Lattice Study of Finite Density QCD

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QCD at finite density is expected to have a rich phase structure. So far most of the analyses were based on NJL type model, or the perturbative calculation which valid only at extremely high density. Therefore it is very desireble to study the quark/gluon system around the critical density by lattice QCD. However, when we introduce the chemical potential, the action becomes complex in color SU(3) system, and the standard Monte Carlo method does not work. Only a work with color SU(2) was performed[1].

Due to the recent progress in analytical investigations, we can hopefully obtain some information on real SU(3)-QCD through the investigation of the finite density region of the "QCD-like" theories [2]. The QCD-like theories, such as SU(2)-QCD, quark model in the adjoint representation and QCD at finite isospin density, are expected to have less difficulties in numerical analyses. In these years, there are indeed high activities in Monte Carlo calculations with dynamical quark of such kinds of models [3].

We investigate high density state of SU(2) QCD by using Lattice QCD simulation with Wilson fermions. The ratio of fermion determinants is evaluated at each step of the Metropolis link update by Woodbury formula. We calculate the baryon number density, the Polyakov lines, and the energy density of gluon sector with chemical potential on the $4^3\times(4,8,12)$ lattice with the periodic and antiperiodic boundary condition. For the gauge action, we employ standard plaquette action and Iwasaki improved action. We investigate mainly $N_f = 2$ case, but also $N_f = 3$ case is studied. Behavior of the meson propagators and diquark propagators with finite chemical potential are also investigated.

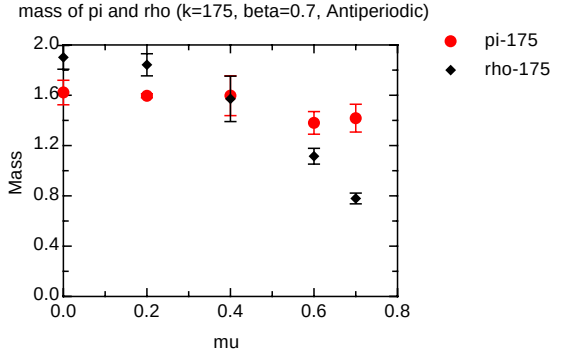

Figure 1: π, ρ masses as a function of μ. Anti-periodic boundary condition

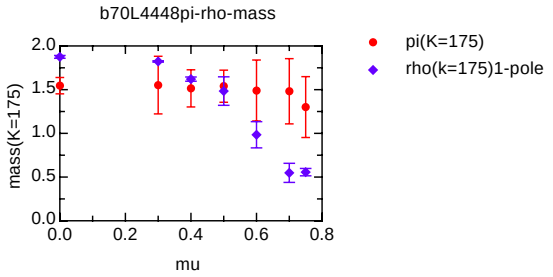

Figure 2: π, ρ masses as a function of μ. Periodic boundary condition

The chemical potential, μ, is introduced in the fermion action, $\bar{\psi}W\psi$, as [9],

$$W(x,x') = \delta_{x,x'} - \kappa\sum_{i=1}^{3}\Big\{(1-\gamma_i)U_i(x)\delta_{x',x+\hat{i}} + (1+\gamma_i)U_i^{\dagger}(x')\delta_{x',x-\hat{i}}\Big\}$$

$$-\kappa\left\{e^{+\mu a}(1-\gamma_4)U_4(x)\delta_{x',x+\hat{4}}+e^{-\mu a}(1+\gamma_4)U_4^{\dagger}(x')\delta_{x',x-\hat{4}}\right\}. \qquad (1)$$

Little is known about the behavior of dynamical fermion simulations when the chemical potential is introduced. For $\mu \neq 0$, the relation $W^{\dagger} = \gamma_5 W \gamma 5$ does not hold, and hence det$W$ is in general not real. Differing essentially from the SU(3) case, action of the SU(2)-QCD is real with chemical potential.

However, numerical simulation is not straightforward and instability occurs with large chemical potential, which makes lattice simulation difficult[10]. Therefore, we need careful treatment for the updation of the configulation. We here adopt locally updating exact algorithm based on the Woodbery formula [11]

We have investigated phase diagram in (κ, μ) for $\beta = 0.7$ with periodic and anti-periodic boundary condition for $4^3 \times 4, 8, 12$ lattice. We have observed that around μ_c, vector meson masse decreases rapidly. See Fig.1 and 2. This may an indication of "vector manifestation" [12].

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