# Rare radiative $B$ decays to orbitally excited $K$ mesons and heavy-to-light form factor relations at large recoil 

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Rare radiative decays of $B$ mesons represent an important test of the standard model of electroweak interactions. These transitions are induced by flavour changing neutral currents and thus they are sensitive probes of new physics beyond the standard model. Such decays are governed by one-loop (penguin) diagrams with the leading contribution from a virtual top quark and a $W$ boson. Therefore, they provide valuable information about the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_{t s}$ and $V_{t b}$. The increased statistics of rare radiative $B$ decays allowed a significantly more precise determination of exclusive and inclusive branching fractions. The first observation of the rare $B$ decays to the orbitally excited strange mesons has been reported by CLEO and then confirmed by Belle. The data for the other decay channels will be available soon. This significant experimental progress provides a challenge to the theory. We investigated rare radiative $B$ decays to orbitally excited $K^{* *}$ mesons in the framework of the relativistic quark model[1]. The large value of the recoil momentum $|\boldsymbol{\Delta}| \sim m_{b} / 2$ makes relativistic effects to play a significant role and strongly increases the energy of the final meson. This effect considerably simplifies the analysis since it allows to make an expansion both in inverse powers of the large $b$ quark mass and in the large recoil momentum of the light final meson. Such an expansion has more firm theoretical grounds than the previously used expansion in inverse powers of the $s$ quark mass, which is not heavy enough. We carried out this expansion up to the second order and calculated resulting form factors in our relativistic quark model. Rare radiative $B$ decays to axial-vector $K_{1}^{(*)}$ and tensor $K_{2}^{*}$ mesons have been considered. We present the calculated decay branching ratios in Table 1. The relativistic effects substantially influence decay form factors. Thus, the Wigner rotation of the light quark spin gives an important contribution, which leads to the suppression of the $B \rightarrow K_{1}^{*}(1270) \gamma$ decay rate. In the nonrelativistic quark model, where these effects are missing, the ratio of branching fractions $B R\left(B \rightarrow K_{1}^{*}(1270) \gamma\right) / B R\left(B \rightarrow K_{1}(1400) \gamma\right)$ is equal to 2 , while in our model it is substantially smaller and equals to $0.7 \pm 0.3$. It will be very interesting to test this conclusion experimentally.

Our predictions for the branching fractions $B \rightarrow K^{*} \gamma$ and $B \rightarrow K_{2}^{*} \gamma$ as well as their ratio are in a good agreement with recent CLEO and Belle data.

The form factor relations for $B$ decays to light mesons arising in the heavy quark and large recoil energy limits were derived [2]. The decays both to ground state and radially and orbitally excited light mesons were considered. The main attention was paid to the complete accounting for corrections of second order in the ratio of the light meson mass to the large recoil energy. Such corrections are especially important for decays to excited light mesons, since their masses are of order of the charmed quark mass. The correction to the effective Lagrangian quadratic in the final meson mass was obtained. It was found that this correction does not violate the symmetry of the leading order Lagrangian, since it has the same Dirac structure as the leading contribution. Therefore the corrections to the weak current matrix

Table 1: Theoretical predictions and experimental data for the branching fractions $\left(\times 10^{-5}\right)$ and their ratios $R_{K^{*}} \equiv B R\left(B \rightarrow K^{*} \gamma\right) / B R\left(B \rightarrow X_{s} \gamma\right), \quad R_{K_{i}^{(*)}} \equiv$

| $B R\left(B \rightarrow K_{i}^{(*)} \gamma\right) / B R\left(B \rightarrow X_{s} \gamma\right)(i=1,2), r \equiv B R\left(B \rightarrow K_{2}^{*} \gamma\right) / B R\left(B \rightarrow K^{*} \gamma\right)$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Value | theory 11$]$ | Exp. CLEO | Exp. Belle |
| $B R\left(B \rightarrow K^{*}(892) \gamma\right)$ | $4.5 \pm 1.5$ | $4.55_{-0.68}^{+0.72} \pm 0.34$ | $4.96 \pm 0.67 \pm 0.45$ |
| $R_{K^{*}}(\%)$ | $15 \pm 3$ |  |  |
| $B \rightarrow K_{0}^{*}(1430) \gamma$ | forbidden |  |  |
| $B R\left(B \rightarrow K_{1}^{*}(1270) \gamma\right)$ | $0.45 \pm 0.15$ |  |  |
| $R_{K_{1}^{*}}(\%)$ | $1.5 \pm 0.5$ |  |  |
| $B R\left(B \rightarrow K_{1}(1400) \gamma\right)$ | $0.78 \pm 0.18$ |  |  |
| $R_{K_{1}}(\%)$ | $2.6 \pm 0.6$ |  |  |
| $B R\left(B \rightarrow K_{2}^{*}(1430) \gamma\right)$ | $1.7 \pm 0.6$ | $1.66_{-0.53}^{+0.59} \pm 0.13$ | $1.89 \pm 0.56 \pm 0.18$ |
| $R_{K_{2}^{*}}(\%)$ | $5.7 \pm 1.2$ |  |  |
| $r$ | $0.38 \pm 0.08$ | $0.39_{-0.13}^{+0.15}$ |  |

elements quadratic in the ratio of the final meson mass to the large recoil energy do not lead to the introduction of additional invariant functions. Their inclusion requires a more accurate consideration of the decay kinematics, keeping all final meson masses finite. The heavy quark and large recoil symmetries substantially constrain the number of independent form factors. Thus, for $B$ decays to pseudoscalar (scalar) light mesons three decay form factors can be parametrized by one invariant function $\zeta_{P, S}\left(q^{2}\right)$, and for $B$ decays to vector (axial-vector, tensor) light mesons seven decay form factors can be expressed through two invariant functions $\zeta_{V, A, T}^{\perp}\left(q^{2}\right)$ and $\zeta_{V, A, T}^{\|}\left(q^{2}\right)$ for each decay, respectively. This establishes relations between decay form factors at the large recoil of the final light meson. These relations were obtained with the complete account of second order corrections in the light meson to $B$ meson mass ratio.

The important consequence of these equations are the well known Isgur-Wise relations between form factors of semileptonic and rare radiative $B$ decays, which were originally obtained for small values of the recoil momentum, and now they are established near the point of maximum recoil if all contributions quadratic in final meson mass are included.

We tested the fulfilment of the large recoil symmetry relations between the $B$ decay form factors in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. It was found that they are exactly satisfied in the appropriate limits and corresponding invariant functions were determined.

## References

[1] D. Ebert, R. N. Faustov, V. O. Galkin and H. Toki, Phys. Rev. D 64 (2001) 054001.
[2] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 64 (2001) 094022.

