Chiral Symmetry and Collective Excitations in Anisotropic Superconductors

Tadafumi Ohsaku

Research Center for Nuclear Physics, Osaka University, Osaka, Japan

Motivated by recent experimental observations of coexistence of antiferromagnetic and superconducting orders [1], Franz and Tesanovic, and independently Herbut found that the dynamical chiral symmetry breaking (DχSB) is realized in d-wave copper oxide superconductors [2]. They introduced a four-component Dirac field Ψ to describe the nodal excitation of quasiparticles in d-wave superconductors. After taking into account the coupling between the quasiparticles and fluctuating vortices of the system by a gauge interaction, the low energy effective theory becomes the (2+1)-dimensional two-flavor massless quantum electrodymanics (QED_3). It is well-known that the four-component QED_3 dymanically generates a parity-conserving Dirac mass. Then they discussed the chiral symmetry breaking in the low-energy effective theory, and they observed that the chiral condensate $\langle\bar{\Psi}\Psi\rangle$ is an alternating spin density wave (SDW). Based on the result, they argued that the d-wave superconductor has an antiferromagnetic instability, and explained the existence of the antiferromagnetic order in the phase diagram of copper oxide.

The essential part of their discussions and conclusions, especially about the phenomenon of DχSB can also be obtained by the following Lagrangian:

$$\bar{\Psi}_n i\gamma^\mu\partial_\mu\Psi_n + G_0^{(3)}[(\bar{\Psi}_n\Psi_n)^2 + (\bar{\Psi}_n i\gamma_5\Psi_n)^2]. \tag{1}$$

It is clear from the logic, both the QED_3 model and our model (1) can be applied to *all* d-wave superconductors. By introducing the local one-particle density matrix $Q(x) = -\langle\Psi(x)\bar{\Psi}(x)\rangle$, we proceed to perform the group-theoretical classification for the order parameter developed from our theory [3]. Because $Q(x)$ is a 4×4 matrix, we expand it by 16-dimensional complete set of gamma matrices: $Q = Q^S\hat{1} + Q^V_\mu\gamma^\mu + Q^T_{\mu\nu}\sigma^{\mu\nu} + Q^A_\mu\gamma_5\gamma^\mu + Q^P i\gamma_5$. SDW corresponds to the scalar density Q^S. If we examine each component of the matrix Q more in detail, we can discuss the possibility of the appearances of other types of order. Now we study the problem, and intend to publish our results elsewhere.

On the other hand, a point-like-node p-wave superconductor has two Fermi points in a specific direction in momentum space, and quasiparticles are easily excited near the Fermi points. If we describe the low-energy long-wavelength excitation by $\psi_{R\sigma}(z)e^{ik_Fz} + \psi_{L\sigma}(z)e^{-ik_Fz}$, ($R$; a right mover, L; a left mover and σ; a spin quantum number) we will obtain a (1+1)-dimensional two-flavor massless Dirac fermion model: $\sum_\sigma \bar{\psi}_\sigma i\gamma^\mu\partial_\mu\psi_\sigma$ (here, $\gamma_0 = \sigma_1$, $\gamma_1 = -i\sigma_2$, $\gamma_5 = \gamma_0\gamma_1 = \sigma_3$). We take the definition of the two-component Dirac field as $\psi_\sigma = (\psi_{R\sigma}(z), \psi_{L\sigma}(z))$. This model also has the chiral symmetry. Add a chiral invariant four-body interaction to the Dirac kinetic term, we get

$$\sum_{n=1}^{2}(\bar{\psi}_n i\gamma^\mu\partial_\mu\psi_n + G_0^{(2)}[(\bar{\psi}_n\psi_n)^2 + (\bar{\psi}_n i\gamma_5\psi_n)^2]). \tag{2}$$

In the one-dimensional case, the non-Abelian bosonization procedure [4] should be employed. After incorporating the band multiplicity in our model, our Hamiltonian will be decoupled to three sectors: $U(1)$ (charge), $SU(2)$ (spin) and $SU(N)$ (orbital or band multiplicity). Then we can write down the bosonized Hamiltonian in the Sugawara form [5]:

$$H = H_{U(1)} + H_{SU(2)} + H_{SU(N)}, \tag{3}$$

$$H_{U(1)} = 2\pi v_{charge} \int dx(: J_R J_R : + : J_L J_L : + G : J_R J_L :), \tag{4}$$

$$H_{SU(2)} = \frac{2\pi}{2+N} v_{spin} \sum_{a=1}^{3} \int dx(: J^a_R J^a_R : + : J^a_L J^a_L : - G : J^a_R J^a_L :), \tag{5}$$

$$H_{SU(N)} = \frac{2\pi}{2+N} v_{orb} \sum_{A=1}^{N^2-1} \int dx(: J^A_R J^A_R : + : J^A_L J^A_L :), \tag{6}$$

In the expression given above, the spin-charge-orbital separation was occured. By using the conformal field theoretical techniques with renormalization group approach, we can predict that the excitation in each sector becomes massless or massive [4]. Then we determine what kind of order (CDW, SDW and "orbital wave") will emerge. For example, when the spectrum of the charge sector is massless, CDW will arise, while it is massive, CDW will not appear. It is clear from our discussion, this model can be applied to *all* systems which have point-like p-wave nodes. The excitation of p-wave superconductors will be described by the chiral invariant model, and when a kind of perturbation (interaction between particles) is applied, CDW or SDW may appear/disappear. Because of the dimensionality of the nodal excitation in p-wave systems, CDW and/or SDW can emerge.

In real substances, coexistences of CDW and SDW in $(\mathrm{TMTTF})_2\mathrm{Br}$, $(\mathrm{TMTSF})_2\mathrm{PF}_6$ (p- or f-wave superconductors) and $\alpha-(\mathrm{BEDT}-\mathrm{TTF})_2\mathrm{MHg}(\mathrm{SCN})_4$ (non-pure s-wave) were observed. The phese diagram of $(\mathrm{BEDT}-\mathrm{TTF})_3\mathrm{Cl}_2(\mathrm{H}_2\mathrm{O})_2$ has a CDW phase neighbor its superconducting phase. The importance of charge fluctuation with ferromagnetic spin fluctuation in $\mathrm{Sr}_2\mathrm{RuO}_4$ (p- or f-wave) was pointed out by Takimoto [6]. Kuroki et al. performed a theoretical investigation about the effect of the coexistence of CDW and SDW in $(\mathrm{TMTSF})_2\mathrm{PF}_6$ [7]. Neighbor the superconducting phase of UGe_2 (p- or f-wave), there is a CDW-SDW coexistent phase. Almost all of these substances are p- or f-wave superconductors. We speculate that the CDW phase or CDW-SDW coexistent phase may emerge by the mechanism of DχSB in two-dimensional system, or by collective excitations of charge and spin in one-dimensinal system. We would like to make an argument that *p-wave, d-wave and f-wave superconductors generally have the SDW/CDW instability.* To the contrary, s-wave superconductors do not have such kind of instability. Usually, the phase diagrams of p-wave, d-wave and f-wave superconductors have several ordered phases, while the phase diagram of s-wave should become a simple one. The chiral symmetry arises from the nodal structure of superconducting gap, and plays the key-role in the coexistence/competition of various phases in superconductors.

References

[1] For example, Gabovich et al. Phys. Rep. **367**, 583 (2002).

[2] M. Franz and Z. Tesanovic, Phys. Rev. Lett. **87**, 257003 (2001), I. F. Herbut, *ibid.* **88**, 047006 (2002).

[3] T. Ohsaku, Thesis, Dept. Phys., Osaka Univ. (2000), Phys. Rev. **B65**, 024512 (2002), *ibid.* **B66**, 054518 (2002), cond-mat/0209352 (submitted for publication).

[4] For example, E. Witten, Commun. Math. Phys. **92**, 455 (1984), I. Affleck, Nucl. Phys. **B265**, 409, 448 (1986).

[5] H. Sugawara, Phys. Rev. **170**, 1659 (1968).

[6] T. Takimoto, Phys. Rev. **B62**, R14641 (2000).

[7] K. Kuroki et al., cond-mat/0006218.