Generalized flux-tube solution in the $[U(1)]^{N-1}$ dual Ginzburg-Landau theory

Y. Koma

Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

It is known that the QCD vacuum can be viewed as a dual superconductor in the maximally Abelian gauge, where the dual Meissner effect is expected to occur. In this context, the dual Ginzburg-Landau (DGL) theory is constructed as the low-energy effective theory of QCD, which possesses dual (Abelian) gauge symmetry corresponding to the Cartan subgroup of the original non-Abelian gauge group. The confining quark-antiquark system is then described by the color-electric flux-tube solution in the DGL theory.

How is the original non-Abelian gauge symmetry related to the properties of the resulting flux tubes ? In order to learn this, we have studied the dual superconducting scenario applied to the arbitrary SU(N) gauge theory by formulating explicitly the $[\mathrm{U}(1)]^{N-1}$ DGL theory [1]. Based on the manifestly Weyl symmetric formulation [2], such the DGL theory consists of the $N(N-1)/2$ types of the dual gauge fields $B_{ij\,\mu}$ and the complex scalar monopole fields χ_{ij} with external color-electric Dirac strings $\Sigma^{(e)}_{k\,\mu\nu}$:

$$\mathcal{L} = \sum_{i<j}^{N} \left[-\frac{1}{2Ng^2} F^2_{ij\,\mu\nu} + |(\partial_\mu + iB_{ij\,\mu})\chi_{ij}|^2 - \lambda \left(|\chi_{ij}|^2 - v^2 \right)^2 \right],$$

$$F_{ij\,\mu\nu} = \partial_\mu B_{ij\,\nu} - \partial_\nu B_{ij\,\mu} + 2\pi \sum_{k=1}^{N} m_{ij\,k} \Sigma^{(e)}_{k\,\mu\nu},$$

where $m_{ij\,k} = \delta_{ik} - \delta_{jk}$ is an integer associated with the Dirac quantization condition for color-electric and color-magnetic charges. Although the fields have many subscripts, the structure is, say, the sum of $N(N-1)/2$ types of the U(1) DGL theory. Due to constraints among the dual gauge fields, the dual gauge symmetry is kept $[\mathrm{U}(1)]^{N-1}$. The characteristic scales are given by the masses of the monopole field $m_\chi = 2\sqrt{\lambda}v$ and of the dual gauge field $m_B = \sqrt{N}gv$. The ratio of masses, $\kappa \equiv m_\chi/m_B$, is the Ginzburg-Landau parameter which classifies the type of the vacuum: $\kappa < 1$ (type-I) and $\kappa > 1$ (type-II).

We have found that the string tensions of the flux tubes associated with static charges in various SU(N) representations at $\kappa = 1$ becomes

$$\sigma_D = 2\pi v^2 \sum_{k=1}^{N-1} k(N-k)p_k,$$

where integers $[p_1, p_2, \ldots, p_{N-1}]$ are the Dynkin indices for a D dimensional representation. For instance, the string tension in the fundamental representation σ_F is given by $p_1 = 1$, $p_i = 0$ $(i = 2, \ldots, N-1)$ as $2\pi v^2(N-1)$. At the large N limit, the ratio of string tensions, $d_D \equiv \sigma_D/\sigma_F$, tends to reproduce that of eigenvalues of the quadratic Casimir operator (Casimir scaling).

References

[1] Y. Koma, Phys. Rev. **D66**, 114006 (2002), hep-ph/0208066.

[2] Y. Koma and H. Toki, Phys. Rev. **D62**, 054027 (2000), hep-ph/0004177.