# QCD critical point in the improved ladder approximation

O. Kiriyama

*Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan*

Recently there has been great interest in studying the phase structure of quantum chromodynamics (QCD). We expect that at sufficiently high temperature and/or density the QCD vacuum undergoes a phase transition into a chirally symmetric/deconfinement phase and a color superconducting phase. They may be realized in high-energy heavy-ion collisions at the BNL Relativistic Heavy Ion Collider (RHIC) and CERN Large Hadron Collider (LHC). These phase transitions are also important in the physics of neutron (or quark) stars and the early universe.

In massless two–flavor QCD, as confirmed by using the lattice simulation and several effective theories, the chiral phase transition at high temperature is probably second order. On the other hand, at high density a first order one is expected. These observations indicate the existence of a tricritical point in the $\mu$-$T$ phase diagram for zero current quark masses and a critical point, where the first order transition ends, for nonzero current quark masses. The existence and the location of the critical point has been studied also by using the recently proposed lattice QCD method. It has been proposed that the critical point may lead to characteristic signatures which enable us to explore the phase structure of QCD in heavy-ion collisions. We investigate where the critical point locates by making use of the so-called *QCD in the improved ladder approximation.* In the following we fix the scale parameter of our model by the condition $\Lambda_{QCD} = 1$ except for numerical calculations.

In terms of the quark dynamical mass function $\Sigma(p_E)$, the effective potential is given as follows:

$$V = -2\int \frac{d^4p_E}{(2\pi)^4}\ \ln\frac{\Sigma^2(p_E)+p_E^2}{p_E^2} - \frac{2}{3C_2}\int dp_E^2\ \frac{1}{\Delta(p_E)}\left(\frac{d}{dp_E^2}\Sigma(p_E)\right)^2, \tag{1}$$

where $C_2 = (N_c^2-1)/(2N_c)$ is the quadratic Casimir operator for color $SU(N_c)$ group, $\Delta(p_E) = d(\bar{g}^2(p_E)/p_E^2)/dp_E^2$ with $\bar{g}^2(p_E)$ being the QCD running coupling of one-loop order, and an overall factor (the number of light flavors times the number of colors) is omitted.

We calculate the effective potential at finite temperature and density by making use of the imaginary time formalism and the following functions:

$$\mathcal{D}_{T,\mu}(p) = \frac{2\pi^2 a}{\ln(\omega_n^2+|\vec{p}|^2+p_R^2)}\ \frac{1}{\omega_n^2+|\vec{p}|^2}, \tag{2}$$

$$\Sigma_{T,\mu}(p) = m_R\left[\ln(\omega_n^2+|\vec{p}|^2+p_R^2)\right]^{-a/2} + \frac{\sigma}{\omega_n^2+|\vec{p}|^2+p_R^2}\left[\ln(\omega_n^2+|\vec{p}|^2+p_R^2)\right]^{a/2-1}, \tag{3}$$

where $p_R$ is an infrared regularization parameter to keep the QCD running coupling from blowing up at $p_E = 1(\Lambda_{QCD})$, and $a = 8/9$ (we use three–flavor, three–color effective running coupling).

In numerical calculations, we fix $\ln(p_R^2/\Lambda_{QCD}^2) = 0.1$ and the value of $\Lambda_{QCD}$ is determined by the condition $f_\pi = 93$ MeV at $T = \mu = 0$ and in the chiral limit; i.e., $m_R = 0$. Figure 1 shows the phase diagram in the chiral limit in $n_B$-$T$ plane. At $T = 0$, there is a mixed phase that consists of massive quarks with $n_B^{(-)} = 1.5n_0$ and massless quarks with $n_B^{(+)} = 4.3n_0$ where $n_0 = 0.17\text{fm}^{-3}$ is normal nuclear matter density. The phase diagram for $m_u(1\text{GeV}) =$

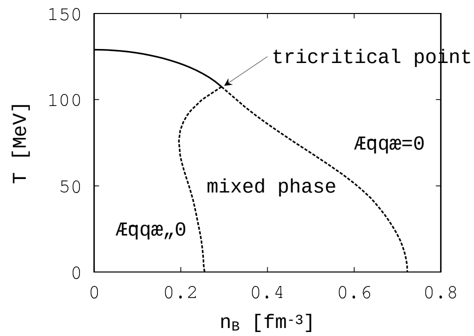


Figure 1: The phase diagram in $n_B$-$T$ plane.

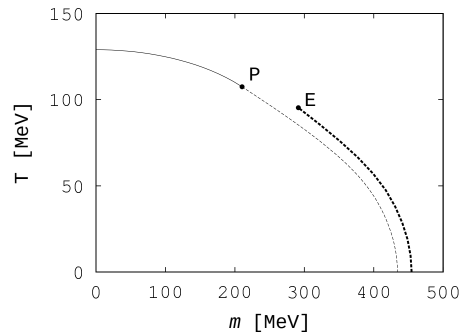


Figure 2: The phase diagram in $\mu$-$T$ plane.

7MeV case is shown in Fig. 2. The critical point $E$ where the first order phase transition ends is found at $\mu_E \simeq 290$ MeV, $T_E \simeq 95$ MeV;

$$\frac{\mu_E}{\mu_{crit}} \simeq 0.64, \ \frac{T_E}{T_{cross}} \simeq 0.70, \tag{4}$$

where $\mu_{crit}$ is the critical chemical potential at $T = 0$, and $T_{cross}$ is the temperature where $M_\sigma$ is minimized and $M_\pi$ starts to increase at $\mu = 0$. We have confirmed that by the finite current quark mass the critical point is moved from the tricritical point, which we have found in the previous paper[1], toward larger value of $\mu$ and smaller value of $T$. The value of $\mu_E$ seems to be too large in light of the experiments at RHIC and LHC. By including strange quark mass, however, it would be reduced.

Finally, some comments are in order. (1) We did not take into account the screening of the gluon, that is to say, the Debye screening for the electric gluons. (2) At finite temperature, the wave function renormalization may affect the precise location of the critical point. In any case, it is preferable to take into account the screening of the gluon, wave function renormalization and the effects of $s$ quark in the future work.

# References

[1] O. Kiriyama, M. Maruyama, and F. Takagi, Phys. Rev. D **62** (2000) 105008, *ibid.* **63** (2001), 116009, and the references cited therein.