Color Superconductivity and Chiral Symmetry Breaking at high Density and Temperature in Nambu–Jona-Lasinio Model

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The motivation of this master thesis is that we understand color superconductivity in quark matter in finite tmperature and density with Nambu–Jona-Lasinio (NJL) model.

It is important to consider properties of quark matter. Ordinary condensed matter is based on atoms, ions and electrons governed by QED. However quark matter is based on another elements–quarks, which are described by QCD. Quark matter physics has possibility to bring us new aspects in condensed matter physics. In fact quark matter is thought to be realized in heavy ion collisions, core of neutron stars and early universe. Recently color superconductivity (CSC) in quark matter is given much attention. Color superconductivity is a condensation in quark–quark correlation. On the other hand quark–anti-quark correlation is given through chiral symmetry breaking (CSB) which is one of important phenomena in low energy QCD.

Color superconductivity was first discussed by B. C. Barrois [1] in 1977 and B. C. Bailin and A. Love [2] in 1984. They solved Schwinger-Dyson equation by rainbow approximation in gluon propagator but gap was 1 MeV and too samall. Magnetic gluon mass becomes zero within purturbation and has infrared divergence. They introduced magnetic mass by hand in order to avoid the divergence. However their results did not have much attention, because the value of gap energy is almost 1 MeV and too small. In 1999 D. T. Son [3] showed that gap could be larger by long range interaction of magnetic gluon. His result was confirmed by other people. These discussions are systematic calculation based on QCD. However because puturbation is valid only in small coupling region (high density region), we can not say anything in low or middle density region. In lower density region peolpe use effective models. In 1998 R. Rapp, T. Schaefer, E. V. Shuryak, and M. Velkovsky [6] used NJL model including instanton effects and got 100 MeV gap energy. J. Berges and K. Rajagopal [4] and K. Rajagopal, E. V. Shuryak, and S. P. Klevansky [5] also dicussed in NJL model.

We discussed CSC anc CSB phase in NJL model [7]. We are able to handle with both phases in parallel easily because interaction in NJL model does not depend on momentum. In our discussion the degree of freedom of flavour is fixed to u and d. The interaction term of NJL lagrangian is chiral symmetric 4-point interaction . NJL lagrangian has symmetry of $SU(2)_L \otimes SU(2)_R \otimes SU(3)_c$. Here $SU(3)_c$ is global symmetry. The form of NJL lagrarian is

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + G_1[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \boldsymbol{\tau}\psi)^2].$$

Chiral symmetry breaking is caused by the force of $\bar{q}q$ channel. This strength is G_1 from above NJL lagrangian. Color superconductivity is realized in qq channel. We can get many types of qq channel by Fierz transfomation from NJL lagrangian. For simplicity we consider only qq channel of Dirac scalar, flavor asymmetry and color asymmtery. The strength of this qq channel is

$$G_2 = \frac{1}{4} G_1.$$

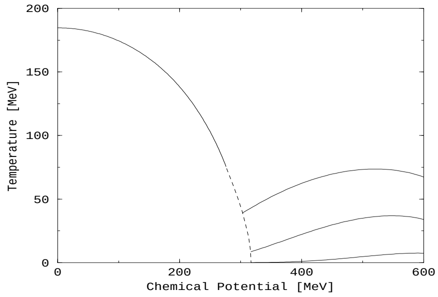

Figure 1: Left side region shows CSB phase. Right side shows CSC phase. $G_2/G_1 = 3/4, 1/2, 1/4$ from up to dowm. Real lines are second order and dashed line is first order phase transition.

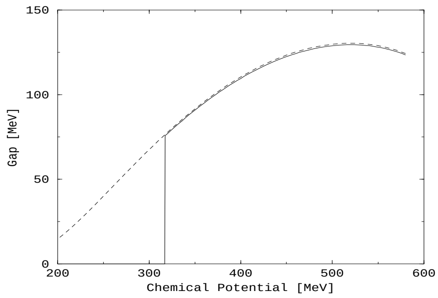

Figure 2: Real line is affected by CSB. Dashed line is in case of no CSB (m=0). T=0.

Then we calculated effective potential $V(m, \Delta)$ up to leading order in loop expansion. This effective potential has two oreder parameters – dynamical mass m and gap Δ. The true vacuum is characterized by m and Δ which minimize effective potential.

CSB and CSC phase are plotted on $T-\mu$ plane [Fig. 1]. Chial symmetry is broken in low temperature and density region. On the other hand color superconductivity is realized in low temperature but high density region. The other G_2 strength cases $G_2/G_1 = 3/4$ [4], 1/2 [6] are also shown. The gap is very sensitive to the value of G_2.

We also have to note that there is no mixing phase of CSB ans CSC in NJL model [Fig. 2]. There is no qq correlation when $\bar{q}q$ correlates and vice versa. In case of that we do not consider CSB ($m = 0$), gap goes to zero continuously as $\mu \to 0$ and qq correlation exists even in low density. However gap is suppressed to zero when mass is included. On the other hand mass is not affected by the existence of gap.

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