Chiral Phase Properties of Finite Size Quark Droplets in the Nambu–Jona-Lasinio Model

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The behavior of (finite lumps of) quark matter is of great interests in cosmology, compact stars, cosmic ray physics and high-energy heavy-ion collisions. Absolutely stable nonstrange quark matter contradicts ordinary nuclei consisting of neutrons and protons. However, the existence of absolutely stable strange quark matter is still open question and it may be realized in the form of strangelets. The stability of finite quark lumps has been studied by making use of the various QCD motivated models, e.g., the MIT bag model and the Fermi gas model. However, most of the previous studies do not take account of the phase structure of dense QCD. In this work [1], as the first attempt to tackle this long standing problem, we investigate the chiral phase properties of finite size quark droplets in the two-flavor Nambu–Jona-Lasinio model.

We start with the following Lagrangian,

$$\mathcal{L} = \bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi + G \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \right],$$

where ψ denotes a quark field with two flavors ($N_f = 2$) and three colors ($N_c = 3$), the Pauli matrices $\vec{\tau}$ act in the flavor space and the coupling constant G has a dimension $[G] = [\text{mass}]^{-2}$.

In the mean-field approximation, the thermodynamic potential ω is obtained as

$$\omega = \frac{m^2}{4G} - \nu \int \frac{k^2 dk}{2\pi^2} E_k - \nu T \int \frac{k^2 dk}{2\pi^2} \ln \left[1 + e^{-\beta(E_k + \mu)}\right] \left[1 + e^{-\beta(E_k - \mu)}\right],$$

where $\nu = 2 N_f N_c$, $E_k = \sqrt{k^2 + m^2}$, $\beta = 1/T$, with m being the dynamically generated (constituent) quark mass. Since the model is not renormalizable, we have to specify a regularization scheme. Throughout this work, we use a sharp cutoff $\Lambda = 600$ MeV in the 3D momentum space with the coupling constant $G\Lambda^2 = 2.45$.

In order to take into account finite size effects, we use the so-called multiple reflection expansion (MRE). In the MRE framework, the density of states for a spherical system is written as $k^2 \rho_{\rm MRE}/2\pi^2$, where $\rho_{\rm MRE} = \rho_{\rm MRE}(k, m, R)$ is given by

$$\rho_{\rm MRE} = 1 + \frac{6\pi^2}{kR} f_S\left(\frac{k}{m}\right) + \frac{12\pi^2}{(kR)^2} f_C\left(\frac{k}{m}\right),$$

with R being the radius of the sphere. The functions $f_S(k/m)$ and $f_C(k/m)$ represent the surface and the curvature contribution to the fermionic density of states, respectively. Their functional forms are given by

$$f_S\left(\frac{k}{m}\right) = -\frac{1}{8\pi}\left(1 - \frac{2}{\pi}\arctan\frac{k}{m}\right), \quad f_C\left(\frac{k}{m}\right) = \frac{1}{12\pi^2}\left[1 - \frac{3k}{2m}\left(\frac{\pi}{2} - \arctan\frac{k}{m}\right)\right].$$

The thermodynamic potential for spherical quark droplets is then obtained as

$$\omega = \frac{m^2}{4G} - \nu \int \frac{k^2 dk}{2\pi^2} \rho_{\rm MRE} E_k - \nu T \int \frac{k^2 dk}{2\pi^2} \rho_{\rm MRE} \ln \left[1 + e^{-\beta(E_k + \mu)}\right] \left[1 + e^{-\beta(E_k - \mu)}\right].$$

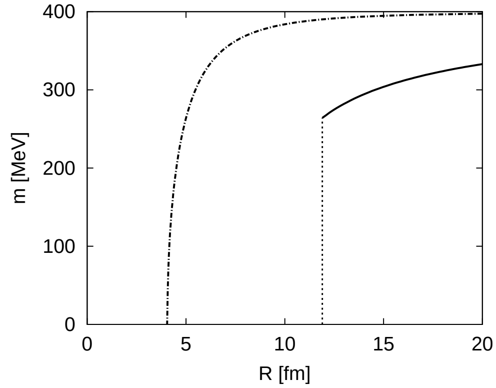

Figure 1: Radius dependence of the quark mass for $A = 100$ (solid line). For comparison, the results without the MRE is represented by a dot-dashed line.

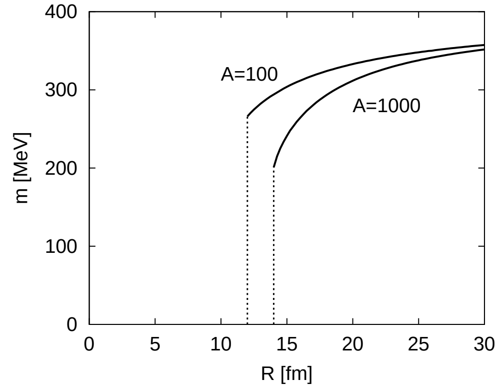

Figure 2: The radius dependence of the quark mass for the cases of $A = 100$ and $A = 1000$. For a relatively large baryon number, the first-order transition is weakened.

Henceforth, we restrict ourselves to zero temperature. Minimizing ω with respect to m, we obtain the Schwinger-Dyson equation (SDE) for the quark mass. In addition, we fix the baryon number of the droplet to A. The resulting coupled equations are given by

$$m = 2G\nu \frac{\partial}{\partial m} \int_{k_F}^{\Lambda} \frac{k^2 dk}{2\pi^2} \rho_{\rm MRE} E_k \ , \quad V \frac{\nu}{3} \int_0^{k_F} \frac{k^2 dk}{2\pi^2} \rho_{\rm MRE} = A.$$

The set of the coupled equations is solved numerically and, then, it turns out that the finite size effects enhance the restoration of chiral symmetry (see Fig. 1). Figure 2 shows the quark mass as a function of the radius for the cases of $A = 100$ and $A = 1000$. It implies that, for relatively large baryon numbers, the first order phase transition is weakened and that the finite size effects becomes less important.

Finally, we comment on the outlook for future studies. It is likely that the stable droplets is in the color superconducting (CSC) phase. Thus, it is interesting to include CSC which is expected to give large contribution to the stability of the finite quark lumps.

References

[1] See, O. Kiriyama and A. Hosaka, Phys. Rev. D **67**, 085010 (2003), and the references cited therein.