Effective string action of the dual Ginzburg-Landau theory beyond the London limit

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The dual Ginzburg-Landau (DGL) theory can sketch the dual superconductor scenario of quark confinement mechanism intuitively by the formation of a flux tube due to the dual Meissner effect. Since the flux tube corresponds to the hadronic object, the construction of the effective string action of the flux tube is one of the interesting applications of the DGL theory to the hadron physics. In this brief report, we discuss the structure of the effective string action of the $U(1)$ version of the DGL theory by using path-integral analysis [1]. In particular, we pay attention to the effect of the finite thickness of the flux tube to the form of the string action.

The $U(1)$ DGL action in the differential form has the following form:

$$S_{\rm DGL} = \frac{\beta_g}{2}(F)^2 + ((d-iB)\chi^*, (d+iB)\chi) + \lambda(|\chi|^2 - v^2)^2, \tag{1}$$

where B is the dual gauge field (1-form) and $\chi = \phi\exp(i\eta)$ (ϕ, $\eta \in \Re$) the complex-scalar monopole field (0-form). The dual gauge coupling, the strength of the self-coupling and the monopole condensate are denoted by $\beta_g = 1/g^2$, λ and v, respectively. They are related to the two mass scales of the theory: the mass of the dual gauge field $m_B = \sqrt{2}\,gv$ and that of the monopole field $m_\chi = 2\sqrt{\lambda}\,v$. The inverses of these masses correspond to the penetration depth and coherence length, respectively.

In the presence of external quark sources, the dual field strength F (2-from) is expressed as $F = dB – 2\pi * \Sigma$. In a dual model, external q-$\bar{q}$ sources are described as an *open* color-electric Dirac string Σ (2-form), whose boundary gives a quark current j (1-form). By the Hodge decomposition, the dual gauge field is decomposed into two parts as $B = B^{\rm reg} + B^{\rm sing}$, where $B^{\rm sing} = 2\pi\Delta^{-1}\delta * \Sigma$. Here, Δ^{-1} denotes a Coulomb propagator, the inverse of the Laplacian $\Delta \equiv \delta d + d\delta$. Then, the dual field strength can be rewritten as $F = dB^{\rm reg} + 2\pi d\Delta^{-1}\delta * j$. From the square of the second term of the dual field strength, one gets the Coulombic interaction between electric currents, which remains even if there is no monopole field. The rest of the effective action comes through the interaction between the monopole field and the dual gauge field.

After the functional path-integration over $B^{\rm reg}$ and η, introducing 2-form Kalb-Ramond (KR) field h, we obtain the action in terms of h and ϕ. We divide it into three parts as

$$S^{(1)} = 2\pi^2\beta_g(j, \Delta^{-1}j), \tag{2}$$

$$S^{(2)} = (d\phi)^2 + \frac{1}{4}(dh, \left\{\frac{1}{\phi^2} - \frac{1}{v^2}\right\} dh) + \lambda(\phi^2 - v^2)^2, \tag{3}$$

$$S^{(3)} = \frac{1}{4v^2}(dh)^2 + \frac{1}{2\beta_g}\{(h)^2 - (\delta h, \Delta^{-1}\delta h)\} - 2\pi i\{(h,\Sigma) - (\delta h, \Delta^{-1}j)\}. \tag{4}$$

Here $S^{(1)}$ is the pure Coulombic interaction from the dual field strength as discussed above, $S^{(2)}$ contains monopole modulus ϕ, which plays a role only in the core region of the flux tube, where $\phi \neq v$ or $d\phi \neq 0$, and $S^{(3)}$ is the rest, which remains even where $\phi = v$. From the structure of a classical flux-tube solution, we know that ϕ varies from 0 to v at the region $0 \le \rho \le m_\chi^{-1}$, where ρ denotes transverse distance from the position of the Dirac string Σ. This region, characterized by m_χ^{-1}, is nothing but the core region. Therefore, we consider m_χ as an effective cutoff and evaluate the action $S^{(2)}$ and $S^{(3)}$ above and below this cutoff. The $S^{(1)}$ does not depend on m_χ.

To treat the ultraviolet scale with respect to the cutoff m_χ (higher resolution than m_χ^{-1}), one must evaluate the action $S^{(2)} + S^{(3)}_{<m_\chi^{-1}}$, since the detailed structure of the flux tube, variation of ϕ, is visible. From the knowledge of the classical solution, we speculate that this part gives a Nambu-Goto action with a certain string tension $\sigma_{\rm core}$ and non-confining potential $S_{\rm core}(j)$.

On the other hand, for the infrared scale (lower resolution than m_χ^{-1}), one can only see the surface of the flux tube and cannot recognize inside of it. Thus $S^{(2)}$ gives no contribution and $S^{(3)}_{>m_\chi^{-1}}$ are essential to describe the system. In this case, further path-integration over the KR field h is possible, which yields

$$S^{(3)}_{>m_\chi^{-1}}(\Sigma, j) \;=\; \left[2\pi^2\beta_g(j, [D - \Delta^{-1}]j) + 4\pi^2 v^2(\Sigma, D\Sigma)\right]_{>m_\chi^{-1}}, \tag{5}$$

where $D \equiv (\Delta + m_B^2)^{-1}$ is the propagator of the massive KR field. The last term gives another Nambu-Goto action with a string tension $\sigma_{\rm surf} = \pi v^2 \ln((m_B^2 + m_\chi^2)/m_B^2)$ as a leading contribution of derivative expansion, together with corrections, for instance, a rigidity term with the negative coefficient. By adding all contributions, we arrive at the effective string action of the DGL theory as

$$S_{\rm eff}(\Sigma, j) = S(j) + (\sigma_{\rm core} + \sigma_{\rm surf})\int d^2\xi \sqrt{g(\xi)} + (\text{corrections}), \tag{6}$$

where $\xi^a (a = 1, 2)$ parametrize the string world sheet described by the coordinate $\tilde{x}_\mu(\xi)$ and $g(\xi)$ the determinant of the induced metric $g_{ab} \equiv \frac{\partial \tilde{x}_\mu(\xi)}{\partial \xi^a}\frac{\partial \tilde{x}_\mu(\xi)}{\partial \xi^b}$. The non-confining part of the effective action is given by

$$S(j) = 2\pi^2\beta_g(j, \Delta^{-1}j) + S_{\rm core}(j) + \left[2\pi^2\beta_g(j, [D - \Delta^{-1}]j)\right]_{>m_\chi^{-1}}. \tag{7}$$

We find that the Coulombic interaction from $S^{(1)}$ is partially cancelled by that from $S^{(3)}_{>m_\chi^{-1}}$. In the London limit, $m_\chi \to \infty$, the complete cancellation of the Coulombic term takes place and $S_{\rm core}(j)$ varnishes. Thus only the Yukawa term from $S^{(3)}$ remains.

To summarize, we have studied the effective string action of the $U(1)$ DGL theory. The effect of the finite thickness appears not only in the string tension but also in the shape of the non-confining potential, which becomes not pure Yukawa nor pure Coulomb potential, but something in between for a finite monopole mass m_χ. This feature is the same even in the the $U(1)\times U(1)$ DGL theory, which corresponds to an infrared effective theory of $SU(3)$ Yang-Mills theory. In fact, an extension of these path-integral analyses is straightforward by using the Weyl symmetric representation of the $U(1)\times U(1)$ DGL action [2].

References

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[2] Y. Koma, M. Koma, D. Ebert and H. Toki, Nucl. Phys. **B648**, 189 (2003), hep-th/0206074.