Quenched charmonium spectrum

QCD-TARO Collaboration: S. Choe[a], Ph. de Forcrand[b,c], M. García Pérez[d], S. Hioki[e],
Y. Liu[a], A. Nakamura[f], I.-O. Stamatescu[g,h], T. Takaishi[i], and T. Umeda[j]

[a] Department of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan
[b] Institut für Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
[c] Theory Division, CERN, CH-1211 Geneva 23, Switzerland
[d] IFT, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
[e] Department of Physics, Tezukayama University, Nara 631-8501, Japan
[f] IMC, Hiroshima University, Higashi-Hiroshima 739-8521, Japan
[g] Institut für Theoretische Physik, Universität Heidelberg, D-69120 Heidelberg, Germany
[h] FEST, Schmeilweg 5, D-69118 Heidelberg, Germany
[i] Hiroshima University of Economics, Hiroshima 731-0192, Japan
[j] Center for Computational Physics, University of Tsukuba, Tsukuba 305-8577, Japan

Heavy quarkonium systems have been thoroughly studied within the heavy quark non-relativistic approximation (NRQCD). This approach, however, generically fails for charmonium. An exponent of this failure is the hyperfine splitting $\Delta M \equiv M(J/\Psi - \eta_c) = 117$ MeV. NRQCD predicts $\Delta M = 55(5)$ MeV [1] far below the experimental value. The discrepancy could be blamed on the non-relativistic approximation but all other relativistic lattice determinations also underestimate ΔM by 30–50% [2, 3]. Almost all lattice calculations up to now have been performed within the quenched and OZI approximations. Quenching effects have been estimated from the form of ΔM in the heavy quark approximation, to be as large as 40% [4]. However, first numerical results with dynamical quarks indicate a much milder N_f dependence [5]. The contribution of Zweig-rule forbidden diagrams is expected to be small for charmonium. We stress, however, that they may amount to several MeV, perhaps not negligible given the smallness of ΔM. They could be also enhanced by mixing with glueballs.

The difficulty in determining ΔM by means of purely non-perturbative relativistic calculations is that charm is too heavy. Calculations are typically done at not so small values of the lattice spacing, raising some worry on their reliability. One reason of concern is the claim that continuum extrapolations of ΔM depend quite strongly on the choice of Dirac operator [1, 3, 5]. Our methodology, in order to address this problem, is to compute ΔM on very fine *isotropic* lattices (a =0.1 to 0.04 fm) within the quenched OZI relativistic formalism. Preliminary results have been given in [6], the final ones will appear in [7].

We present in Fig. 1 (Top) our continuum extrapolations of ΔM, fixing the charm point by setting the J/Ψ mass equal to the experimental value. Extrapolations are linear in a for Wilson, linear in a^2 for the tree-level and non-perturbatively improved clover Dirac operators. The splitting extracted from the latter shows very little dependence on the lattice spacing for $a \leq 0.1$fm. The dependence is much stronger for the other operators, but they still yield consistent extrapolations if $a \leq 0.07$ fm ($aM(\eta_c) \leq 1$). We consider our Fig. 1 as a spectacular advertisement for non-perturbative improvement. It is possible to extract comparable results from the other discretizations, but very fine lattices are needed in order to remove the ambiguities inherent in the extrapolation procedure. Our final result is $\Delta M = 77(2)(6)$ MeV (r_0 fixes the scale). This value is about 30% below experiment. Dynamical quark effects might be particularly large for this quantity but the remaining discrepancy might also be due to the OZI approximation. We have also computed the P-wave spectrum. Deviations from experiment amount to 22% for the ${}^3P_0$ to J/Ψ splitting, being considerably reduced for the P_1 to J/Ψ splittings which are consistent within errors with experiment.

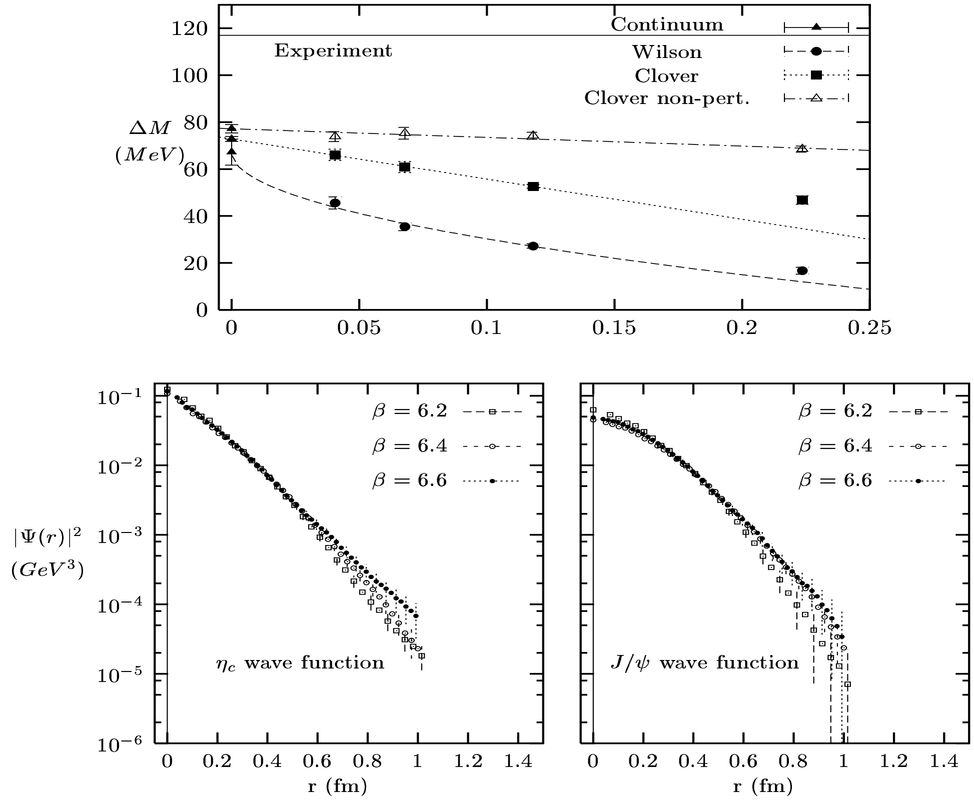

Figure 1: Top: Continuum extrapolation of ΔM from different Dirac operators. Bottom: Scaling analysis of pseudoscalar (left) and vector (right) matter wave functions.

The η_c and J/ψ wave functions, using the non-perturbatively improved clover Dirac operator, are presented in Fig. 1 (Bottom). The observed pattern corroborates qualitatively the predictions of the heavy-quark model.

Acknowledgments: We would like to acknowledge the use of the SX5 at the Research Center for Nuclear Physics at Osaka University where the simulations have been performed.

References

[1] H. D. Trottier, Phys. Rev. D**55** (1997) 6844.

[2] P. Boyle (UKQCD Collaboration), hep-lat/9903017; Nucl. Phys. B (Proc. Suppl.) **63** (1998) 314. T. R. Klassen,Nucl. Phys. B (Proc. Suppl.) **73** (1999) 918. CP-PACS Collaboration (A. Ali Khan e.a.), Nucl. Phys. B (Proc. Suppl.) **94** (2001) 325. P. Chen, Phys. Rev. D**64** (2001) 034509. P. Chen e.a.,Nucl. Phys. B (Proc. Suppl.) **94** (2001) 342.

[3] CP-PACS Collaboration (M. Okamoto e.a.), Phys. Rev. D **65** (2002) 094508.

[4] A. X. El-Khadra, Nucl. Phys. Proc. Suppl. **30** (1993) 449.

[5] C. Stewart and R. Koniuk, Phys. Rev. D**63** (2001) 054503. A. X. El-Khadra e.a., Nucl. Phys. Proc. Suppl. **83** (2000) 283. M. Di Pierro *et al.*, hep-lat/0210051.

[6] S. Choe e.a. [QCD-TARO Collaboration], Nucl. Phys. Proc. Suppl. **106** (2002) 361;

[7] S. Choe e.a. [QCD-TARO Collaboration], JHEP **0308**, 022 (2003).