Nonperturbative calibration of anisotropic lattice actions

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Anisotropic lattices, on which the temporal lattice spacing a_τ is finer than the spatial one a_σ, have become a convenient tool for various subjects in lattice QCD simulations. Among such applications, the goal of present work is computations of heavy-light matrix elements which are important for the extraction of an effect beyond the standard model from experimental data [1, 3]. Recent experimental progress requires theoretical predictions of matrix elements to a few percent level of precision. We are developing the anisotropic lattices as a candidate of approaches which enable computations to this level [1, 2, 3]. To the level of 10% accuracy, we have verified that this approach successfully reproduces the relativity relation [3] and the heavy-light decay constant [4, 5].

However, the precision achieved in these works are still insufficient for our purpose. To control the systematic uncertainties in the final results at a few percent level, anisotropy parameters must be tuned with much high accuracy, say, 0.2%. In the present work, we are implementing calibration procedures to achieve this level of accuracy for both the gauge and quark field actions in the quenched approximation.

(a) Gauge field action. So far Klassen's result for the anisotropy parameter has been used most frequently for the quenched Wilson gauge action [6]. However, the precision of this work, 1% level, no longer meets the present requirement. We propose to calibrate the gauge anisotropy parameter through the hadronic radius r_0 which is defined as $r_0^2 F(r_0) = 1.65$, where $F(r)$ is the force between static quark and antiquark and is determined from the static quark potential [7]. r_0 is frequently used to set the lattice scale via phenomenological identification $r_0 \simeq 0.5$ fm. Measuring r_0 in the spatial (coarse) and temporal (fine) directions, the renormalized anisotropy $\xi = a_\sigma/a_\tau$ is defined as the ratio of them. Recently, a high precision computation algorithm has been developed for computations of the static quark potential by Lüscher and Weisz [8]. This method is based on a multilevel scheme that exploits the locality of the theory and can exponentially reduce the statistical errors. One can then obtain a relation between γ_G, the bare anisotropy parameter in the gauge action, and ξ with high precision.

We apply the Lüscher-Weisz algorithm to a computation of static potential on anisotropic lattices and verified that the above procedure can indeed determine ξ at 0.2% accuracy. In the range of $a_\sigma \simeq 1$–2 GeV, we determine the bare anisotropy parameter γ_G which gives $\xi = 4$ with less than 1% error. Further precise calibration in this region and extended calculation for higher values of β are in progress.

(b) Quark field action. As the quark action, we adopt the $O(a)$-improved Wilson quark action. To achieve the aforementioned accuracy, $O(a)$-improving coefficients c_E and c_B must be tuned nonperturbatively in addition to the bare anisotropy parameter γ_F. We apply the nonperturbative renormalization technique [9] which has been successfully applied to isotropic lattices. For the tuning of c_E, we verified that this method indeed achieve the required accuracy, while for c_B some improved procedure should be developed. The accuracy

of γ_F is also to be increased from the level of Ref. [2]. Although present approach using the ratio of meson masses in the coarse and fine directions already gives statistical error less than 1%, further accurate determination is under investigation.

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