Non-abelian plane waves and stochastic regimes for (2+1)-dimensional gauge field models with the Chern-Simons term

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Various exact solutions are known of the classical gauge field equations in (3+1)- and (2+1)-dimensional space-time with or without Higgs fields, demonstrating nontrivial topological properties (see, e.g., [1]) and chaotic behavior (see, e.g., [2] – [4]). We consider here a generalization of the plane-wave solutions, found earlier in the $d = 3+1$-case (see [5] – [7]), to the case of a (2+1)-dimensional gauge field theory with a topological Chern-Simons term and discuss an important case of spontaneous symmetry breaking in the CS-Georgi-Glashow model [8]. The dynamics of a system of coupled gauge and Higgs fields in this model is investigated in the general case, where fairly large values of the energy are admitted and, in contrast to earlier works, no assumptions are made that the system is near a stable solution (near the "minimum" critical point of the effective potential). We demonstrate that the dynamics of 3-d YM fields with a CS topological mass interacting with Higgs fields is described by solutions that, generally speaking, are not regular, but rather obtain ergodic properties.

1. The 3-dimensional $SU(2)$ gauge field Lagrangian with a Chern-Simons term can be written as follows

$$\mathcal{L}_g = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{\mu}{4}\epsilon^{\mu\nu\alpha}[F^a_{\mu\nu}A^a_\alpha - \frac{g}{3}\epsilon^{abc}A^a_\mu A^b_\nu A^c_\alpha], \tag{1}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu$ is the gauge field tensor. The last term in (1) is the CS term for the gauge field topological mass μ.

We seek solutions as functions of time only, according to the following ansatz $A^a_\mu = \delta^a_\mu f_a(t)$, $A^{\mu a} = g^{\mu a} f_a(t), f_1 = -\frac{\mu}{2g}, f_2 = f_3 = f(t)$, where $a = 1,2,3$, $\mu = 1,2,3$ $(x_1 = t, x_2 = x, x_3 = y), g_{\mu\nu} = \mathrm{diag}(+,-,-)$, and no summation over a is assumed (this case formally corresponds to a mechanical system).

The corresponding equations of motion were integrated to give the following solution [8]:

$$f(t) = \frac{\mu}{\sqrt{2}g}\frac{k}{\sqrt{1-2k^2}}\,\mathrm{cn}\left((\frac{\mu^4}{16} + 2\mathcal{E}g^2)^{\frac{1}{4}}(t-t_0), k\right),\ k = \sqrt{\frac{1}{2}\left(1 - \frac{\mu^2}{4(\frac{\mu^4}{16} + 2\mathcal{E}g^2)^{\frac{1}{2}}}\right)}, \tag{2}$$

where $\mathrm{cn}(x,k)$ is the elliptic Jacobi *cosine* function of the argument x and the module k. This solution is periodical with the period $T = 4\mathrm{K}(k)/(\frac{\mu^4}{16} + 2\mathcal{E}g^2)^{\frac{1}{4}}$, where $\mathrm{K}(x)$ is the full elliptic integral of the first kind.

The solution (2) describes a nonlinear standing wave. By applying a Lorentz boost $x'_\mu = a^\nu_\mu(\vec{v})x_\nu$, $kx = k'x'$, with $k'_0 = M\gamma, k'_i = Mv_i\gamma, (\gamma = (1-v^2)^{-1/2})$, we obtain nonlinear propagating waves with the finite mass $M \sim (\frac{\mu^4}{16} + 2\mathcal{E}g^2)^{\frac{1}{4}}$. As expected, the effective mass includes both the energy density of the solution and the topological mass of the gauge field. We expect that further investigations of realistic field configurations of this type should concentrate on the search for possible non-abelian solutions that describe localized field configurations with a finite energy at rest.

2. Including a Higgs field contribution leads to a D=3 Georgi-Glashow model with the CS term ("CS-Georgi-Glashow model")

$$\mathcal{L} = \mathcal{L}_g + \frac{1}{2}(D_\mu \Phi^a)(D^\mu \Phi^a) + \frac{m^2}{2}\Phi^a\Phi^a - \frac{\lambda}{4}(\Phi^a\Phi^a)^2, \tag{3}$$

where $\mathcal{L}_g$ is the gauge field Lagrangian (1), and $\Phi^a (a = 1, 2, 3)$ is the scalar Higgs field in the adjoint representation ($D_\mu \Phi^a = \partial_\mu + g\epsilon^{abc}A^b_\mu \Phi^c$). We again seek solutions that depend only on the time with the above ansatz for $A^a_\mu(t)$: $f_1 \neq f_2 = f_3 = f(t)$, and the following for the Higgs field $\Phi_1 = \Phi(t), \Phi_2 = \Phi_3 = 0$.

We have numerically analyzed the behavior of the effective mechanical system with coordinates Φ, f and the Lagrangian [8]

$$L = \dot{f}^2 - \frac{\mu^2}{4}f^2 - \frac{g^2}{2}f^4 + \frac{\dot{\Phi}^2}{2} + \frac{m^2}{2}\Phi^2 - \lambda\frac{\Phi^4}{4} - g^2 f^2\Phi^2 = L_0 - g^2 f^2 \Phi^2 \tag{4}$$

with the last term as a perturbation. For the system with the unperturbed Lagrangian L_0 all trajectories are either periodical or quasi-periodical. In order to describe the role of the perturbation, the KAM-theorem can be applied. To examine the trajectories in detail, we considered intersections of trajectories and the plane $f = 0$ under the condition $f' > 0$. Plotting the intersection points in the so called Poincaré surfaces of intersection, we can study the character of motion for the mechanical system equivalent to our interacting gauge and Higgs fields for different values of energy $\mathcal{E}$. For comparatively low energies, well below the "saddle" point, all the trajectories form closed curves, hence, according to KAM-theorem, practically the entire phase space consists of toruses, which corresponds to quasiperiodical motion. For higher energies and larger perturbations the situation is completely different: there arise regions of ergodic behavior (the intersection points cover densely a finite area). For still higher energies the motion of the system again becomes more and more stabilized. Near the saddle point the character of motion changes, corresponding to the symmetry breaking in the Lagrangian. The motion becomes more unstable and its stochastic character more pronounced when the energy is closer to the saddle point. It is well known that the Yang-Mills field system is not exactly solvable. We confirm this conclusion for the case of a 3-d gauge field model with a topological CS term and an additional interaction with a dynamical Higgs field. Detailed analysis shows, however, that the presence of the CS topological mass in the 3-d case is conducive to the restoration of symmetry and to stabilizing the system.

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