Generalized Husimi distribution and complexity of many-body quantum states

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Although quantum manifestation of chaos has been extensively studied over the past few decades, to define "quantum chaos" is still the main problem in this field. The direct extension of the definition of classical chaos seems to fail because of the linearity of the Schrödinger equation. Since quantum mechanics contains classical mechanics as a limit, however, there must be something in quantum mechanics that produces chaos in the classical limit.

We propose the second moment of the Husimi distribution as a measure of complexity (or chaoticity) of quantum states. The Husimi distribution of a quantum state is defined as the square of the absolute value of the overlap between the quantum state and coherent states. In one-body systems, the Husimi function is a kind of coarse-grained Wigner function, and regarded as a probability distribution in phase space. Since chaoticity of classical systems can be characterized by delocalization of orbits, delocalization of the Husimi distribution is a sign of quantum chaos. In [1], we have shown that the second moment of the Husimi distribution represents the delocalization, and works as a good measure of complexity of quantum states.

We can easily extend the idea of [1] to many-body systems using generalized coherent states [2]. In many-body cases, coherent states are independent particle states, and, at the same time, the most localized states in the Husimi representation. Therefore delocalization of the Husimi distribution is a sign of many-body correlation (or entanglement). The second moment of the Husimi distribution is a quantitative measure of many-body correlation, and easy to calculate. We apply this measure to bosons, fermions, and distinguishable particles like qubits in quantum computers.

Let us illustrate the meaning of the generalized Husimi distribution with a simple example. Fig. 1 shows the Husimi distribution of two bosons in two single-particle states. In this case, the particle-hole excitation operators form Lie algebra of $U(2)$. The two single particle states form the fundamental representation, and many-body states form the three-dimensional (spin 1) representation. A coherent state of the $U(2)$ is a product of single particle states, which is represented as a minimum wave packet on the phase space S^2. If a state can not be written as a product, it is represented by a broad state in the Husimi distribution. The broadening can be measured by the second moment, whose explicit form is

$$M^{(2)}_{|\varphi\rangle} = |c_1|^4 + |c_{-1}|^4 + 2|c_1|^2|c_0|^2 + 2|c_0|^2|c_{-1}|^2 + \frac{2}{3}|c_1 c_{-1} + c_0^2|^2, \tag{1}$$

where

$$|\varphi\rangle = c_1|2,0\rangle + c_0|1,1\rangle + c_{-1}|0,2\rangle. \tag{2}$$

The second moment M_2 takes the maximum value 1 for $|2,0\rangle$ and $|0,2\rangle$, while $M_2 = 2/3$ for $|1,1\rangle$. This result shows that $|1,1\rangle$ is more complex than $|2,0\rangle$ and $|0,2\rangle$, in the sense that it can not be written as a product of single-particle states.

Similar analyses can be applied also to fermions and qubits. See [2] for details.

The numerical calculations in [1] have been done on NEC SX-5 at Research Center for Nuclear Physics (RCNP), Osaka University.

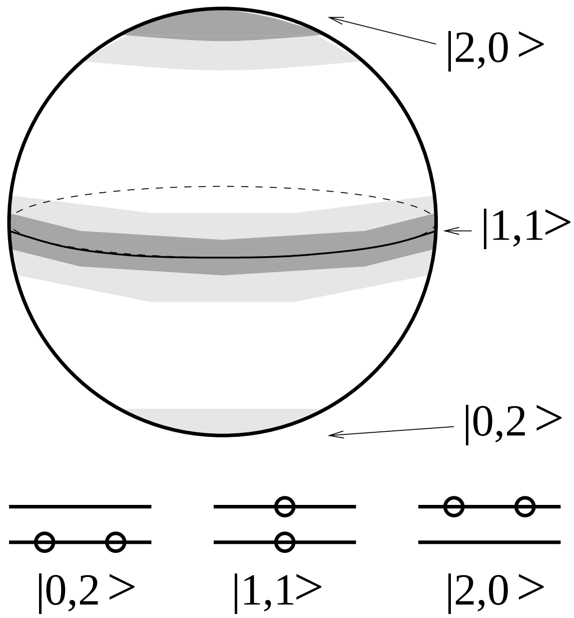

Figure 1: Schematic picture of the Husimi distribution for the system with two bosons in two single-particle states. $|2,0\rangle$ and $|0,2\rangle$ are coherent states, but $|1,1\rangle$ is not. Therefore $|1,1\rangle$ has a rather extended distribution along the equator, whose second moment is the minimum value 2/3. This result shows that $|1,1\rangle$ can not be written as a product of single-particle states.

References

[1] A. Sugita and H. Aiba, Phys. Rev. E **65** (2002) 036205 (10 pages).
[2] A. Sugita, arXiv/nlin.CD/0112042, submitted to Phys. Rev. E.