Lattice QCD analysis of Q-$\bar{\text{Q}}$ and 3Q potentials at zero and finite temperatures

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The strong interaction in hadrons or nuclei is fundamentally governed by quantum chromodynamics (QCD). In the spirit of the elementary particle physics, it is desired to understand hadron physics and nuclear physics at the level of quarks and gluons. However it still remains a difficult problem to grasp hadron physics directly based on QCD, due to its strong-coupling nature in the infrared region. Lately, the lattice QCD Monte Carlo calculation has been adopted as a reliable method for the nonperturbative analysis on QCD. The three-quark (3Q) potential is directly responsible for the baryon properties and of great importance in the hadron physics. In contrast with lots of lattice studies on the quark-antiquark (Q$\bar{\text{Q}}$) potential, which is responsible for the meson features, there is no reliable lattice studies on the 3Q potential so far. The 3Q potential has been treated hypothetically and phenomenologically for a long time.

We perform the accurate measurement and detailed analysis of the 3Q potential $V_{3\text{Q}}$ in SU(3) lattice QCD using the smearing method[1-5], which reduces the excited-state contaminations. For more than 300 patterns of the 3Q configurations in total, we investigate $V_{3\text{Q}}$ on $12^3 \times 24$ lattice at $\beta = 5.7$ and $16^3 \times 32$ lattices at $\beta = 5.8$ and 6.0. We observe that the 3Q potential is well reproduced by a sum of a constant, the perturbative two-body Coulomb term due to the one-gluon-exchange (OGE) and the linear confinement three-body term proportional to the total length of the flux tube as

$$V_{3\text{Q}} = -A_{3\text{Q}} \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3\text{Q}} L_{\text{min}} + C_{3\text{Q}} \tag{1}$$

within a few % deviation. Here, L_{min} denotes the minimal length of the total flux tube. As the remarkable features, we observe the universality of the string tension as $\sigma_{3\text{Q}} \simeq \sigma_{\text{Q}\bar{\text{Q}}}$ and the consistency with perturbative QCD as $A_{3\text{Q}} \simeq \frac{1}{2} A_{\text{Q}\bar{\text{Q}}}$. We list the best fit parameters in Table 1.

In addition, we try a more generalized ansatz which includes the Y ansatz and the Δ ansatz in some limits[5]. The Δ ansatz is an interesting candidate[6,7] as well as the Y ansatz, which is expressed as $V_{3\text{Q}} = -A^{\Delta}_{3\text{Q}} \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma^{\Delta}_{3\text{Q}} \sum_{i<j} |\mathbf{r}_i - \mathbf{r}_j| + C^{\Delta}_{3\text{Q}}$ and has been adopted in various model calculations for its simplicity. So far, the lattice QCD data support the Y ansatz rather than Δ ansatz in terms of the fit analysis. However, it is not trivial whether the Y ansatz holds in the intermediate region. There remains the possibility of the Δ ansatz contaminations in this region, which was in fact conjectured by Cornwall. Moreover, this generalized ansatz would be able to contain the flux-tube core effect. For instance, in the dual superconductor picture, the hadron flux tube has an intrinsic structure of the core inside. In table 2, we list the fitted parameters. The universality of the string tension ($\sigma^{\text{GY}}_{3\text{Q}} \simeq \sigma_{\text{Q}\bar{\text{Q}}}$) and the consistency with perturbative calculation ($A^{\text{GY}}_{3\text{Q}} \simeq \frac{1}{2} A_{\text{Q}\bar{\text{Q}}}$) still

hold This result shows $R_{\rm core}$ is rather small and seems to support the Y ansatz rather than Δ ansatz in the hadronic scale as $r >> 0.1$ fm.

We study also the inter-quark potential at finite temperature. The quark-gluon-plasma (QGP), which is expected to appear above the critical temperature T_c, is now energetically investigated as a new phase of the matter in the RHIC project. The thermal inter-quark potential is considered to be one of the key quantities, for an important signal such as the J/ψ suppression

We investigate the 3Q potential at $T = 1.17T_c$ and $T = 1.4T_c$, where there may remain some nonperturbative effects, from the Polyakov loop correlations using anisotropic lattice QCD. We observe that the 3Q potential can be expressed as the sum of the two-body Yukawa potential with the Debye screening mass of m as

$$V_{\rm 3Q} = -A_{\rm 3Q} \sum_{i<j} \frac{\exp(-m|\mathbf{r}_i - \mathbf{r}_j|)}{|\mathbf{r}_i - \mathbf{r}_j|}. \quad (2)$$

even just above T_c.

	σ	A	C	$\chi^2/N_{\rm DF}$
$3{\rm Q}_{\rm Y}$ (β=5.8)	0.1027(6)	0.1230(20)	0.9085(55)	5.03
$3{\rm Q}_{\rm Y}$ (β=6.0)	0.0460(4)	0.1366(11)	0.9599(35)	2.81
${\rm Q\bar{Q}}$ (β=5.8)	0.1079(28)	0.2607(174)	0.6115(197)	0.92
${\rm Q\bar{Q}}$ (β=6.0)	0.0506(7)	0.2768(24)	0.6374(30)	3.56

Table 1: The main result on the fit analysis of the lattice QCD data with the Y-ansatz at each β. We list the best-fit parameter set (σ, A, C) in the function form as $V_{\rm 3Q} = -A_{\rm 3Q}\sum_{i<j}\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|} + \sigma_{\rm 3Q} L_{\rm min} + C_{\rm 3Q}$, where $L_{\rm min}$ denotes the minimal value of the Y-type flux-tube length. The similar fit on the Q-$\bar{\rm Q}$ potential is also listed. The listed values are measured in the lattice unit.

β	$\sigma_{\rm GY}$	$A_{\rm GY}$	$C_{\rm GY}$	$R_{\rm core}$ $[a]$	$R_{\rm core}$ [fm]	$\chi^2/N_{\rm DF}$
5.8	0.1054(6)	0.1354(18)	0.9569(53)	0.57	0.08	2.63
6.0	0.0480(4)	0.1451(11)	0.9837(33)	0.79	0.08	1.23

Table 2: The fit analysis of the lattice QCD data $V_{\rm 3Q}^{\rm latt}$ with the generalized Y-ansatz at each β. We list the best-fit parameter set $(\sigma_{\rm GY}, A_{\rm GY}, C_{\rm GY}, R_{\rm core})$ in the lattice unit at β=5.8 and 6.0. The flux-tube core radius $R_{\rm core}$ in the physical unit is added.

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