Five-Body Calculation of ${}^{5}_{\Lambda}$He with Explicit Σ Admixture

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Few-body calculations for s-shell hypernuclei with mass number $A = 3-5$ are important not only to explore exotic nuclear structure, including the strangeness degrees of freedom, but also to clarify the characteristic features of the hyperon-nucleon (YN) interaction. The complete set of the observed separation energies (B_Λ's) for s-shell Λ hypernuclei are expected to provide important information on the YN interaction, because the relative strength of the spin-dependent term or of the $\Lambda N - \Sigma N$ coupling term is affected from system to system. For example, since ${}^{4}_{\Lambda}$He and ^{3}He have both the isospin $T = \frac{1}{2}$, both $[\phi_c \otimes \psi_\Lambda]_{T=\frac{1}{2}}$ and $[\phi_c \otimes \psi_\Sigma]_{T=\frac{1}{2}}$ components would contribute to the bound states, where ϕ_c is the wave function of the core nucleus (^{3}He) and ψ_Λ (ψ_Σ) is the wave function of the relative motion between the core nucleus and Λ (Σ). On the other hand, since the ground states of ${}^{5}_{\Lambda}$He and ^{4}He are both $T = 0$, admixture of Σ component (e.g. $[\phi'_c \otimes \psi_\Sigma]_{T=0}$) would be suppressed in ${}^{5}_{\Lambda}$He, where ϕ'_c (with $T = 1$) is not ever equal to the wave function of the ^{4}He (with $T = 0$). Although few-body studies for $A = 3, 4$ hypernuclei have been conducted using modern YN interactions[1, 2, 3], these calculations have not yet reached a stage to calculate the ${}^{5}_{\Lambda}$He as five-body system.

The NN tensor interaction due to a one-pion-exchange mechanism is the most important ingredient for the binding mechanisms of light nuclei. More than a third, or about one half, of the interaction energy comes from the tensor force for the ^{4}He[4, 5, 6]. Since the pion-(or kaon-) exchange also induces the $\Lambda N - \Sigma N$ transition for the YN sector, both the NN and $\Lambda N - \Sigma N$ tensor interactions may also play important roles for light hypernuclei. If this is the case, the structure of the core nucleus (^{4}He) in the hypernucleus (${}^{5}_{\Lambda}$He) would be strongly influenced by the presence of a Λ particle.

Recently, we performed *ab initio*-type variational calculations for the complete set of the s-shell Λ hypernuclei by explicitly including Σ degrees of freedom[7]. Four sets of YN interactions (SC97d(S), SC97e(S), SC97f(S) and SC89(S))[8] and a set of NN interaction (G3RS)[9] were used. The SC97d-f(S) and SC89(S) are set to reproduce the low-energy S matrix of the original Nijmegen YN potentials[10]. Table 1 lists the results of the Λ separation energies,

$$B_\Lambda({}^{A}_{\Lambda}\mathrm{Z}) = B({}^{A}_{\Lambda}\mathrm{Z}) - B({}^{A-1}\mathrm{Z}). \tag{1}$$

Table 1: Λ separation energies, given in units of MeV, of $A = 3-5$ Λ-hypernuclei.

	$B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$	$B_\Lambda({}^{4}_{\Lambda}\mathrm{H})$	$B_\Lambda({}^{4}_{\Lambda}\mathrm{H}^*)$	$B_\Lambda({}^{4}_{\Lambda}\mathrm{He})$	$B_\Lambda({}^{4}_{\Lambda}\mathrm{He}^*)$	$B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$	χ^2
SC97d(S)	0.01	1.67	1.20	1.62	1.17	3.06	0.79
SC97e(S)	0.10	2.06	0.92	2.02	0.90	2.59	0.54
SC97f(S)	0.18	2.16	0.63	2.11	0.62	1.96	1.96
SC89(S)	0.37	2.48	unbound	2.40	unbound	0.14	(> 11.67)
Expt.	0.13(5)	2.04(4)	1.00(4)	2.39(3)	1.24(4)	3.12(2)	

The χ^2 is defined by

$$\chi^2 = \sum_{\mathrm{X}} \left(B_{\Lambda}^{(\mathrm{calc})}(\mathrm{X}) - B_{\Lambda}^{(\mathrm{expt})}(\mathrm{X}) \right)^2 , \tag{2}$$

where the summation is taken over $\mathrm{X} = ({}^{3}_{\Lambda}\mathrm{H}, {}^{4}_{\Lambda}\mathrm{H}, {}^{4}_{\Lambda}\mathrm{H}^{*}, {}^{4}_{\Lambda}\mathrm{He}, {}^{4}_{\Lambda}\mathrm{He}^{*}, {}^{5}_{\Lambda}\mathrm{He})$. The bound-state solution of ${}^{5}_{\Lambda}\mathrm{He}$ was obtained, which is a *first ab initio* calculation including explicit Σ admixture. The number of $\chi^2 = 0.54$ by using the SC97e(S) is the smallest among the YN interactions employed in the calculation.

Table 2 lists the probability, P_Σ (in percentage), of finding a Σ particle in the system. A sizable amount of $P_\Sigma({}^{5}_{\Lambda}\mathrm{He})$ was found; The $P_\Sigma({}^{5}_{\Lambda}\mathrm{He}) = 1.52\%$ of the SC97e(S) is slightly larger than the $P_\Sigma({}^{4}_{\Lambda}\mathrm{H}) = 1.49\%$ of the SC97e(S). This large $P_\Sigma({}^{5}_{\Lambda}\mathrm{He})$ value is due to the tensor $\Lambda N - \Sigma N$ transition potential. The tensor $\Lambda N - \Sigma N$ transition potential contributes dominantly to the energy expectation value of the YN interaction. In the case of ${}^{5}_{\Lambda}\mathrm{He}$ using SC97e(S), the energy expectation value of the YN interaction is about -21 MeV, while the contribution from the tensor $\Lambda N - \Sigma N$ transition part is -19.26 MeV. Therefore, the $\Lambda - \Sigma$ coupling is considerably important to make the ${}^{5}_{\Lambda}\mathrm{He}$ bound. The naive picture as $\alpha + \Lambda$ is no longer suitable for a realistic description of ${}^{5}_{\Lambda}\mathrm{He}$. Regarding this kind of $\Lambda - \Sigma$ coupling, another interesting light hypernucleus is ${}^{6}_{\Lambda\Lambda}\mathrm{He}$. Investigations into the strength of the $\Lambda\Lambda$ interaction based on the experimental binding energy of ${}^{6}_{\Lambda\Lambda}\mathrm{He}$[11] should take account of explicit Σ (and probably Ξ) admixture(s). The RCNP's SX-5 is a efficient computer for these five- and six-body calculations.

Table 2: Probabilities of finding a Σ particle, given in percentage, of $A = 3-5$ Λ-hypernuclei.

	$P_\Sigma({}^{3}_{\Lambda}\mathrm{H})$	$P_\Sigma({}^{4}_{\Lambda}\mathrm{H})$	$P_\Sigma({}^{4}_{\Lambda}\mathrm{H}^{*})$	$P_\Sigma({}^{4}_{\Lambda}\mathrm{He})$	$P_\Sigma({}^{4}_{\Lambda}\mathrm{He}^{*})$	$P_\Sigma({}^{5}_{\Lambda}\mathrm{He})$
SC97d(S)	0.06	1.27	1.37	1.24	1.35	2.02
SC97e(S)	0.15	1.49	0.98	1.45	0.96	1.52
SC97f(S)	0.23	1.88	1.09	1.83	1.08	1.83
SC89(S)	0.65	3.64	unbound	3.50	unbound	1.20

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