Nucleon axial coupling constant g_A in linear representations of chiral symmetry

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The axial coupling constant g_A of the nucleon is one of fundamental quantities of the nucleon; experimentally, it is $g_A = 1.25$. Sometimes, the fact that g_A is close to unity is considered as an evidence of partially conserved axial vector current (PCAC), and hence it would become one in the limit that the axial current is conserved. This, however, is not correct, and g_A takes any value, as in the non-linear sigma model. Even in the linear sigma model, g_A can be arbitrary if higher derivative terms are added.

The arbitrariness of g_A is related to the fact that chiral symmetry is broken spontaneously [1]. Particles are classified only in terms of isospin (flavor) and it is not necessary to refer to the full chiral symmetry including axial symmetry. Instead, the axial symmetry is realized non-linearly including pion fields and their covariant derivatives.

Now, in QCD, it is postulated that quarks are in linear representations of chiral symmetry. Since hadrons are their composite states, it is natural to consider that particles are classified as linear representations of chiral symmetry. In fact, consideration based on linear representations was initiated by Weinberg prior to QCD, where he derived commutation relations as consistency conditions among scattering amplitudes. The amplitudes that are computed using a low energy chiral lagrangian behave badly for large momenta, which can be fixed by the consistency condition. The idea is similar to the Adler-Weisberger sum rule where current commutators are applied in the dispersion theory. The two methods are closely related, since the dispersion theory is based on analytic properties of scattering amplitudes and maintains correct large momentum behavior.

In this report, we construct several solutions of the Weinberg's consistency conditions which are essentially the chiral algebra where nucleons are given as linear representations of chiral symmetry. The original derivation of the Weinberg seems convenient, since the algebraic relation is given in terms of physical axial charge of the nucleon.

By considering forward pion-nucleon scattering and imposing correct large momentum behavior for the sum of the three dominant diagrams as shown in Fig. 1, it was shown that commutation relations among the pion-nucleon coupling matrices X_a and isospin charges T^a are derived:

$$\left[X^a,\, X^b\right] = i\epsilon_{abc}T^c\,, \quad \left[T^a,\, T^b\right] = i\epsilon_{abc}T^c\,, \quad \left[T^a,\, X^b\right] = i\epsilon_{abc}X^c\,. \tag{1}$$

The nucleons appear as linear representations $|\alpha\rangle$ which are the bases of the matrix elements $(T^a)_{\alpha\beta}, (X^a)_{\alpha\beta}, \cdots$.

By sandwiching these relations with physical states and saturating the intermediate states, we can solve for the matrix elements $(X^a)_{\alpha\beta}$, which are related to the axial charges of the nucleons. Detailed calculations are presented in ref. [2], but here we give essential results:

1. For a fundamental representation of chiral symmetry, $(1/2, 0) \oplus (0, 1/2)$, $g_A = 1$. This explains the value of g_A in the linear sigma model, when there is no higher order derivative terms. The latter terms introduces higher dimensional representations for the nucleon.

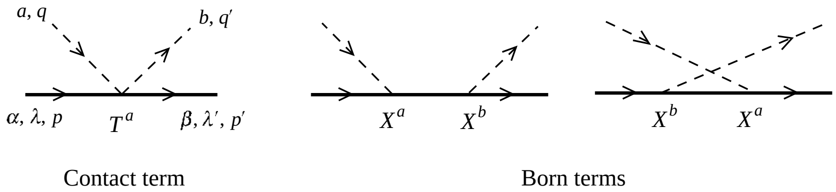

Figure 1: Contact and Born diagrams for the pion-nucleon scatterings.

2. For $(1,1/2)\oplus(1/2,1)$, $g_A = 5/3$. This explains the value in the non-relativistic quark model where the nucleon and the delta are in the same multiplet.

3. For large-N_c, we can consider a representation

$$((N_c+1)/4,(N_c-1)/4)\oplus((N_c-1)/4,(N_c+1)/4)\,, \tag{2}$$

giving $g_A = (N_c+2)/3$ as expected from the large-N_c analysis.

4. One can also extend the representation to the mirror one and their mixing with ordinary one [3]: $\cos\theta((1/2,0)\oplus(0,1/2))\oplus\sin\theta((0,1/2)\oplus(1/2,0))$. In this case,

$$g_A = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{pmatrix} . \tag{3}$$

We have shown that the algebraic constraint due to Weinberg can determine the value of g_A for a given representation of chiral symmetry of the nucleon. This offers different nature of nucleon structure. Among several possibilities, mixing of delta and mirror representations seems interesting for further investigations.

References

[1] S. Weinberg, Phys. Rev. 177 (1969) 2604.
[2] A. Hosaka, D. Jido and M. Oka, Proceedings for international conference of EMI, RCNP Osaka, December (2001).
[3] D. Jido, M. Oka and A. Hosaka, Prog. Theor. Phys. 106 (2001) 823; 873.