Monte Carlo simulation of matrix models

J. Nishimura

High Energy Accelerator Research Organization (KEK),
1-1 Oho, Tsukuba 305-0801, JAPAN

It occurs in many interesting systems ranging from condensed matter physics to high-energy physics that their action has an imaginary part. Some examples for instance in high-energy physics are the finite density QCD, Chern-Simons theories, systems with topological terms (like the θ-term in QCD), and systems with chiral fermions. While this is not a conceptual problem, it poses a technical problem when one attempts to study these systems by Monte Carlo simulations, which would otherwise provide a powerful tool to understand their properties from first principles.

In Ref. [1] we have proposed a new approach to this 'complex-action problem'. Suppose we want to obtain an expectation value of some observable. Then, as a more fundamental object, we consider the distribution function associated with that observable. In general the distribution function has a factorization property, which relates it to the distribution function associated with the same observable but calculated *omitting* the imaginary part of the action. The effect of the imaginary part is represented by a correction factor which can be obtained by a constrained Monte Carlo simulation. One of the virtues of this method is that it removes the so-called overlap problem completely. This problem comes from the fact that the two distribution functions — one for the full model and the other for the model omitting the imaginary part — have little overlap in general. The method avoids this problem by 'forcing' the simulation to sample the important region for the full model.

Here we are concerned with a nonperturbative study of superstring theory using its matrix formulation [2]. Eventually we would like to examine the possibility that our 4-dimensional space time appears dynamically in 10-dimensional string theory [5, 6, 3, 7]. Monte Carlo simulation of the matrix model suffers from the complex action problem, and there are evidences that the imaginary part of the action plays a crucial role in the dynamical reduction of the space-time dimensionality [3]. We will discuss how we can study such an issue by Monte Carlo simulation using the new approach.

As a nonperturbative definition of type IIB superstring theory in 10 dimensions, Ishibashi, Kawai, Kitazawa and Tsuchiya [2] proposed a matrix model, which can be formally obtained by the zero-volume limit of $D = 10$, $\mathcal{N} = 1$, pure super Yang-Mills theory. The partition function of the type IIB matrix model can be written as

$$Z = \int \mathrm{d}A \ \mathrm{e}^{-S_{\mathrm{b}}} \ Z_{\mathrm{f}}[A] \ , \tag{0.1}$$

where A_μ $(\mu = 1, \cdots, D)$ are D bosonic $N \times N$ traceless hermitian matrices, and $S_{\mathrm{b}} = -\frac{1}{4}\mathrm{Tr}\,([A_\mu, A_\nu]^2)$ is the bosonic part of the action. The factor $Z_{\mathrm{f}}[A]$ represents the quantity obtained by integration over the fermionic matrices, and its explicit form is given for example in Refs. [3, 4].

In this model space-time is represented by A_μ, and hence treated dynamically [5]. It is Euclidean as a result of the Wick rotation, which is always necessary in path integral formalisms. Its dimensionality is dynamically determined and can be probed by the moment of inertia tensor defined by [6]

$$T_{\mu\nu} = \frac{1}{N}\mathrm{Tr}\,(A_\mu A_\nu)\ . \tag{0.2}$$

Since $T_{\mu\nu}$ is a $D \times D$ real symmetric matrix, it has D real eigenvalues corresponding to the principal moments of inertia, which we denote as λ_i $(i = 1, \cdots, D)$ with the ordering

$$\lambda_1 > \lambda_2 > \cdots > \lambda_D > 0\ . \tag{0.3}$$

Let us define the VEV $\langle \mathcal{O} \rangle$ with respect to the partition function (0.1). If we find that $\langle \lambda_i \rangle$ with $i = 1, \cdots, d$ is much larger than the others, we may conclude that the dimensionality of the dynamical space-time is d .

In order to obtain the expectation value $\langle \lambda_i \rangle$, we consider the distribution associated with the observable λ_i : $\rho_i(x) \stackrel{\mathrm{def}}{=} \langle \delta(x - \lambda_i) \rangle$. As an important property of the distribution $\rho_i(x)$, it factorizes as

$$\rho_i(x) = \frac{1}{C}\,\rho_i^{(0)}(x)\,w_i(x)\ , \tag{0.4}$$

where C is a normalization constant, and the real positive function $\rho_i^{(0)}(x)$ is the distribution omitting the phase Γ. The function $w_i(x)$ in (0.4) can be regarded as the correction factor representing the effect of Γ, and it is given explicitly as

$$w_i(x) \stackrel{\mathrm{def}}{=} \langle \mathrm{e}^{i\Gamma} \rangle_{i,x} = \langle \cos\Gamma \rangle_{i,x}\ , \tag{0.5}$$

where the symbol $\langle\ \cdot\ \rangle_{i,x}$ denotes a VEV with respect to the partition function

$$Z_{i,x} = \int \mathrm{d}A\, \mathrm{e}^{-S_0}\, \delta(x - \lambda_i)\ . \tag{0.6}$$

We have obtain the function $w_i(x)$ by Monte Carlo simulation, and discussed how it can result in the dynamical generation of our space time in the matrix model.

Our new approach to complex-action systems is based on the factorization property. (0.4), which is quite general. Simulating constrained systems (0.6) is crucial for the complete resolution of the overlap problem. Indeed this method has been applied successfully to various θ-vacuum like systems [8] and to Random Matrix Theory for finite density QCD [9]. We hope that the 'factorization method' allows us to study interesting complex-action systems in various branches of physics.

Acknowledgements : The computation has been done partly on NEC SX5 at Research Center for Nuclear Physics (RCNP) of Osaka University.

References

[1] K. N. Anagnostopoulos and J. Nishimura, Phys. Rev. D **66** (2002) 106008.

[2] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. **B 498**, 467 (1997).

[3] J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000); Phys. Rev. Lett. **85**, 4664 (2000) .

[4] J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, JHEP **0007**, 013 (2000).

[5] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, Prog. Theor. Phys. **99**, 713 (1998).

[6] T. Hotta, J. Nishimura and A. Tsuchiya, Nucl. Phys. **B 545**, 543 (1999).

[7] J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, JHEP **0007**, 011 (2000).

[8] V. Azcoiti, G. Di Carlo, A. Galante and V. Laliena, Phys. Rev. Lett. **89** (2002) 141601.

[9] J. Ambjorn, K. N. Anagnostopoulos, J. Nishimura and J. J. Verbaarschot, JHEP **0210** (2002) 062.