# Determining the $\Theta^{+}$quantum numbers through the $K^{+} p \rightarrow \pi^{+} K^{+} n$ reaction 

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LEPS collaboration has found a clear signal for an $S=+1$ resonance around 1540 MeV at SPring-8 [1], which has been confirmed by several experiments. Here we study the $K^{+} p \rightarrow$ $\pi^{+} K^{+} n$ reaction to determine the $\Theta^{+}$quantum numbers [2,3,4]. A successful model for the reaction was considered in Ref. [5], consisting of the mechanisms depicted in terms of Feynman diagrams in upper panel of Fig. 1. The term (a) (meson pole) and (b) (contact term) are easily obtained from the chiral Lagrangians. We choose a situation where the final pion momentum small compared to the momentum of the initial kaon, such that the diagram (c) can be safely neglected. If there is a resonant state for $K^{+} n$ then this will be seen in the final state interaction of this system. Then we shall have those in lower panel of Fig. 1, in addition to the diagrams (a) and (b). For an $s$-wave $K^{+} n$ resonance we have $J^{P}=1 / 2^{-}$, and for a $p$-wave, $J^{P}=1 / 2^{+}, 3 / 2^{+}$. We write the couplings of the resonance to $K^{+} n$ as $g_{K^{+} n}$, $\bar{g}_{K+n}$ and $\tilde{g}_{K^{+} n}$ for $s$-wave and $p$-wave with $J^{P}=1 / 2^{+}, 3 / 2^{+}$respectively, and relate them to the $\Theta^{+}$width via $g_{K^{+}{ }_{n}}^{2}=\frac{\pi M_{R} \Gamma}{M q}, \bar{g}_{K^{+}{ }_{n}}^{2}=\frac{\pi M_{R} \Gamma}{M q^{3}}$ and $\tilde{g}_{K^{+}{ }_{n}}^{2}=\frac{3 \pi M_{R} \Gamma}{M q^{3}}$.

A straightforward evaluation of the meson pole and contact terms leads to the $K^{+} n \rightarrow$ $\pi^{+} K N$ amplitudes with suitable coefficients $a_{i}, b_{i}[2]$

$$
\begin{equation*}
-i t_{i}=a_{i} \vec{\sigma} \cdot \vec{k}_{i n}+b_{i} \vec{\sigma} \cdot \vec{q}^{\prime}, \tag{1}
\end{equation*}
$$

where $i=1,2$ stands for the final state $K^{+} n, K^{0} p$ respectively, and $k_{i n}$ and $q^{\prime}$ are the initial and final $K^{+}$momenta. Next we turn to the resonance diagrams. When taking into account $K N$ scattering through the $\Theta^{+}$resonance, the $K^{+} p \rightarrow \pi^{+} K^{+} n$ amplitude is given by $-i \tilde{t}=-i t_{1}-i \tilde{t}_{1}-i \tilde{t}_{2}$ where $\tilde{t}_{1}$ and $\tilde{t}_{2}$ account for the scattering terms with intermediate $K^{+} n$ and $K^{0} p$, respectively. They are given by

$$
\begin{equation*}
-i \tilde{t}_{i}^{(s)}=c_{i} \vec{\sigma} \cdot \vec{k}_{i n}, \quad-i \tilde{t}_{i}^{(p, 1 / 2)}=d_{i} \vec{\sigma} \cdot \vec{q}^{\prime}, \quad-i \tilde{t}_{i}^{(p, 3 / 2)}=f_{i} \vec{\sigma} \cdot \vec{k}_{i n}-g_{i} \vec{\sigma} \cdot \vec{q}^{\prime} \tag{2}
\end{equation*}
$$



Figure 1: Upper panel : Feynman diagrams of the reaction $K^{+} p \rightarrow \pi^{+} K^{+} n$ in the model of Ref. [5]. Lower panel : the $\Theta^{+}$resonance contribution.


Figure 2: Left : The double differential cross sections at $\theta=0$ deg (forward direction) for $I=0,1$ and $J^{P}=1 / 2^{-}, 1 / 2^{+}, 3 / 2^{+}$. Right : with polarized amplitude at $\theta=90 \mathrm{deg}$.

We take an initial three momenta of $K^{+}$in the Laboratory frame $k_{i n}(L a b)=850 \mathrm{MeV} / c$ $(\sqrt{s}=1722 \mathrm{MeV})$, which allows us to span $K^{+} n$ invariant masses up to $M_{I}=1580 \mathrm{MeV}$, thus covering the peak of the $\Theta^{+}$, and still is small enough to have negligible $\pi^{+}$momenta with respect to the one of the incoming $K^{+}$. In the left panel of Fig. 2, we show the invariant mass distribution $d^{2} \sigma / d M_{I} d \cos \theta$ in the $K^{+}$forward direction $(\theta=0)$. A resonance signal is always observed, independently of the quantum numbers of $\Theta^{+}$. The signals for the resonance are quite clear for the case of $I, J^{P}=0,1 / 2^{+}$and $I, J^{P}=0,1 / 2^{-}$, while in the other cases the signal is weaker and the background is more important.

Let us now see what one can learn with resorting to polarization measurements. Eqs. (2) tell us that if the $\Theta^{+}$has the negative parity, the amplitude goes as $\vec{\sigma} \cdot \vec{k}_{i n}$ while if it has positive parity, $\vec{\sigma} \cdot \vec{q}$. Consider the measurement of the cross section for initial proton polarization $1 / 2$ in the direction $z\left(\vec{k}_{i n}\right)$ and final neutron polarization $-1 / 2$ (the experiment can be equally done with $K^{0} p$ in the final state, which makes the nucleon detection easier). In this spin flip amplitude $\langle-1 / 2| t|+1 / 2\rangle$, the $\vec{\sigma} \cdot \vec{k}_{i n}$ term vanishes. With this test the resonance signal disappears for the $s$-wave case, while the $\vec{\sigma} \cdot \vec{q}^{\prime}$ operator of the $p$-wave case would have a finite matrix element proportional to $q^{\prime} \sin \theta$. This means, away from the forward direction of the final kaon, the appearance of a resonant peak in the cross section would indicate a $p$-wave coupling and hence a positive parity resonance. In the right panel of Fig. 2, we show the results for the polarized cross section measured at 90 degrees as a function of the invariant mass. The two cases with s-wave do not show any resonant shape since only the background contributes. All the other cross sections are quite reduced to the point that the only sizeable resonant peak comes from the $I, J^{P}=0,1 / 2^{+}$case. A clear experimental signal of the resonance in this observable would unequivocally indicate the quantum numbers as $I, J^{P}=0,1 / 2^{+}$.

## References

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