Role of the relativistic RPA states with negative energy

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Phenomenological relativistic field theories based on hadrons, referred to as quantum hadrodynamics (QHD)[1], have been successful in describing the bulk and single-particle properties of nuclei in the mean field approximation. Nuclear excitations also have been investigated by QHD using the relativistic random-phase approximation(RRPA) with the MFT basis[2, 3]. In the previous calculations in Refs. [2, 4], where the spectral method is used to solve the RRPA equation, the configuration space is restricted to ordinary particle-hole pairs. This seems a reasonable approximation at first sight, since the excitation of the negative-energy states in the Dirac sea to the positive-energy states has an unperturbed energies more than 1 GeV. Due to the huge binding of the negative-energy states in the QHD model, however, it was found that the negative-energy states are needed in RRPA to preserve current conservation for the transition currents, to decouple the spurious translational states and to reproduce the excitation energies and transition form factors obtained by the non-spectral RRPA in which the negative-energy contribution is included automatically[5, 6].

The resulting RRPA equation with configuration including negative-energy states of MFT also has the eigenstates with the negative energy, which have never been discussed. The contribution from these states represents the blocking effect of the nucleon - antinucleon creation due to the states occupied by the spectator nucleon. Therefore, these RRPA states are quite important to construct a completeness. In this study, we discuss the property of the RRPA states with negative energy.

The multipole form factors are defined by the Fourier transform of transition current densities as

$$\langle I'||M_\lambda(q)||I\rangle = \int d\vec{x} j_\lambda(qx)\langle I'||Y_\lambda(\Omega_x)\varrho_N(\vec{x})||I\rangle, \tag{1}$$

$$\langle I'||T_{\lambda L}(q)||I\rangle = \int d\vec{x} j_L(qx)\langle I'||\vec{Y}_{\lambda L}(\Omega_x)\cdot\vec{j}_N(\vec{x})||I\rangle, \tag{2}$$

where $j_\lambda(qx)$ is a spherical Bessel function. $\vec{Y}_{\lambda L}$ is a vector spherical harmonics, and λ is the multipolarity of transition. The longitudinal and transverse form factors for the 1^- state with the transition energy of -1076 MeV is shown in Fig. 1(a). The transverse form factor is an order of magnitude larger than that of the positive-energy 1^- state shown in Fig. 1(b). It is also seen that the form factors of the negative-energy states have a peak in a very lower momentum than the corresponding transition energy, because the states are bound strongly. It would be interesting to investigate the transverse response function in order to observe the strong binding of antinucleon which is the essential feature of the phenomenological QHD model.

Next, we present the energy-weighted sum of the reduced matrix element $B(E\lambda)$ calculated by the relativistic nuclear model. The result for ^{16}O is shown in Table I, where we take the summation over only the positive-energy states in the RRPA excitation. Both in HS

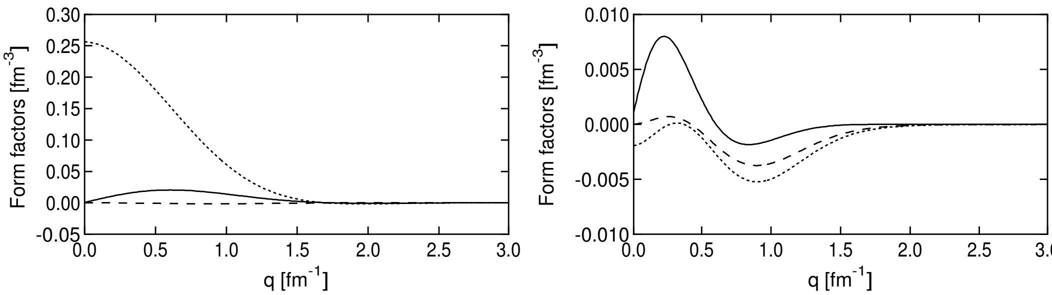

Figure 1: Nuclear form factors for the isoscalar 1^- state obtained in the RRPA calculation with HS: (a) the 1076 MeV highest negative-energy state and (b) the 8.5 MeV lowest positive-energy state in ^{16}O. The longitudinal and transverse form factors $\langle I'||M_1(q)||I\rangle$, $\langle I'||T_{10}(q)||I\rangle$, and $\langle I'||T_{12}(q)||I\rangle$ are displayed by the solid, dotted, and dashed curves, respectively.

Table 1: Energy-weighted sums of $B(E\lambda)$ over positive-energy RRPA states in unit of e^2b^λ·MeV. The classical EWSR values [7] with and without effective mass are also shown for comparison. The effective mass m_N^* is calculated by MFT with the HS parameter set.

$E\lambda$	Present(HS)	Present(NL – SH)	Classical$(m_N)^a$	Classical$(m_N^*)^a$
$E0^b$	0.0416	0.0394	0.0350	0.0460
$E1^c$	0.8052	0.7835	0.5940	0.8637
$E2$	0.5211	0.4940	0.4372	0.5745

^{a} The radial moments $\langle r^\lambda\rangle_p$ in the classical EWSR are calculated with the charge distribution from the MFT with parameter set HS.
^{b} The $E0$ operator is defined as $O(E0)=\sum_p r^2/\sqrt{4\pi}$.
^{c} The $E1$ operator is defined as $O(E1)=\sum_i -1/2\tau_3 r Y_{1\mu}$.

and NL-SH, the present results are somewhat larger than the classical energy-weighted sum rule(EWSR) values of Ref. [7] in any multipole states. The reason has been presumed as due to the effective mass m_N^* in Ref. [5]. Certainly, our EWSR agrees better with the classical EWSR value when the effective mass is used as shown in the fourth column of Table I. In addition, this role of the effective mass explains the result that EWSR values in HS become larger than those in NL-SH; HS provides smaller effective mass as compared to NL-SH in the nuclear matter calculation.

Finally, we mention that since the charge operator of the Dirac current commutes with the single-particle Hamiltonian of nucleus, the EWSR value vanishes in the present calculation if the vacuum polarized states with negative energies are also taken into account in the sum. This is the result that the RRPA states satisfy a completeness relation by including the negative-energy states.

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