is also natural to regard the NRCQM as non-relativistic limit of RCQM. If so, we can expect that static properties in the RCQM should be similar to that of the NRCQM.

The purpose of the present study is to solve the discrepancy between the RCQM and the NRCQM. To do this, we investigate the relativistic three-body system with mass m including the boost effect for low momentum region [3]. First we write the wave function of the three particle system in the center of mass (CM) frame, then obtain the wave function boosted to a desired momentum frame. In this procedure, we will see that not only the relative coordinate, but also the Lorentz boost of the CM coordinate should be taken into account. This fact is not considered seriously in the usual treatment of the RCQM. After boosting both the relative and the CM degrees of freedom, we calculate the electromagnetic matrix element and extract the magnetic moment of the system. We find that the Lorentz boost of the CM degrees of freedom is important to take non-relativistic limit of the matrix element correctly, and to get the magnetic moment of the system.

As a result, with the boosted wave function, magnetic moment is just the sum of the magnetic moment of each particle as we have used in the NRCQM,

$$\boldsymbol{\mu} = \boldsymbol{\mu}_1 + \boldsymbol{\mu}_2 + \boldsymbol{\mu}_3.$$

When we use the wave function without boost on spinors as ordinary RCQM, the CM momentum $\mathbf{P}$ does not appear in the calculation of the matrix element. This leads to a small

Table 1: Magnetic moments of proton and neutron in both non-relativistic and relativistic model are summarized in units of $\mu_N = \frac{e}{2M_N}$.

	NRCQM	RCQM [1]	RCQM [2]	exp.[4]
μ_p	3	2	1.568	2.79
μ_n	-2	$-\frac{4}{3}$	-1.045	-1.91

magnetic moment as

$$\boldsymbol{\mu} = \frac{2}{3}\left(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2 + \boldsymbol{\mu}_3\right)$$

Note that lack of the Lorentz boost reduces the values of the magnetic moment by factor 2/3, which is considered to be the origin of the small magnetic moment in Refs [1, 2].

What we have done in this paper is the $\frac{1}{m}$ expansion of the matrix element of three-body system with electromagnetic interaction. On the other hand, the operator of the magnetic moment in NRCQM is obtained from the $\frac{1}{m}$ expansion of the electromagnetic interaction Hamiltonian, for instance, by using Foldy-Wouthuysen (FW) transformation. Then the consistency between our result and usual NRCQM treatment is natural, rather trivial [1].

We have also found that the wave function obtained by PY method coincides with our relativistic wave function within the first order of the total and the relative momentum after taking loosely bounding limit and non-relativistic limit. In fact, it is enough to reproduce the magnetic moment. However, in Refs.[1, 2], they used the wave function projected to $\mathbf{P}_T = 0$ so that the wave function does not reflect the CM motion. Then, they could not take into account correct kinetic condition of initial and final state, and missed the contribution of the CM motion in the magnetic moment. The wave function with $\mathbf{P}_T = 0$ must be used to obtain the correct spectrum, however, to calculate the electromagnetic properties one needs the wave function in arbitrary momentum frame since the system couples to an external source.

The present argument applies to physical quantities which are expressed as products of upper and lower components and are proportional to the momentum transfer $\vec{q}$. Examples of such quantities are the πNN coupling and axial charges which are the matrix element of $\bar{\psi}\gamma_5\psi$. In the previous calculations [2], those quantities were also underestimated.

To summarize, Lorentz boost of CM motion is important to treat correct kinematics of a relativistic composite system. With this effect, we can understand the success of the NRCQM also in the RCQM framework, and there is still room to discuss baryons as pure relativistic three-body system before considering about meson cloud effect or other complicate pictures for baryons.

References

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[1] At higher order of $\frac{1}{m}$, there are some differences between FW transformation and direct expansion of the matrix element [3].