Continuum pairing in the BCS formalism in the relativistic mean field theory

L.S. Geng[a,b], N. Sandulescu[a,c], H. Toki[a], and J. Meng[b]

[a] *Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan*
[b] *School of Physics, Peking University, Beijing 100871, P.R. China*
[c] *Royal Institute of Technology, SCFAB, SE-10691, Stockholm, Sweden*

As recognized long time ago, the basic features of the superfluidity are the same in atomic nuclei and infinite Fermi systems. Yet, in atomic nuclei the pairing correlations have many features related to the finite size of the system. The way how the finite size affects the pairing correlations depends on the position of the chemical potential. If the chemical potential is deeply bound, like in stable and heavy nuclei, the finite size influences the pairing correlations mainly through the shell structure induced by the spin-orbit interaction. The situation becomes more complex in nuclei close to the drip lines, where the chemical potential approaches the continuum threshold. In this case the inhomogeneity of the pairing field produces strong mixing between the bound and the continuum part of the single-particle spectrum. Due to this mixing the quasi-particle spectrum becomes dominated by resonant quasi-particle states, which originate both from single-particle resonances and deep hole states [1].

The continuum effects on pairing correlations are commonly taken into account in the Hartree-Fock-Bogoliubov (HFB) or Relativistic-Hartree-Bogoliubov (RHB) approach. In most of these calculations the continuum is replaced by a set of positive energy states determined by solving the HFB or RHB equations in coordinate space and with box boundary conditions [2, 3]. Due to this fact the genuine continuum effects, as the widths of the quasi-particle states, are not accounted for in these types of calculations.

The aim of this work is to show how the resonant continuum can be treated accurately in the RMF-BCS approach. The single-particle states belonging to the resonant part of the continuum spectrum will be calculated by solving the RMF equations with scattering-type boundary conditions. Then the resonant continuum will be handled in the BCS equations in a similar way as in the non-relativistic HF-BCS calculations [4]. This approach is applied for the case of Zr isotopes, for which earlier calculations predict a very large neutron skin close to the neutron drip line. In the following, a brief outline of the model will be given. Detailed formalism and results for Zr isotopes can be found in Ref. [5]. At large distances, where both the scalar and the vector mean fields are zero, the radial equations for the upper component G and lower component F of the radial wave function can be written in the form:

$$\frac{d^2G}{dr^2} + (\alpha^2 - \frac{\kappa(\kappa+1)}{r^2})G = 0 \tag{1}$$

$$F = \frac{1}{E+M}(\frac{dG}{dr} + \frac{\kappa}{r}G), \tag{2}$$

where $\alpha^2 = E^2 - M^2$. These equations are suited for fixing the scattering-type boundary conditions for the continuum spectrum. They are given by:

$$G = C\alpha r[cos(\delta)j_l(\alpha r) - sin(\delta)n_l(\alpha r)] \tag{3}$$

$$F = \frac{C\alpha^2 r}{E+M}[cos(\delta)j_{l-1}(\alpha r) - sin(\delta)n_{l-1}(\alpha r)], \tag{4}$$

where j_l and n_l are the Bessel and Neumann functions and δ is the phase shift. The constant C is fixed by the normalisation condition of the scattering wave functions and the phase shift δ is calculated from the matching conditions. In the vicinity of an isolated resonance the derivative of the phase shift has a Breit-Wigner form, i.e.,

$$\frac{d\delta(E)}{dE} = \frac{\Gamma/2}{(E_r - E)^2 + \Gamma^2/4} \tag{5}$$

from which one estimates the energy and the width of the resonance. In the vicinity of a resonance the radial wave functions of the scattering states have a large localisation inside the nucleus. Close to a resonance the energy dependence of both components of the Dirac wave functions can be factorized approximatively by a unique energy dependent function [6]. Using this property all the matrix elements of a two-body interaction between these scattering states can be expressed in term of a unique matrix element, i.e. the one corresponding to the scattering state with energy equal to the energy of the resonance. This property is employed in the present work for the treatment of the resonant continuum in the BCS equations.

The extension of the BCS equations for taking into account the resonant continuum was proposed in Ref. [4]. For the case of a general pairing interaction these equations, read [4]:

$$\Delta_i = \sum_j V_{i\bar{i}j\bar{j}} u_j v_j + \sum_\nu V_{i\bar{i},\nu\epsilon_\nu\overline{\nu\epsilon_\nu}} \int_{I_\nu} g_\nu(\epsilon) u_\nu(\epsilon) v_\nu(\epsilon) d\epsilon \ , \tag{6}$$

$$\Delta_\nu \equiv \sum_j V_{\nu\epsilon_\nu\overline{\nu\epsilon_\nu},j\bar{j}} u_j v_j + \sum_{\nu'} V_{\nu\epsilon_\nu\overline{\nu\epsilon_\nu},\nu'\epsilon_{\nu'}\overline{\nu'\epsilon_{\nu'}}} \int_{I_{\nu'}} g_{\nu'}(\epsilon') u_{\nu'}(\epsilon') v_{\nu'}(\epsilon') d\epsilon' \ , \tag{7}$$

$$N = \sum_i v_i^2 + \sum_\nu \int_{I_\nu} g_\nu^c(\epsilon) v_\nu^2(\epsilon) d\epsilon \ . \tag{8}$$

Here Δ_i are the gaps for the bound states and Δ_ν are the averaged gaps for the resonant states. The quantity $g_\nu^c(\epsilon) = \frac{2j_\nu+1}{\pi}\frac{d\delta_\nu}{d\epsilon}$ is the total level density and δ_ν is the phase shift of angular momentum $(l_\nu j_\nu)$. The factor $g_\nu^c(\epsilon)$ takes into account the variation of the localisation of scattering states in the energy region of a resonance (i.e., the width effect) and goes to a delta function in the limit of a very narrow width. The interaction matrix elements are calculated with the scattering wave functions at resonance energies and normalised inside the volume where the pairing interaction is active. For more details see Ref.[4, 5].

In conclusion, we discussed how the resonant states can be treated accurately in the RMF-BCS approach. This approximation scheme is applied for the neutron-rich Zr isotopes [5]. It is shown that the sudden increase of the neutron radii close to the neutron drip line depends on a few resonant states close to the continuum threshold. Including into the RMF-BCS calculations only these resonant states one gets for the neutron radii and neutron separation energies practically the same results as in the more involved RHB calculations.

References

[1] M. Grasso, N. Sandulescu, Nguyen Van Giai, and R. J. Liotta, Phys. Rev. C 64 (2001) 064321.

[2] J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. **A 422** (1984), 103; J. Dobaczewski et al, Phys. Rev. **C 53** (1996), 2809.

[3] J. Meng and P. Ring, Phys. Rev. Lett **77** (1996) 3963

[4] N. Sandulescu, Nguyen Van Giai, R. J. Liotta, Phys. Rev. **C61** (2000) 061301(R).

[5] N. Sandulescu, L.S. Geng, H. Toki, and G. Hillhouse, *arXiv: nucl-th/0306035.*

[6] A. B. Migdal, A. M. Perelemov, and V. S. Popov, Sov. J. Nucl. Phys. **14** (1971), 488.